Name: _____   Partner(s): _____

Investigation #10   _____

EMISSION SPECTRUM OF ELEMENTAL GASES

After the discovery of the electron in 1898, John Joseph Thompson put forth a model of the atom. Known as the "plum pudding" model, this model essentially described the atom as a positive charged fluid distributed uniformly throughout its volume. The negatively charged electrons were tiny clumps that were dabbled throughout this fluid. In 1910, Ernest Rutherford and his students (Hans Geiger and Ernest Marsden) performed experiments on how recently discovered "α-particles" interacted with matter. In their famous "gold-foil" experiment, these "α-particles" (now known to be Helium-4 nuclei) were aimed at a very thin gold foil. The startling results of the scattering experiments indicated that the structure of an atom was very different from that proposed by Thompson. The results of Rutherford's gold foil experiment indicated that the almost all of the mass of the atom was concentrated into a very tiny positively charged volume he called the *nucleus*. The electrons surrounded the nucleus in some unknown fashion and accounted for the volume of the atom.

The problem with this new "nuclear" model was that the Coulomb force would tend to rip the nucleus apart while simultaneously attracting the electrons inward. Since there are no stable electrostatic configurations, the charges must be moving and accelerating. The well-established laws of electromagnetism predict that an accelerating charged particle (like an electron moving around a nucleus) should emit electromagnetic radiation in a broad range of wavelengths. The orbiting electrons should therefore lose energy and very quickly spiral into the nucleus. In other words, why are atoms stable?

Another issue was the peculiar complexity and regularity of the spectrum of the simplest atom, hydrogen. Light that was emitted (or absorbed) always occurred at discrete wavelengths. How could a nucleus consisting of a single proton and one electron produce the observed spectral lines? Why only discrete wavelengths? Why these particular wavelengths?

In 1885, Johann Balmer discovered (by trial and error) an empirical formula that predicted the observed wavelengths for the spectral lines of hydrogen in the visible part of the electromagnetic spectrum. A few years later Johannes Rydberg generalized Balmer's formula to predict all the transitions that occur in hydrogen. Rydberg's expression is

$$\frac{1}{\lambda} = R_H \left| \frac{1}{n^2} - \frac{1}{m^2} \right| ,$$

where λ is the wavelength, n and m are unequal positive integers, and R_H is the *Rydberg constant*. In this investigation, you will determine the Rydberg constant using an optical grating spectrometer and a light source of excited hydrogen gas. Along with this main objective, we will review the emission spectra of helium and mercury.

Background
The Bohr Model of the Atom

To explain the observed spectrum of the simplest atom, hydrogen, Niels Bohr challenged the assumption that orbiting electrons must radiate. By following Planck's notion of discrete energies, and Einstein's concept of photons, Bohr proposed a new atomic model. Without justification, Bohr made the following assumptions:

- The electron moves in a circular orbit around the nucleus in circle in the classical sense. The centripetal force is the Coulomb force rather than gravity.
- Completely contrary to the laws of electromagnetism, the electron travels only in certain allowed orbits and does not radiate.
- The angular momentum of the electron was quantized in discrete units of Planck's constant divided by 2π. That is, $L = mvr = n\hbar$, where $\hbar = h/2\pi$ and $n = 1, 2, 3, \ldots$
- The electron can "jump" from one allowed orbit to another by either the absorption or emission of a photon whose energy is equal to the difference between the two orbits. That is, $E_{ph} = hf = |E_{final} - E_{initial}|$.

By equating the Coulomb force to the centripetal force

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

and then applying the quantum restriction for the angular momentum, it can be shown (as you will do in the pre-lab) that the "allowed" orbital radii are given by

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}, \ (n = 1, 2, 3, \ldots)$$

Question: Plug in the accepted numerical values for the physical constants and calculate the lowest ($n = 1$) orbital radius for the hydrogen atom. Show your work. (The lowest orbital state is called the *ground state* of the hydrogen atom.)

To determine the total energies, E_n, for these allowed orbital states, simply sum the (nonrelativistic) kinetic energy and the electric potential energy: $E = K + U$. That is,

$$E_n = \frac{1}{2}mv_n^2 - \frac{e^2}{4\pi\varepsilon_0 r_n}$$

By using the result above for the allowed orbital radii and applying energy conservation it can be shown (pre-lab again!) that the total energies of the allowed orbits is given by

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}, \ (n = 1, 2, 3, \ldots).$$

Question: Plug in the accepted numerical values for the physical constants and calculate the ground state energy for the hydrogen atom. Show your work and express your answer in both Joules and electron-Volts.

The Bohr model was a strange mixture of classical physics and quantum concepts. Still, the model accounted for the discrete spectral lines. The model also accurately predicted the observed wavelengths for the spectral lines in hydrogen as well as "hydrogen-like" atoms and ions. The periodicity of chemical properties of elements in the Periodic Table were also uncovered by the Bohr model of the atom. For his work on atomic structure, Bohr was awarded the Nobel Prize for Physics in 1922.

Part I
Calibration of the Diffraction Grating

As stated earlier, you will use an optical grating spectrometer to determine the wavelengths of the emission lines. From your data, you will deduce the integers n and m in the Rydberg formula and determine the value of the Rydberg constant.

A simple drawing of an optical spectrometer is shown in Figure 1. The atomic gases will be placed in an excited state by use of an electrical potential in the light source shown on the left. When the excited electrons return to their ground state, they will emit light at specific frequencies predicted by quantum mechanics. The multi-frequency light is focused and passed through a transmission diffraction grating. This grating has been inscribed with a few thousand small slits per centimeter. When light passes through each of these multiple slits, it will diffract, causing the grating to act light a source of thousands of points of light, all in-phase. Constructive interference of all these "light sources" will cause maxima to appear at different angles on the right side of the grating, depending on the wavelength of the light. The positions of maximum constructive interference will obey the following formula:

$$m\lambda = d\sin\theta, \ (m = \pm1, \pm2, \pm3, \ldots)$$

where λ is the wavelength, d is the spacing between consecutive slits, θ is the angle the outgoing light makes with the incoming light, and m is the "order" of the constructive interference. The positions of maximum constructive interference will appear as bright lines of different colors at different angles in the spectrometer. By searching for these bright lines with the rotating eyepiece and noting the angle at which they are formed, we can determine the wavelengths that make up the visible portion of the spectrum of the excited gas.
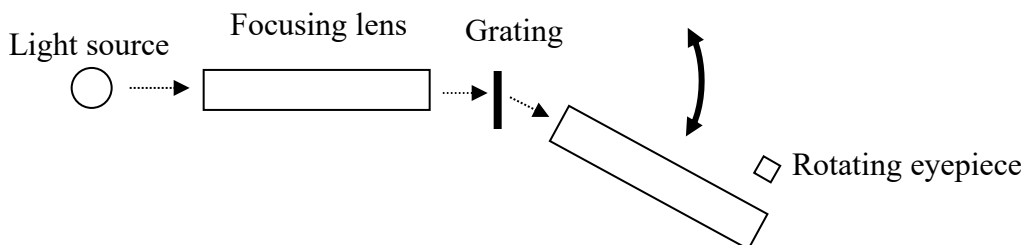
Fig.1: Schematic layout of the optical spectrometer

Your group will need the following materials/equipment for this part:
- 1 optical spectrometer with diffraction grating
- 3 discharge tubes with elemental gases (hydrogen, helium, mercury)

Procedure

1. Place the mercury discharge tube as close as possible with the narrow portion of the gas tube by the slit. Adjust the focusing of your eyepiece on the telescope for clean viewing of both the light and cross hairs as viewed through the scope. If the instrument is calibrated properly the alignment of the scope will register 180° on the angular scale, if it aligns above or below the 180° mark you will have to add or subtract this correction from your angular reading. The 180° mark is your "zero reference line" from which you will be recording angles of respective spectral lines.
2. Moving the scope (pivot) to the left slowly you should begin to see colors of bright lines appearing in your field. Slowly move the scope and its pivoting support so that the cross hairs split the first observed spectral line. (Any line that is observed within 1º of 180º is <u>not</u> an indication of constructive interference. You are merely looking directly into the lamp.)
3. Find the green emission line from mercury gas on the left side and determine the angle for its first position. Record the magnitude of the angle displacement from the zero line (180º mark) to the nearest *arc minute* (1/60th of a degree) in Table 1.
4. Repeat Step 3 by moving scope to the right of the zero line. In an ideal situation the same colors and angle displacement should be found on the right side, however, due to possible alignment error this may not be true. Not knowing if the left or right side is more accurate, the angle that you will use in the constructive interference equation will be the average of left and right side measurements. The known wavelength of

the green spectral line of mercury is 5461 *angstroms* (abbreviated Å). (Note: 1 Å = 10^{-10} m = 0.1 nm.)

Table 1: Reference Line of Mercury Spectrum

Line	Left Angle (º)	Right Angle (º)	Average Angle (º)	Wavelength (nm)
"Green"				546.1

5. Using the average angle from the zero line and the equation for constructive interference, determine the slit width (*d*-spacing) of the diffraction grating. Show your work.

d-spacing of diffraction grating = _____ m

Checkpoint: Consult with you instructor before proceeding. **Instructor's OK:** _____

Part II
Observation of Emission Spectra

6. Continue with mercury and find the position of all other spectral lines on both the left side and right side. Record the colors and angles Table 2 on the next page. Calculate the wavelengths of the spectral lines.

Note: There may be more rows in the data tables than spectral lines actually seen.

7. Repeat Step 6 for helium gas and hydrogen gas.

Question: For each element, list what colors you observed and the order in which you observed according to increasing angle from "straight-on."

Mercury:

Helium:

Hydrogen:

Checkpoint: Consult with you instructor before proceeding. **Instructor's OK:** _____

Table 2: Mercury Spectrum

Line	Left Angle (º)	Right Angle (º)	Average Angle (º)	Wavelength (nm)
1				
2				
3				
4				
5				
6				

Table 3: Helium Spectrum

Line	Left Angle (º)	Right Angle (º)	Average Angle (º)	Wavelength (nm)
1				
2				
3				
4				
5				
6				

Table 4: Hydrogen Spectrum

Line	Left Angle (º)	Right Angle (º)	Average Angle (º)	Wavelength (nm)
1				
2				
3				
4				
5				

Part III
Determining the Rydberg Constant

The Rydberg constant, R, is used in the formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The Rydberg formula allows you to calculate the wavelength of light that is emitted (or absorbed) when an electron makes a transition from one energy level to another. These wavelengths are categorized into series:

Series	n_f	n_i
Lyman	1	2, 3, 4…
Balmer	2	3, 4, 5…
Paschen	3	4, 5, 6…
Bracket	4	5, 6, 7…

Based on the above formula, determine the average value of the Rydberg constant from all of your hydrogen data. (Note: Determination of n_f and n_i may involve trial and error.)

Hydrogen:

Line	Wavelength (nm)	$n_{initial}$	n_{final}	R_H (m⁻¹)
1				
2				
3				
4				
			Average:	
			Accepted Value:	1.097776×10^7

% Error = |(Average – Accepted)/Accepted|*100% = _____ %

Question: What is the final value of n for all transitions that you observed in the hydrogen spectrum? How did you arrive at this value? Based on the value of n_f, what line series did you observe?

Question: For the greatest frequency you observed, what is the initial value of n? Show your calculation.

Checkout: Consult with you instructor before exiting the lab. **Instructor's OK:** _____