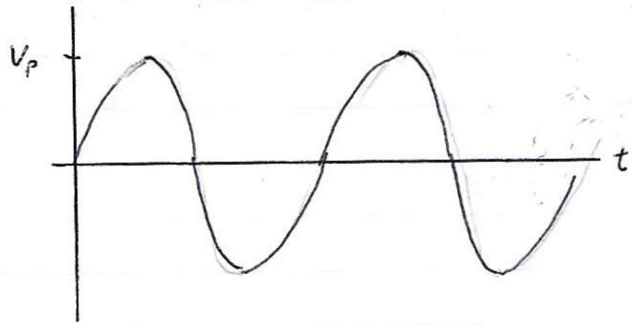
AC Circuits



We'll see why this is a useful quantity later

$$\underbrace{V_{rms}}_{\text{root mean square}} = \frac{V_p}{\sqrt{2}}$$

$$= \left[\text{average of } V_p^2(t)\right]^{1/2} = \sqrt{\overline{V_p^2}}$$

$$= \left[\frac{1}{T}\int_0^T V_p^2 \sin^2 \omega t \, dt\right.$$

ω = ang. freq. $= 2\pi f$

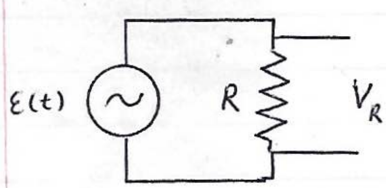
In general, $V(t) = V_p \sin(\omega t + \phi_V)$

other texts use $\mathcal{E}_p$, $\mathcal{E}_{max}$, V_{max} for V_p.

Similarly, $I(t) = I_p \sin(\omega t \pm \phi_I)$

traditionally written with '−' sign

Resistive Load



(Let $\phi_V = 0$)

$\mathcal{E}(t) = \mathcal{E}_{max} \sin \omega_d t$

$I = I_{max} \sin(\omega_d t - \phi)$

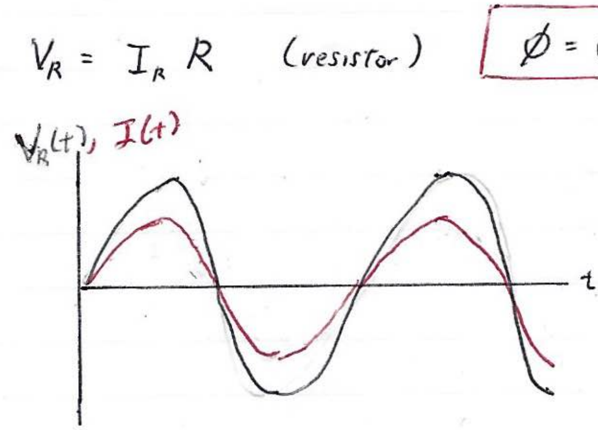
$\phi = \phi_V - \phi_I$

$\mathcal{E} - V_R = 0$

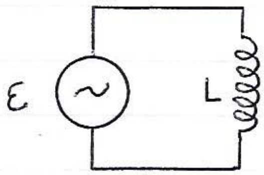
$\therefore V_R = \mathcal{E} = \mathcal{E}_{max} \sin \omega_d t$

So $V_R = \cancel{V_{R\,max}}^{\,\mathcal{E}_{max}} \sin \omega_d t$ and $I_R = \frac{V_R}{R} = \frac{V_{R\,max}}{R} \sin \omega_d t$

$V_R = I_R R$ (resistor) $\boxed{\phi = 0^\circ}$ Current is <u>in phase</u> with V_R.



Inductive Load



V_L

$V_L = V_{L\,max} \sin \omega_d t$ Let $\phi_v = 0$

$= L \frac{dI}{dt}$

$$\frac{dI}{dt} = \frac{V_{L\,max}}{L}$$

$$I_L = \int dI = \frac{V_{L\,max}}{L} \int \sin \omega_d t \, dt = -\frac{V_{L\,max}}{L\omega_d} \cos \omega_d t$$

$$= -\frac{V_{L\,max}}{X_L} \cos \omega_d t$$

where $\boxed{X_L = \omega_d L}$ = inductive reactance

Units of "Ω"

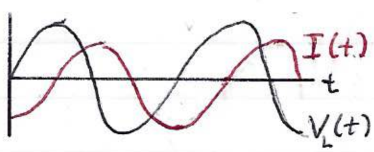
$$= \frac{V_L}{X_L} \sin(\omega_d t - 90^\circ)$$

Again : $\phi = \phi_v - \phi_I = -\phi_I$ $I_L = I_{max} \sin \omega_d t - 90^\circ$

$I = I_{max} \sin(\omega_d t + \phi_I)$

$= \quad \text{''} \quad \text{''} \; (\quad -\phi)$ ← $\boxed{\phi = +90^\circ}$ for pure inductive load

$V_L = I_L X_L$



I_L lags V_L (or V leads I) since changing current induces a "back EMF"

Before I can build up significantly, there must first be a V across the inductor

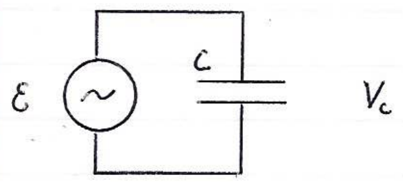
$X_L = \omega L$ As $f \to 0$, $X_L \to 0$. Since rate of change of current $\to 0$

'L' behaves like a SHORT CIRCUIT

As $f \to \infty$, $X_L \to \infty$. The faster I changes, the more vigorously the L opposes that change.

"L" behaves like an OPEN CIRCUIT

Capacitive Load



$V_c = V_{c\,max} \sin \omega_d t \qquad$ Let $\phi_v = 0°$

$q_c = CV_c = CV_{c\,max} \sin \omega_d t$

$I_c = \frac{dq}{dt} = \omega_d C V_{c\,max} \cos \omega_d t$

$= \omega_d C V_{c\,max} \sin(\omega_d t + 90°)$

$I_c = \frac{V_c}{X_c} \sin(\omega_d t + 90°)$ where $\boxed{X_c = \frac{1}{\omega_d C}}$ = capacitive reactance

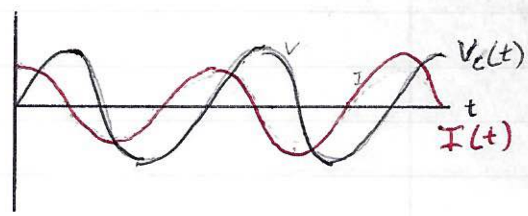
Units of "Ω"

Note: $I = I_{max} \sin(\omega_d t + \phi_I)$

$\therefore \phi = \phi_v - \phi_I = \boxed{-90°}$ for pure capacitive load

(Let $\phi_v = 0$)

$I = I_{max} \sin(\omega_d t - \phi)$



$X_c = \frac{1}{\omega_d C}$, $\omega = 2\pi f$

As $f \to 0$, $X_c \to \infty$

$f = 0 \Rightarrow$ No change in V_c

$\therefore$ No charge on or off

C behaves like an OPEN CIRCUIT

As $f \to \infty$, $X_c \to 0$

Current moves charge on & off in increasingly shorter times.

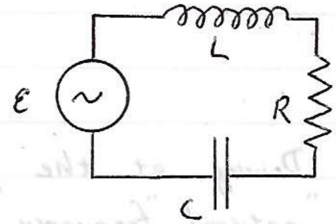
$\therefore$ C behaves like a SHORT CIRCUIT

I leads V since $V_c \propto q$ and it takes current to move the charge

$\therefore$ I flows before V changes significantly.

https://mathlets.org/mathlets/
series-rlc-circuit

Series RLC Circuit



$$\mathcal{E}(t) = \mathcal{E}_{max} \sin \omega_d t$$

If all elements in series;
all have same current

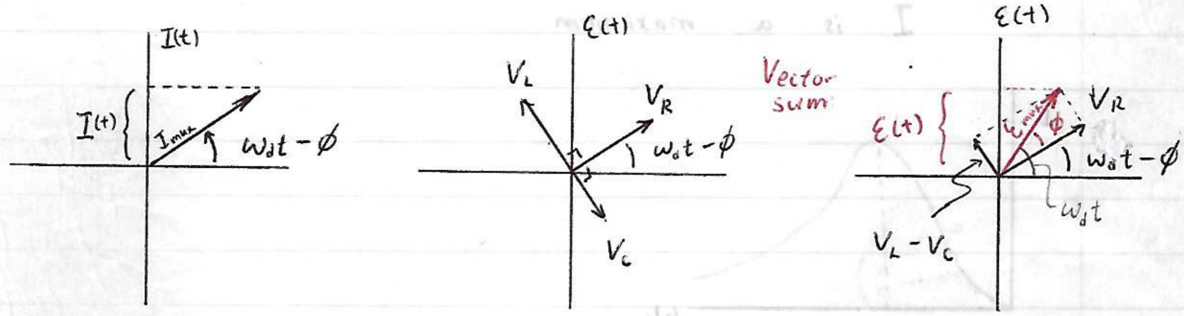
$$I_L = I_R = I_C = I = I_{max} \sin(\omega_d t - \phi)$$

(Again, let $\phi_v = 0$)

Kirchoff:

$$\mathcal{E} = V_L + V_R + V_C = \mathcal{E}_{max} \sin \omega_d t$$

Phasors



$$\mathcal{E}^2 = V_R^2 + (V_L - V_C)^2 = (IR)^2 + [(IX_L) - (IX_C)]^2$$

$$\therefore \quad I = \frac{\mathcal{E}}{[R^2 + (X_L - X_C)^2]^{1/2}} = \frac{\mathcal{E}}{Z}$$

Define impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}$$

Also, $\tan \phi = \dfrac{V_L - V_C}{V_R} = \dfrac{IX_L - IX_C}{IR} = \dfrac{X_L - X_C}{R}$

If $X_L > X_C$ $\quad \phi > 0 \quad$ I_{max} rotates behind $\mathcal{E}_{max}$: inductive circuit

$X_L < X_C$ $\quad \phi < 0 \quad$ I_{max} " ahead of $\mathcal{E}_{max}$: capacitive circuit

$X_L = X_C$ $\quad \phi = 0 \quad$ I_{max} & $\mathcal{E}_{max}$ rotate together in phase : resonance

Use: "ELI $\overset{\phi}{\text{positively}}$ is the ICE man"

Resonance

$X_L = X_C$, $\phi = 0$

$\omega_d L = \frac{1}{\omega_d C} \longrightarrow \omega_d = \frac{1}{\sqrt{LC}}$

Driving at the "natural frequency" of the circuit

Use "'L' is in the loft and 'C' is in the cellar."

At resonance: $Z = Z_{min}$ since $Z = [R^2 + \cancel{(X_L - X_C)^2}]^{1/2}$

$\therefore$ I is a maximum