21 $\begin{cases} x' = y - x^3 y^2 \\ y' = -x^3 - y^3 x^2 \end{cases}$

a)

INDEED $(0,0)$ IS AN EQUILIBRIUM. DEFINE

$$A = \begin{bmatrix} -3x^2y^2 & 1 - x^3(2y) \\ -3x^2 - 2y^3x & -3y^2x^2 \end{bmatrix}\Bigg|_{(0,0)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

THEN $\lambda_1(A) = 0$ AND $\lambda_2(A) = 0$. SINCE 0 IS NOT IN THE JORDAN DIAGONALIZED BLOCK $(0,0)$ IS (NS).

b) INDEED $(0,0)$ IS AN EQUILIBRIUM. LET $V = ax^4 + by^2$ THEN

$$\dot{V} = 4ax^3(y - x^3y^2) + 2by(-x^3 - y^3x^2)$$
$$= 4ax^3y - 4ax^6y^2 - 2byx^3 - 2by^4x^2$$
$$= -4ax^6y^2 - 2by^4x^2 + x^3y(4a - 2b)$$

LET $a = 1$, $b = 2$. THEN $V = x^4 + 2y^2$ IS (+D) AND

$-4x^2y^2(x^4+y^2) = \dot{V} = -(4x^6y^2 + 4y^4x^2)$ IS (-SD). THEREFORE, BY THM $(0,0)$ IS (AS). $\therefore$ (A) $(0,0)$ IS (NS) (B) (AS).

22 a) $\begin{cases} x' = x^3(y - b) \\ y' = y^3(x - a)^5 \end{cases}$ $a, b > 0$

INDEED $(0,0)$ IS AN EQUILIBRIUM. LET $V = x^2 + y^2$ THEN

$$\dot{V} = 2x(x^3(y-b)) + 2y(y^3(x-a)^5)$$
$$= 2x^4(y-b) + 2y^4(x-a)^5$$

WE KNOW $2x^4 \geq 0$ $\forall x$ AND $2y^4 \geq 0$ $\forall y$.

WTS $(y-b) < 0$ AND $(x-a)^5 < 0$ $\forall x, y$ SMALL AND $a, b > 0$.

FIX ANY $a, b > 0$ AND LET $(x,y) \in \mathbb{R}^2$. THEN $\exists r > 0$ S.t. $(a,b) \notin B((0,0), r)$ WHERE $B((0,0), r)$ IS THE OPEN BALL IN $\mathbb{R}$ CENTERED @ $(0,0)$ W/ RADIUS r. THEN ~~THEREFORE~~, FOR SUFFICIENTLY SMALL (x,y) WE HAVE $|x| < a$ AND $|y| < b$.

THUS, $x - a < 0$ AND $y - b < 0$. THEREFORE, $V = x^2 + y^2$ IS (+D) AND $\dot{V}$ IS (-D). BY THM $(0,0)$ IS (U) AND (AS).

FACT $B((0,0), r) \neq \emptyset$.