Magnetism
&
Magnetic Fields

Defining Magnetic Field

Recall…

$\vec{E}$-field $\equiv \dfrac{\textit{Electric force}}{\textit{unit charge}}$ acting on a test charge.

$\vec{g}$-field $\equiv \dfrac{\textit{Gravitational force}}{\textit{unit mass}}$ acting on a test mass.

Similarly, we define a magnetic field, $\vec{B}$, in terms of the magnetic force acting on some appropriate test object.

The test object is a charge q moving with velocity $\vec{v}$.

Experiments show…

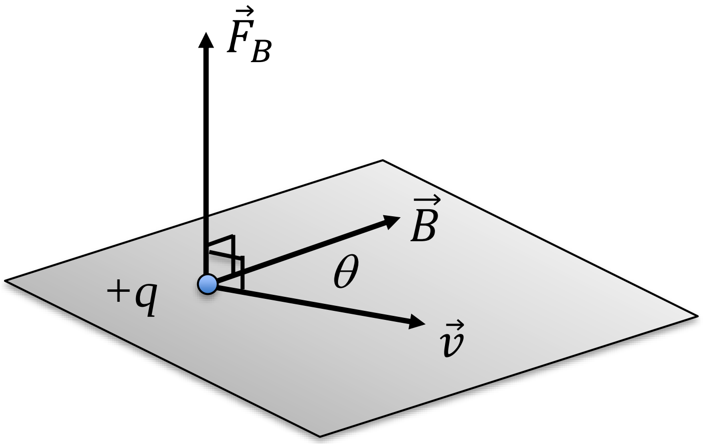
1. $\vec{F}_{mag} \propto q$, $\vec{F}_{mag} \propto |\vec{v}|$ and $\vec{F}_{mag} \propto |\vec{B}|$.
2. $\vec{F}_{mag} = 0$ if $\vec{v} \parallel \vec{B}$.
3. When $\vec{v}$ and $\vec{B}$ make an angle θ, $\vec{F}_{mag} \perp \vec{v}$ and $\vec{B}$.
4. When $\vec{v}$ and $\vec{B}$ make an angle θ, $\vec{F}_{mag} \propto \sin\theta$.
5. $\vec{F}_{mag}$ on a negative (–) charge is opposite to that acting on a positive (+) charge.

Summarizing these result compactly: $\vec{F}_{mag} = q(\vec{v} \times \vec{B})$

SI Unit of $\vec{B}$: $1\,\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}} = 1\,\frac{\text{N}}{\text{A}\cdot\text{m}} \equiv 1$ “Tesla” (T)

Recall the RHR for Cross Products

$$\vec{F}_{mag} = q(\vec{v} \times \vec{B})$$



Consider the plane defined by $\vec{v}$ and $\vec{B}$.

Point the fingers of your right hand in the direction of the first vector ($\vec{v}$). Curl you fingers toward the second vector ($\vec{B}$) through the smaller angle θ. Your thumb will point in the direction of the cross product ($\vec{F}$).

(Note that for a –q, the force will point in the opposite direction than that of the +q.)

Important Differences between $\vec{E}$ and $\vec{B}$

1. $\vec{F}_{elec}$ is always along the direction of $\vec{E}$, whereas $\vec{F}_{mag}$ is always $\perp$ $\vec{B}$.

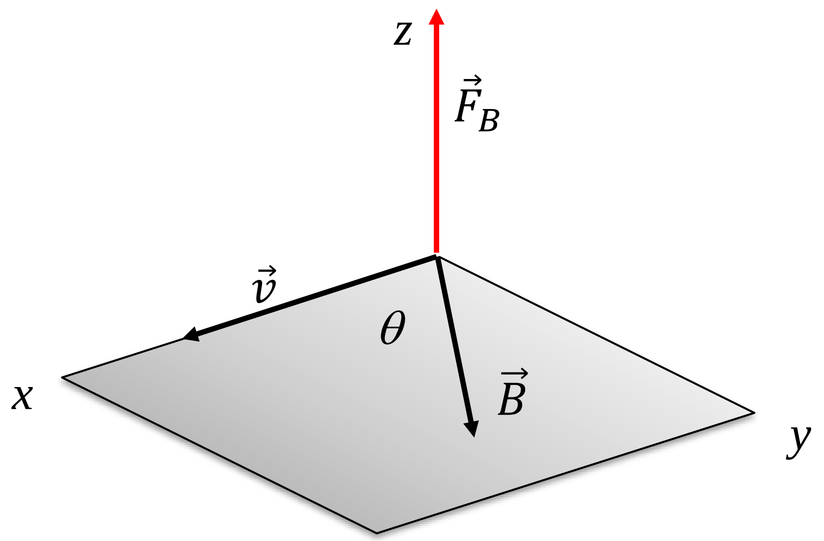
2. $\vec{F}_{elec}$ on a q is independent of q's velocity, whereas $\vec{F}_{mag}$ exists only when $\vec{v} \neq 0$.

3. $\vec{F}_{elec}$ does work on q when q is displaced, whereas $\vec{F}_{mag}$ due to a steady $\vec{B}$ does NO work on q. (Why?)

   Answer: Because $\vec{F}_{mag}$ is always $\perp$ $\vec{v}$.

Example #1

Calculate the magnetic force on a proton at the instant when its velocity is 8.00 x 10^6 m/s along the +*x*-axis in a region where the magnetic field is 2.50 T directed 60^o from +*x*-axis in the *xy*-plane.



$\vec{F}_B = qvB\sin\theta$ directed along the +*z*-axis = 2.77 x 10^{-27} N $\hat{k}$.

Calculate the acceleration of the proton at this instant.

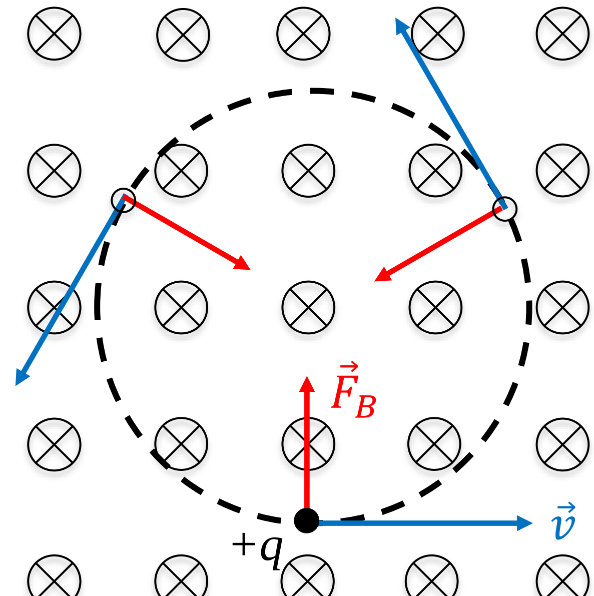
(m_p = 1.67 x 10^{-27} kg)

$$\vec{a} = \frac{\vec{F}_B}{m_p} = 1.66 \text{ x } 10^{15} \text{ m/s}^2\ \hat{k}.$$

Motion of Charge Particle in a $\vec{B}$-field

Recall that $\vec{F}_B$ on a charge q moving in a $\vec{B}$-field is always ⊥ $\vec{v}$ due to the cross-product. As a result, the work done by $\vec{F}_B$ is zero.

That is, static magnetic fields can only change the direction of v, but not the speed or the kinetic energy.



Special case: $\vec{v} \perp \vec{B}$:

Because $\vec{F}_B \perp \vec{v}$ and $\vec{B}$, and has a constant magnitude (= qvB), F_B is a centripetal force causing the charge to execute uniform circular motion.

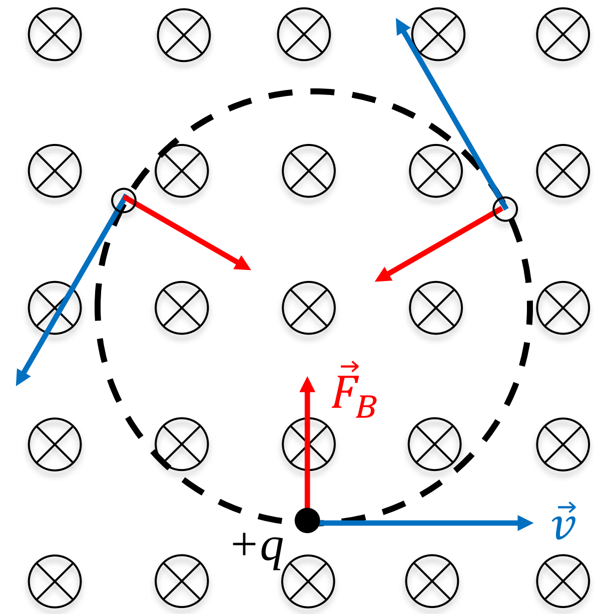
Motion in a $\vec{B}$-field (cont'd)

With $\vec{v} \perp \vec{B}$, $F_B = qvB$ and with $F_B = F_{cen} = m\frac{v^2}{r}$,

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}.$$

Orbital period, $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{qB}{m}$



ω is often referred to as the "cyclotron frequency."

Note that T and ω are independent of the velocity of the charge.

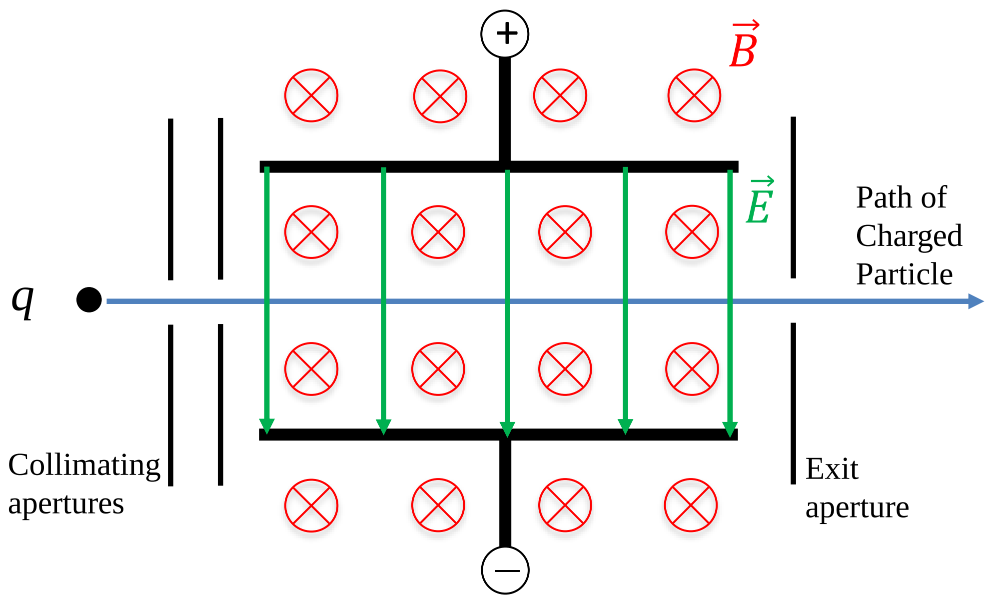
The *Lorentz Force*

In general, a charged particle may simultaneously experience the effects of an electric field and a magnetic field. If so, the total force is…

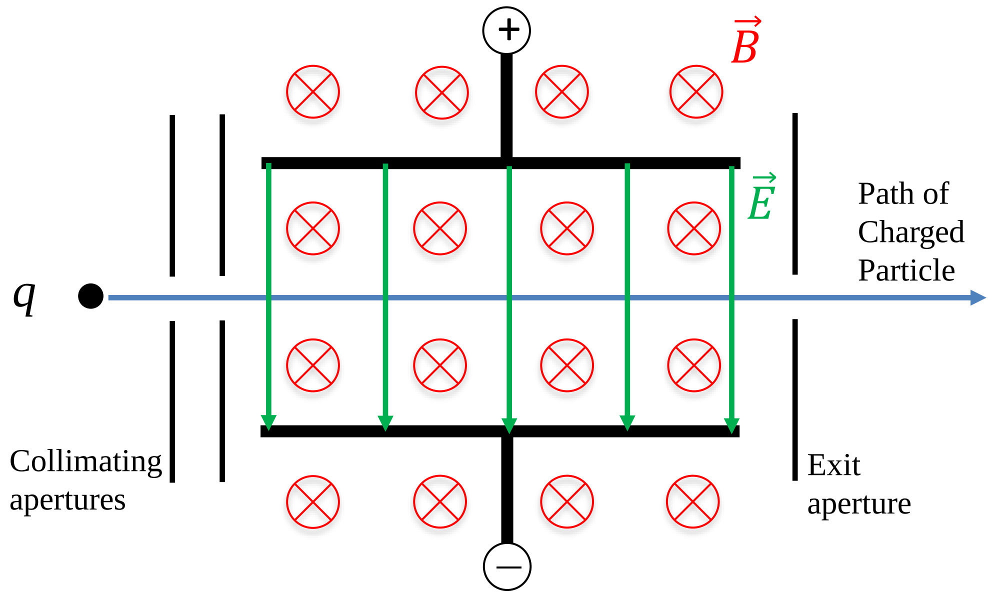
$$\vec{F}_{Lorentz} = \vec{F}_{elec} + \vec{F}_{mag} = q(\vec{E} + \vec{v} \times \vec{B})$$

Consider the special case of crossed fields

This particular arrangement is used as a "velocity filter."



The *Lorentz Force* (con'd)



If q > 0, $\vec{F}_{elec}$ is down and $\vec{F}_{mag}$ is up.

If q < 0, $\vec{F}_{elec}$ is up and $\vec{F}_{mag}$ is down.

To keep particle in a straight line. $\vec{F}_{Lorentz}$ = 0. Then $(\vec{E} + \vec{v} \times \vec{B}) = 0$.

Then, $v = \frac{E}{B}$.

That is, only particles with this speed will emerge from the exit aperture.

Magnetic Force on a Current Carrying Wire

Since a magnetic force can be exerted on a charged particle moving through a magnetic field, it should not be surprise that a current carrying wire in a magnetic field also experiences a force.

In most applications, the charges move through conductors (wires).

For a metal wire, the charges are electrons (as we'll see later) moving with a drift v_{drift}. For a single charge, $\vec{F}_{mag} = q(\vec{v} \times \vec{B})$.

The total number of charges in a wire of length L and cross sectional area A is nAL, where n is the number density of charges.

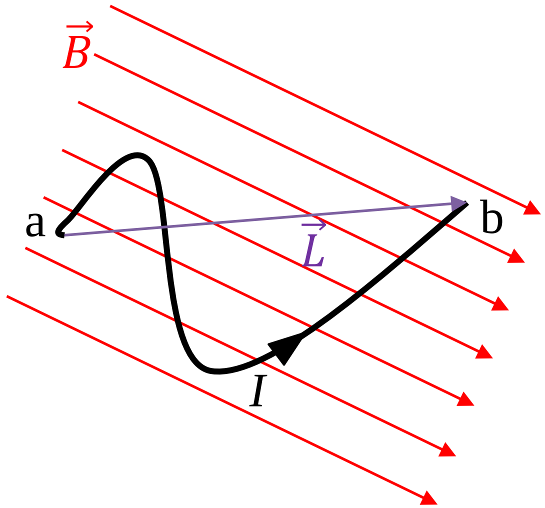
The total force becomes $\vec{F}_{mag} = e(\vec{v}_{drift} \times \vec{B})(nAL)$ since $q = e$.

Recall that $I = JA = nev_{drift}A$. Then the total force becomes:

$\vec{F}_{mag} = I\vec{L} \times \vec{B}$ (The force on a straight wire in a uniform $\vec{B}$-field.)

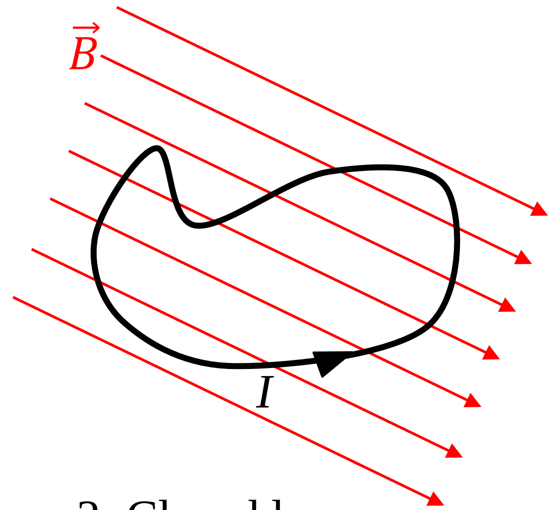
For an arbitrarily shaped wire of uniform cross section in an arbitrary magnetic field, then the total force would be found by integrating $d\vec{F}$ over the path of the wire using the value of $\vec{B}$ appropriate for each infinitesimal segment $d\vec{L}$:

$$\vec{F} = \int d\vec{F} = = \int_a^b I d\vec{L} \times \vec{B}$$



Case 1: Constant I
in a uniform $\vec{B}$-field.

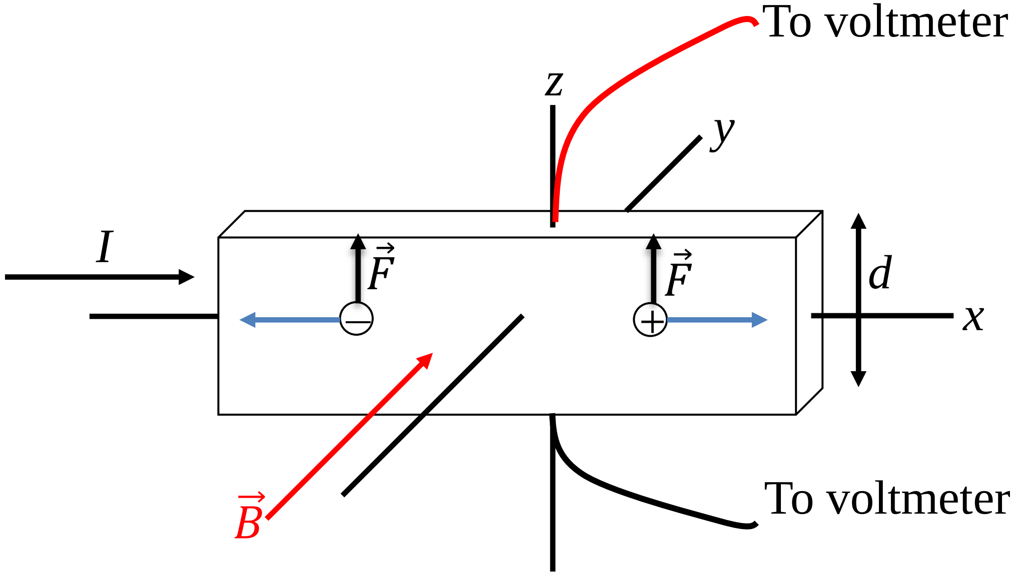
$$\vec{F} = I\left(\int_a^b d\vec{L}\right) \times \vec{B} = I\vec{L} \times \vec{B}$$



Case 2: Closed loop
in a uniform $\vec{B}$-field.

$$\vec{F} = I\left(\oint d\vec{L}\right) \times \vec{B} = 0$$

Hall Effect (1879 Edwin Hall)



Proved that the charges moving in a metal were negative. How?

A current in the +*x*-direction is either due to + charges moving in the +*x*-direction or due to – charges moving in the –*x*-direction.

The metal was placed in a uniform B-field. The moving charges experience a magnetic force. For both signs of charge, the force is directed upward in this scenario. Based on the voltmeter connection (which assumes that the red lead is at the higher potential), the voltmeter will read a $+\Delta V$ if the moving charges are + and a $-\Delta V$ if the moving charges are –.

Experiment showed that this arrangement produced a negative voltage reading. Hence the moving charges are in fact negatively charged electrons.

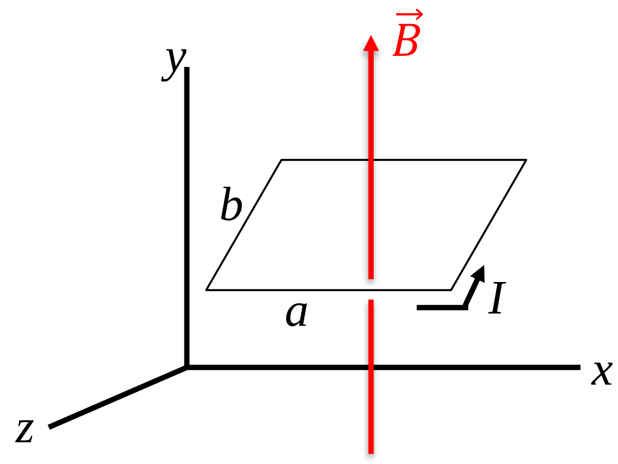
Measuring and ΔV, the *z*-dimension of the metal slab, the current, and magnetic field can also determine *n*, the number density of charges in the metal, however this only works well for monovalent metals(such as Li, Na, Cu, Ag) but not for transition metals (such as Fe, Bi, Cd) nor semiconductors due to quantum mechanical effects.

Torque on a Current Loop in a Uniform $\vec{B}$-field

We have seen that the net force on a current loop in a uniform B-field is zero. However, that does not mean that the torque is zero.

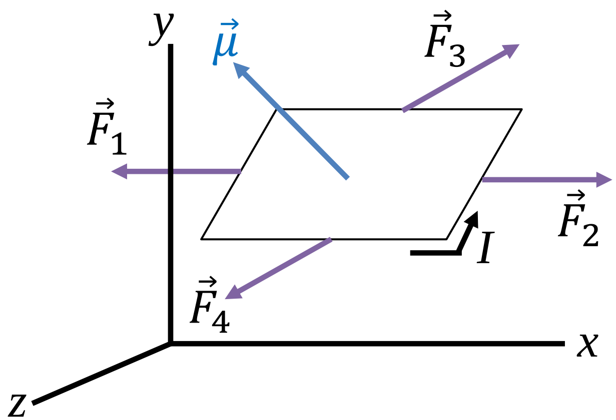
(Recall the behavior of an electric dipole in a uniform electric field.)

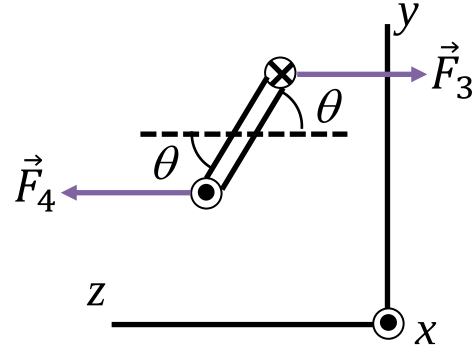
Consider a rectangular current loop having dimensions a and b, in a uniform magnetic field directed in the +y-direction as shown below.



Torque on a Current Loop (cont'd)

Using $\vec{F} = I\vec{L} \times \vec{B}$, the loop will experience the forces:





where, $\vec{F}_1 = IbB\sin(90° + \theta)(-\hat{\imath})$, $\vec{F}_2 = IbB\sin(90° - \theta)(+\hat{\imath})$ cancel and $\vec{F}_3 = IaB\sin(90°)(-\hat{k})$, and $\vec{F}_4 = IaB\sin(90°)(-\hat{k})$.

The torques abut the center due to F_1 and F_2 cancel, whereas the torques due to F_3 and F_4 add to give $\tau = IaB(b/2)\sin(\theta) + IaB(b/2)\sin(\theta)$.

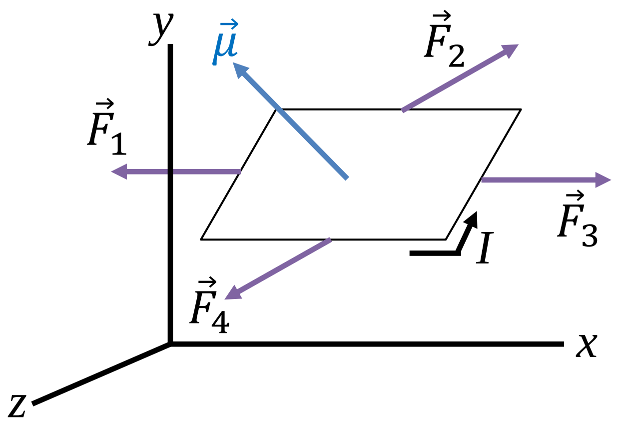
With direction, the total torque is: $\vec{\tau} = IabB\sin(q)(-\hat{\imath}) = \vec{\mu} \times \vec{B}$,

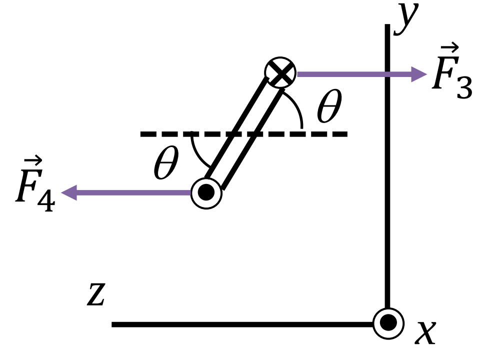
where $\vec{\mu} \equiv (I)(A_{loop}) = Iab$, and whose direction is given by the "right-hand-rule."

$\vec{\mu}$ is called the "*magnetic dipole moment*." In short, the external magnetic field exerts a torque on the current loop to align the magnetic dipole moment with the field.

Torque on a Current Loop (cont'd)

Using $\vec{F} = I\vec{L} \times \vec{B}$, the loop will experience the forces:





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Energy of a Current Loop in a Uniform $\vec{B}$-field

The differential amount of work done by the magnetic field to rotate the magnetic dipole moment of the current loop through an angle $d\theta$ is:

$$dW_{field} = -\int_{\theta_i}^{\theta_f} \tau d\theta$$

(The minus sign is because the torque by the field tend to reduce the angle.)

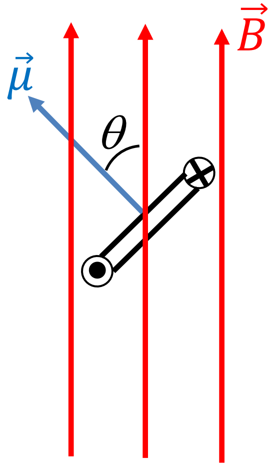
The change in (magnetic) potential energy is the negative of the work:

$$\Delta U = -W_{field} = +\int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} \mu B \sin\theta \, d\theta = -\mu B \left(\cos\theta_f - \cos\theta_i\right)$$

Choosing θ_i to by 90° the expression reduces to

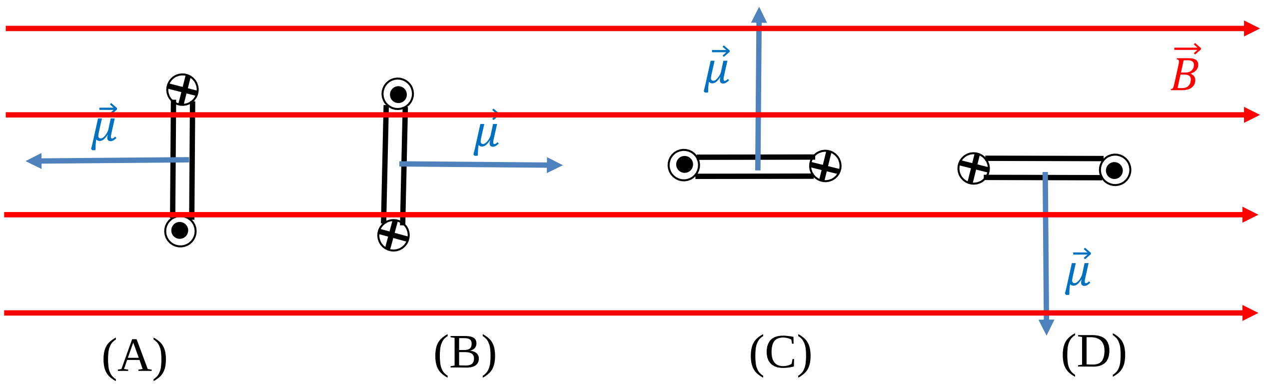
$-\mu B\,(\cos\theta)$, which can be written as: $U = -\vec{\mu} \cdot \vec{B}$

as long as θ is the angle between $\vec{\mu}$ and $\vec{B}$ as shown:



Energy of a Current Loop (cont'd)

Consider the four current loops shown below:



For which case(s) is the potential energy a maximum? (Case A)

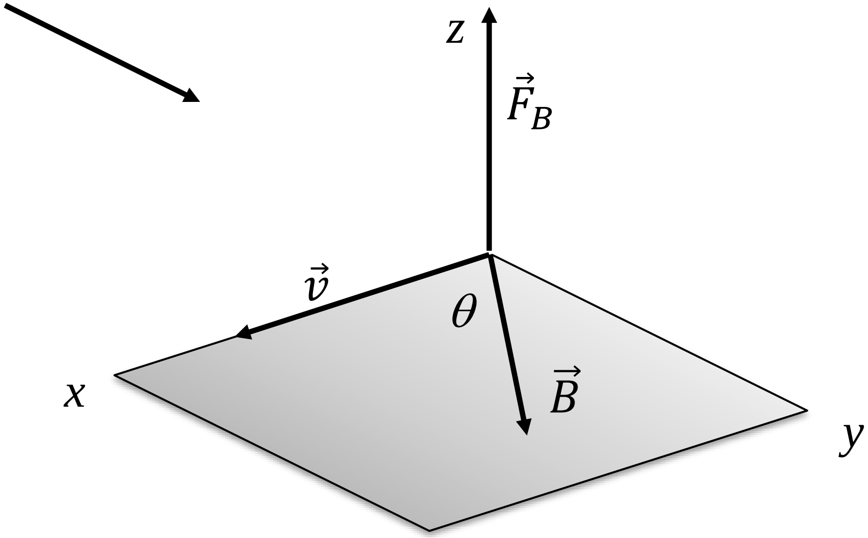
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \sin 180° = +\mu B$$

For which case(s) is the potential energy a maximum? (Case B)

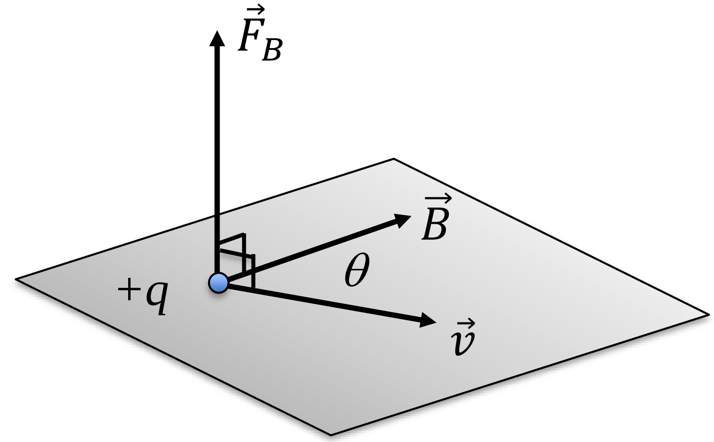
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \sin 0° = -\mu B$$

For which case(s) is the potential energy a maximum? (Cases C & D)

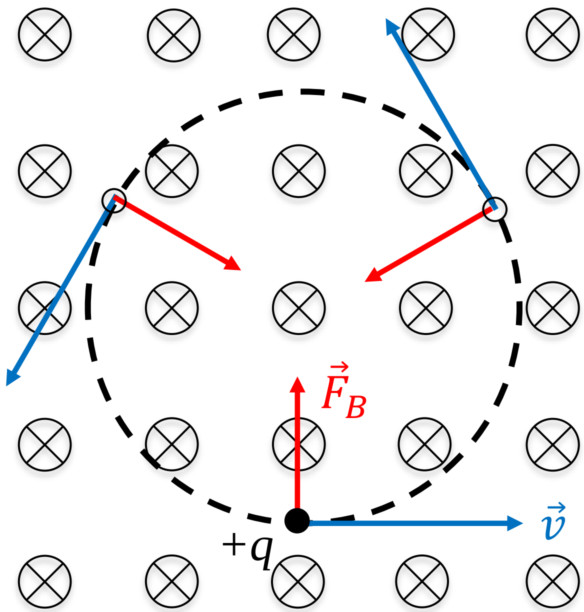
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \sin 90° = 0$$



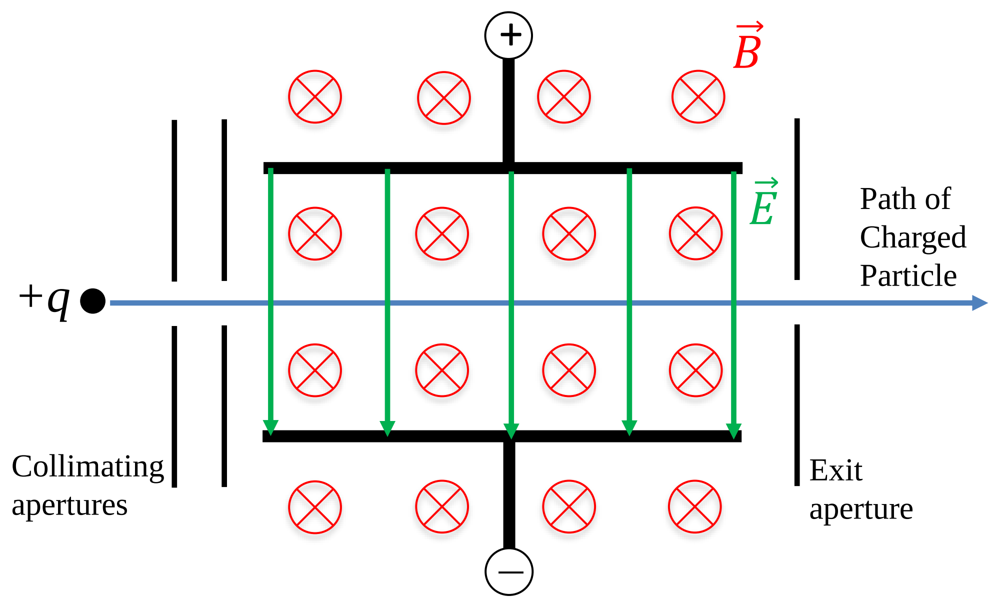
z
$\vec{F}_B$
$\vec{v}$
θ
$\vec{B}$
x
y



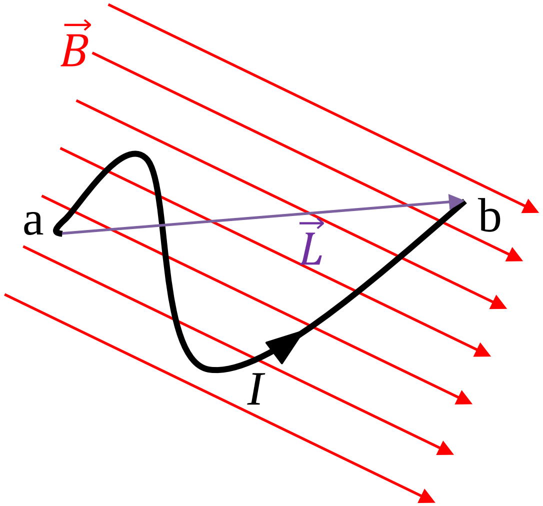
$\vec{F}_B$
$\vec{B}$
θ
+q
$\vec{v}$

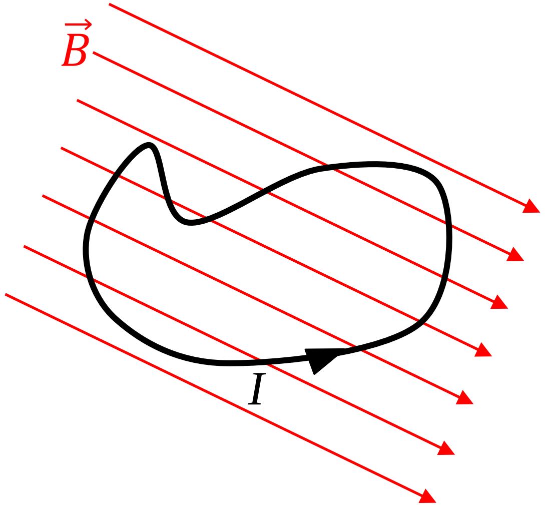


$\vec{F}_B$
+q
$\vec{v}$



+
$\vec{B}$
$\vec{E}$
Path of
Charged
Particle
+q
Collimating
apertures
Exit
aperture
−





Case 1: Constant I
in a uniform $\vec{B}$-field.

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Case 2: Closed loop
in a uniform $\vec{B}$-field.

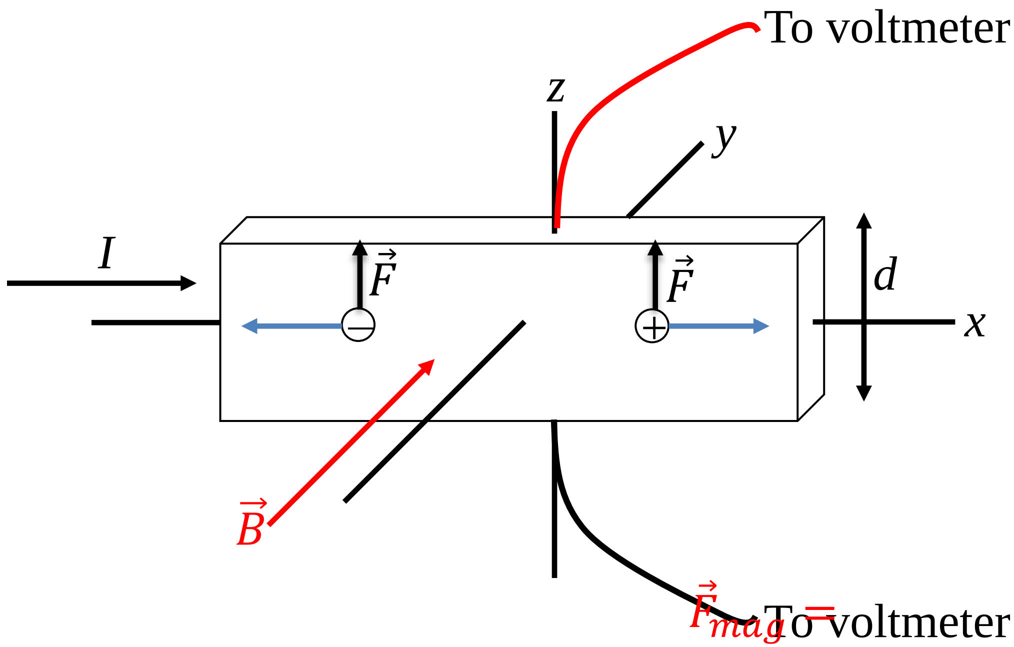
$$\vec{F} = I \left(\int_a^b d\vec{L}\right) \times \vec{B}$$

$$\vec{F} = I \left(\oint d\vec{L}\right) \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\vec{F} = 0$$

$\oplus$


To voltmeter
z
y
I
F
F
d
x
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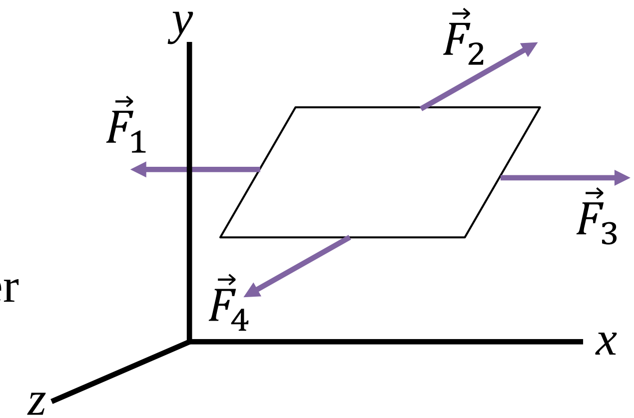
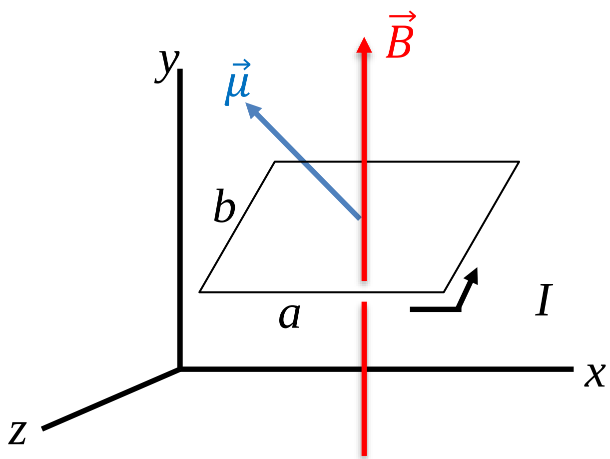
F mag =
To voltmeter


y
F2
F1
F3
F4
x
z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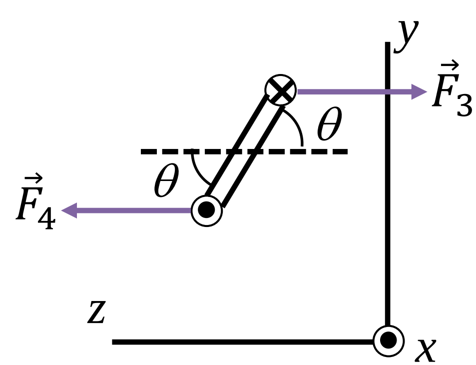
y
F3
θ
θ
F4
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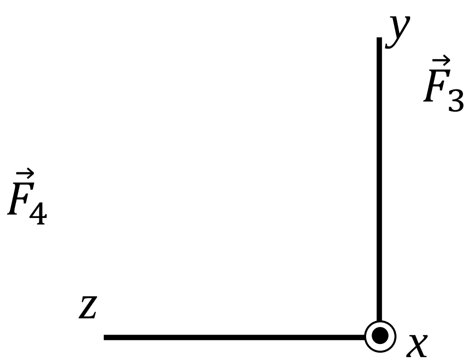
x



θ