EM Waves

Combine Faraday's Law: $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$

and Ampere / Maxwell Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{en} + \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$

In empty space (vacuum) $Q = 0$ and $I = 0$.

So, these become: $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_E}{dt}$ and $\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$

Note: To avoid the mathematical complexity which is beyond the scope of this class, we shall assume that the $\vec{E}$, $\vec{B}$ vectors have specific spatial & temporal behavior that is consistent with Maxwell's Equations.

For EM waves in vacuum, it turns out that $\vec{E} \perp \vec{B}$ and $\vec{E}$ & $\vec{B}$ $\perp$ direction of propagation of the wave.

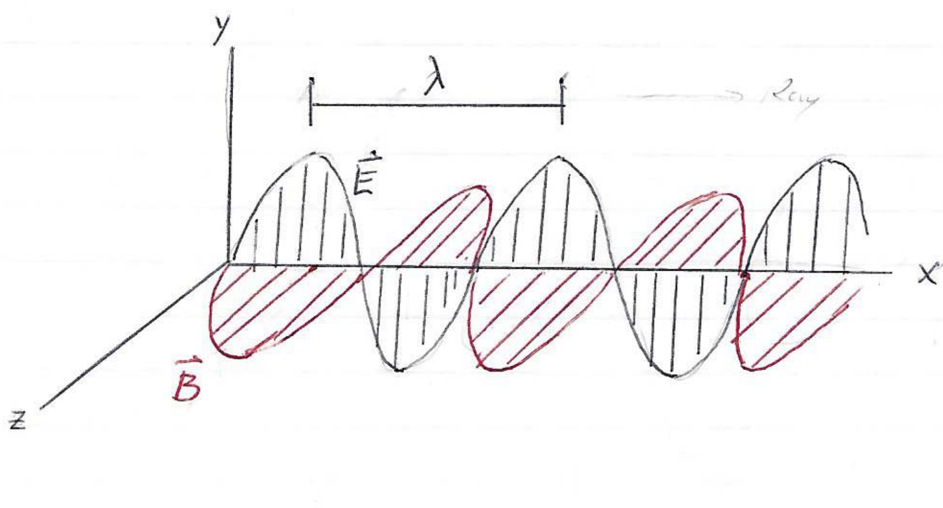
Key features:

① Wave travels in x-direction
$\vec{E}$ oscillates in y-direction
$\vec{B}$ oscillates in z-direction
} Transverse waves since ...

② $\vec{E} \perp \vec{B}$

③ $\vec{E} \times \vec{B}$ gives direction of propagation

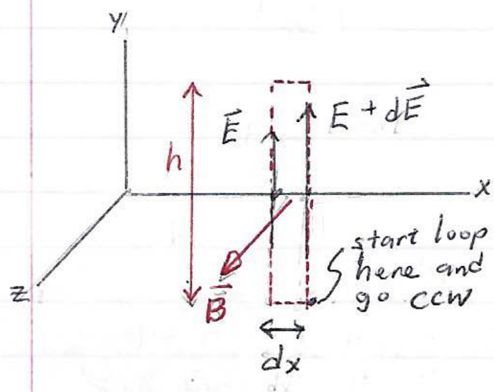
④ $\vec{E}$ & $\vec{B}$-field vary sinusoidally & are in phase with each other.



$E_{(y)} = E_{max} \sin(kx - \omega t)$ and $B_{(z)} = B_{max} \sin(kx - \omega t)$

Recall $k \equiv$ angular wave # and $\omega \equiv$ angular frequency

Speed of wave propagation $= \dfrac{\omega}{k}$



$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

If we evaluate LHS ccw around the rectangle shown:

$$= \oint \vec{E} \cdot d\vec{s} = (E + dE)h + 0 - Eh + 0 = h\,dE$$

$$\text{RHS} = -\frac{d}{dt}(B\,h\,dx) = -h\,dx\,\frac{dB}{dt}$$

$$\therefore \cancel{h}\,dE = -\cancel{h}\,dx\,\frac{dB}{dt} \longrightarrow \frac{dE}{dx} = -\frac{dB}{dt}$$

Evaluated at an instant in time (t = constant)

Evaluated at a particular location (x = constant)

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

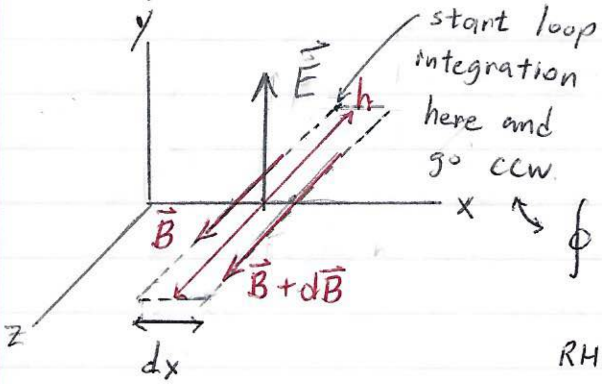
$$\frac{\partial E}{\partial x} = kE_{max}\cancel{\cos(kx - \omega t)}$$

$$-\frac{\partial B}{\partial t} = +\omega B_{max}\cancel{\cos(kx - \omega t)}$$

$$\left.\right\} kE_{max} = \omega B_{max}$$

$$\therefore \frac{E_{max}}{B_{max}} = \frac{\omega}{k} = \text{speed of EM wave} = c$$

Similarly ,



$$\oint B \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Evaluate LHS CCW around the rectangle shown:

$$\oint \vec{B} \cdot d\vec{s} = Bh + 0 - (B + dB)h + 0 = -h\,dB$$

$$RHS = \frac{d}{dt}(E\,h\,dx) = h\,dx\,\frac{dE}{dt}$$

$$\therefore \; -\cancel{h}\,dB = \mu_0 \varepsilon_0 \left(\cancel{h}\,dx\,\frac{dE}{dt}\right) \longrightarrow -\frac{dB}{dx} = \mu_0 \varepsilon_0 \frac{dE}{dt}$$

Similarly as above : $\boxed{-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}}$

$$\left.\begin{aligned} -\frac{\partial B}{\partial x} &= -k B_{max} \cancel{\cos(kx - \omega t)} \\ \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} &= \mu_0 \varepsilon_0 (-\omega) E_{max} \cancel{\cos(kx - \omega t)} \end{aligned}\right\} +k B_{max} = +\mu_0 \varepsilon_0 \omega E_{max}$$

$$\therefore \; \frac{E_{max}}{B_{max}} = \underbrace{\cancel{\left(\frac{k}{\omega}\right)}}_{\frac{1}{c}}\left(\frac{1}{\mu_0 \varepsilon_0}\right) = \text{Recall from above} = \text{speed of EM wave}$$

$$c = \frac{\omega}{k} = \frac{k}{\omega} \cdot \frac{1}{\mu_0 \varepsilon_0} = \frac{1}{\mu_0 \varepsilon_0 c}$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0} \longrightarrow \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

Energy Transport

EM waves carry energy. Flow rate is described by **Poynting vector**, $\vec{S}$

$$\boxed{\vec{S} \equiv \frac{1}{\mu_0}(\vec{E} \times \vec{B})}$$

$|\vec{S}|$ describes the rate at which energy flows through a unit surface area perpendicular to the flow.

Units: $\frac{J/s}{m^2} = \frac{Watts}{m^2}$

$\vec{S}$ points along the direction of propagation.

Since $\vec{E} \perp \vec{B}$, $|\vec{S}| = S = \frac{EB}{\mu_0}$ and since $B = \frac{E}{c}$, $\boxed{S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}}$

Applies at any instant

However, we a typically more interested in the average of S taken over one or more cycles : wave intensity, I

$$\boxed{I = S_{ave} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c B_{max}^2}{2\mu_0}}$$

Recall $E^2 \propto \cos^2(kx - \omega t)$

$E_{ave}^2 = \frac{1}{2} E_{max}$

Recall $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0 c^2 B^2 = \frac{1}{2}\cancel{\varepsilon_0}\left(\frac{1}{\mu_0 \cancel{\varepsilon_0}}\right)B^2 = \frac{B^2}{2\mu_0} = u_B$

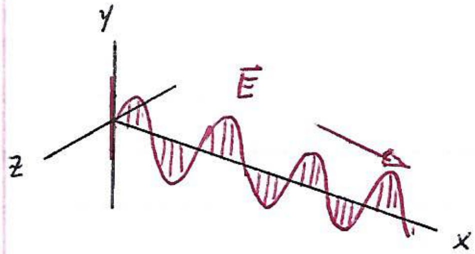
$\boxed{u_E = u_B}$ So total $u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{\mu_0 c}$

Point source : $I = \frac{Power}{Area} = \frac{P}{4\pi r^2}$

Polarization

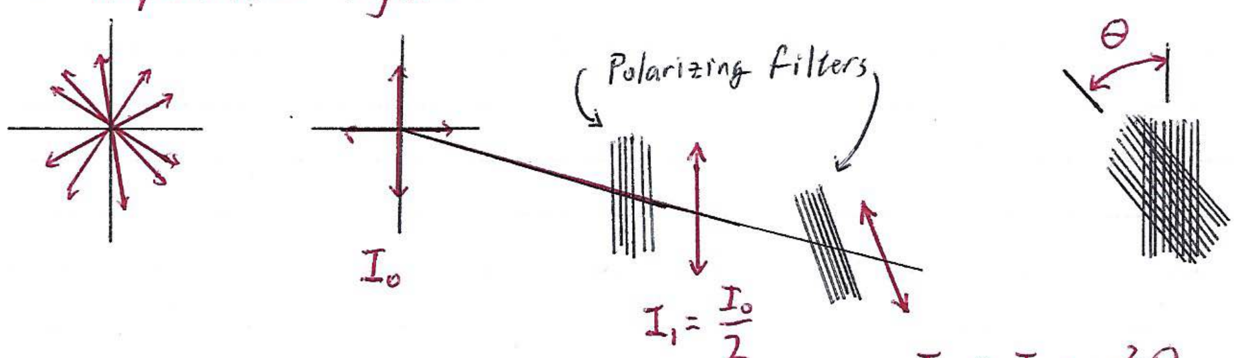
EM Waves are <u>transverse</u> waves (because the $\vec{E}$ and $\vec{B}$-fields are oscillating $\perp$ to the direction of propagation.

By convention, the direction of the electric vibration is generally refered to as the direction of polarization.



In example shown,
the direction of <u>propagation</u> is $+x$.
and
the direction of <u>polarization</u> is y.

Ordinary visible light sources the electric vibrations are emitted randomly in all possible directions. Such sources emit unpolarized light:



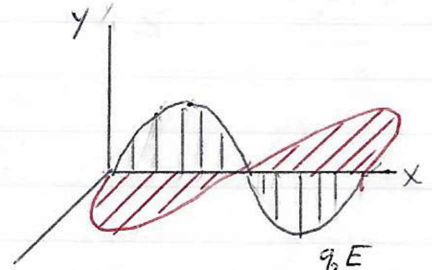
$I_2 = I_1 \cos^2\theta$

Malus' law

(As long as no two <u>consecutive</u> polarizers are oriented 90° apart, light will penetrate the whole arrangement. This demonstrates the transverse wave nature of light.)

Radiation Pressure

If an object is free to move and absorbs all radiation incident on it, then the object gains ΔK energy in time Δt and the object gains a linear momentum $\Delta p = \frac{\Delta K}{c}$



$F_E = qE = ma = F_y$

$F_B = q v_y B = F_x$

For a (−) charge, F_y is ↓ but F_x is still →

$$V_y = a_y t = \frac{qE}{m} t$$

$$(\Delta)K = \frac{1}{2} m v_{(y)}^2 = \frac{1}{2} \cancel{m} \left(\frac{q^2 E^2 t^2}{m^{\cancel{2}}} \right)$$

$$F_x = q V_{(y)} B = q \left(\frac{qEt}{m} \right) B$$

Impulse
↳ Δ(Momentum): $\Delta p_x = \int_0^t F_x \, dt = \frac{q^2 EB}{m} \int_0^t t \, dt = \frac{q^2 E B}{m} \cdot \frac{t^2}{2}$

$B = \frac{E}{c}$

$$= \frac{1}{c} \cdot \left(\frac{1}{2} \frac{q^2 E^2 t^2}{m} \right) = \frac{\Delta K}{c}$$

If an object is free to move and reflects all radiation incident on it... $\Delta p = \frac{2\Delta K}{c}$

Newton's 2nd law : $F = \frac{\Delta p}{\Delta t}$ Recall $I = \frac{\text{Power}}{\text{Area}} = \frac{\Delta K / \Delta t}{\text{Area}}$

$$\Delta K = I A \Delta t$$

$$\therefore F = \frac{(2)\Delta K}{c \Delta t} = \frac{(2) I A \cancel{\Delta t}}{c \cancel{\Delta t}}$$

"2" is there if radiation is all reflected and not there if radiation is all absorbed.

$$\text{Pressure} \equiv \frac{F}{A} = \boxed{\frac{I}{c}} \; ; \; \boxed{\frac{2I}{c}}$$

All Absorbed — All reflected (180° back)

Example 1 "Stranded Astronaut"

A stranded astronaut in their disabled ship decides to use a laser to propel the ship.

If the mass of the ship + astronaut is 1500 kg and the laser has a power rating of 10 kWatt, how fast will the craft be moving after 1 day of continuous use?

$P = 10^4\ \text{Watt} \qquad F = \frac{IA}{c} = \frac{P}{c} = \frac{10^4\ \text{Watt}}{3\times10^8\ \text{m/s}} = 3.3\times10^{-5}\ N$

$F = ma = m\frac{\Delta v}{\Delta t} \qquad (a = \text{constant})$

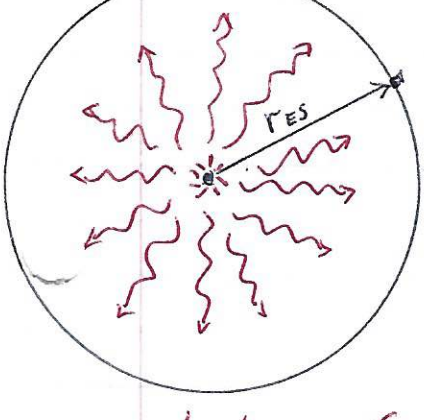
So, $v = \frac{F\Delta t}{m} = \frac{(3.3\times10^{-5}\ N)(1\ d)}{1500\ kg}\left(\frac{24\ hr}{1\ d}\right)\left(\frac{3600 s}{1\ hr}\right) = 0.0019\ \frac{m}{s}$

(1.9 mm/s)

(After 1 year: $\Delta t = 3.15\times10^7 s \rightarrow v = 0.7\ \frac{m}{s}$)

Example 2 "Solar Sailing"

To hold a ship a distance from the Sun equal to the radius of Earth's orbit, how large (what area, A) sail is needed if the mass of the ship is 1500 kg $= m$



$\vec{F}_{grav} + \vec{F}_{rad} = 0 \qquad F_{grav} = F_{rad}$

assume 100% reflection

$\frac{G m M_{sun}}{r_{ES}} = \frac{2IA}{c}$ where $I = \frac{P_{sun}}{4\pi r_{ES}^2}$

Solve for $A = \frac{4\pi G m M_{sun} c}{2 P_{sun}} = \cdots = 9.8\times10^5\ m^2$

$(\approx 10^6 m^2 = 1\ km$

Look up values: G, M_{sun}, c, P_{sun}, r_{ES}