

$$22. (1) \begin{cases} x' = x^2(y-b) \\ y' = y^3(x-a)^5 \end{cases}$$

INDEED (a,b) IS AN EQUILIBRIUM ^{OF (1)}. LET $u = y-b$ AND $w = x-a$. THEN $x = w+a$, $y = u+b$, $y' = u'$, AND $x' = w'$.
 DEFINE (2) $\begin{cases} u' = (u+b)^2(w)^5 \\ w' = (w+a)^3(u) \end{cases}$

INDEED $(0,0)$ IS AN EQUILIBRIUM OF (2). LET $V = uw$
 THEN

$$\begin{aligned} \dot{V} &= V_u f_1 + V_w f_2 = w(u+b)^2 w^5 + u(w+a)^3 u \\ &= w^6(u+b)^2 + u^2(w+a)^3 \end{aligned}$$

AS SHOWN $|u| < b$ AND $|w| < a$ BY REPLACING x WITH w AND y WITH u FROM PREVIOUS ARGUMENT. THEREFORE, FOR ANY FIXED $a, b > 0$ AND (x,y) IN A NEIGH OF $(0,0)$ WE HAVE $u+b > 0$ AND $w+a > 0$. THUS, \dot{V} IS $(+D)$.

WE NOTE $V = uw$ IS (I.D). THEREFORE, BY THM $(0,0)$ OF (2) IS (NS) \Rightarrow (a,b) OF (1) IS (NS).

$$23. \begin{cases} x' = y + 2y^3 \\ y' = -x - 2x^3 \end{cases}$$

OR

INDEED $(0,0)$ IS AN EQUILIBRIUM. LET $V = ax^2 + bx^4 + cy^2 + dy^4$
 THEN

$$\begin{aligned} \dot{V} &= 2ax(y + 2y^3) + 4bx^3(y + 2y^3) + 2cy(-x - 2x^3) + 4dy^3(-x - 2x^3) \\ &= xy(2a - 2c) + xy^3(4a - 4d) + x^3y(4a - 4c) + x^3y^3(8b - 8d) \end{aligned}$$

LET $a=b=c=d=1$. THEN $V = x^2 + x^4 + y^2 + y^4$ IS $(+D)$.

AND $\dot{V} = 0$ IS $(-SD)$. THEREFORE, BY THM $(0,0)$ OF (A) IS (S) \Rightarrow $(0,0)$ IS (LS).

$$24. \begin{cases} x' = y \\ y' = z \\ z' = -y \end{cases}$$

INDEED $(0,0,0)$ IS AN EQUILIBRIUM. LET $V = (x+z)^2 + y^2 + z^2$
 THEN

$$\dot{V} = 2(x+z)^2(y-y) + 2yz - 2yz = 0$$

SO, V IS $(+D)$ AND \dot{V} IS $(-SD)$. THEREFORE, $(0,0,0)$ IS (S) \Rightarrow $(0,0,0)$ IS (LS).