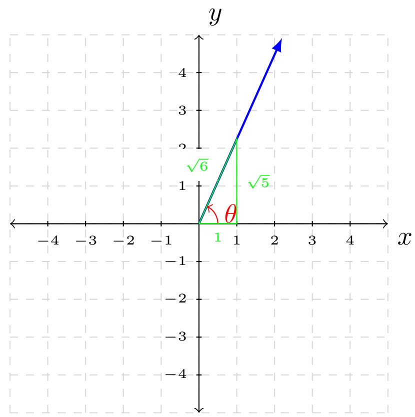
Example 1 (Section 1.3)**.** Sketch the least positive angle θ and find the values of th esix trigonometric functions of θ if the terminal side of an angle θ in standard position is defined by

$$-\sqrt{5}x + y = 0$$

where $x \geq 0$.

The graph of $-\sqrt{5}x + y = 0$ is...



The legs of the triangle is $x = 1$ and $y = \sqrt{5}$. The hypotenuse of the triangle is

$$r = \sqrt{1^2 + \left(\sqrt{5}\right)^2} = \sqrt{6}.$$

$$\cos(\theta) = \frac{1}{\sqrt{6}} \qquad \sin(\theta) = \frac{\sqrt{5}}{\sqrt{6}}$$

$$\tan(\theta) = \frac{\sqrt{5}}{1} \qquad \cot(\theta) = \frac{1}{\sqrt{5}}$$

$$\sec(\theta) = \frac{\sqrt{6}}{1} \qquad \csc(\theta) = \frac{\sqrt{6}}{\sqrt{5}}$$

Example 2 (Section 1.4)**.** Find the exact value of each of the remaining trigonometric functions of θ.

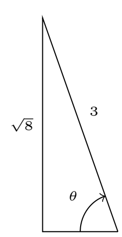
$$\sec(\theta) = -3$$

where $\sin(\theta) > 0$.

Since $\sec(\theta) = -3$ and $\sin(\theta) > 0$ we know that $x = -1$ and $r = 3$. To find y we solve the equation

$$(-1)^2 + y^2 = 3^2$$

and we will get $y = \sqrt{8}$ or $2\sqrt{2}$.



With this triangle we have:

$$\cos(\theta) = \frac{-1}{3} \qquad \sin(\theta) = \frac{\sqrt{8}}{3}$$

$$\tan(\theta) = \frac{\sqrt{8}}{-1} \qquad \cot(\theta) = \frac{-1}{\sqrt{8}}$$

$$\sec(\theta) = \frac{3}{-1} \qquad \csc(\theta) = \frac{3}{\sqrt{8}}$$

Example 3 (Section 2.1)**.** Write the following function in terms of its cofunction. Assume that all angles in which an unknown appears are acute angels.

$$\sec(\beta + 15^\circ)$$

We know that $\cos(\theta) = \sin(90 - \theta)$. Further, we know $\cos(A) = \sin(B)$ when $A + B = 90$. Here we have an $A = \beta + 15$ need to find B.

$$\begin{aligned} A + B &= 90 \\ (\beta + 15) + B &= 90 \\ B &= 90 - \beta - 15 \\ B &= 75 - \beta \end{aligned}$$

$$\begin{aligned} \sec(\beta + 15) &= \frac{1}{\cos(\beta + 15)} \\ &= \frac{1}{\sin(75 - \beta)} \\ &= \csc(75 - \beta) \end{aligned}$$

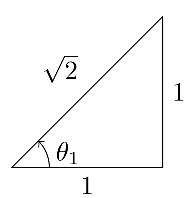
Example 4 (Section 2.2)**.** Find reference angle:

- The reference angle for 92° is $180 - 92 = 88$.
- The reference angle for 218° is $218 - 180 = 38$.
- The coterminal angle for -150° is $-150 + 360 = 210$ and the reference angle for -150° is $210 - 180 = 30$.
- The coterminal angle for -45° is $-45 + 360 = 315$ and the reference angle for -45° is $360 - 315 = 45$.

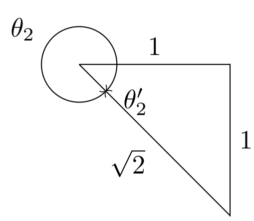
Example 5 (Section 2.2)**.** Find all values of θ, if $\theta \in [0, 360)$ and has the given function values.

$$\sec(\theta) = \sqrt{2}$$

Here we have the following triangle:



Since secant is associated with cosine we also have the following triangle:



Notice that $\theta_1 = \theta_2' = 45^\circ$. A reference angle of 45° in quadrant four is:

$$360 - \theta_2 = \theta_2' \rightarrow \theta_2 = 315$$

Therefore, the solution is $\theta = \{45^\circ, 315^\circ\}$.