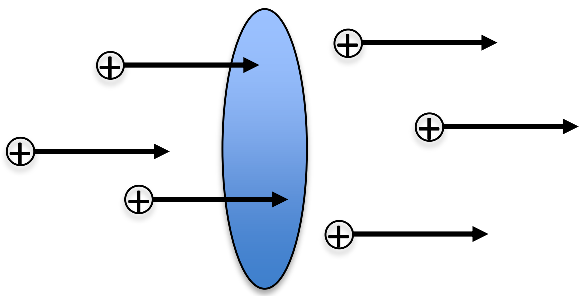
Electric Current
&
Resistance

Electric Current



Defining current:

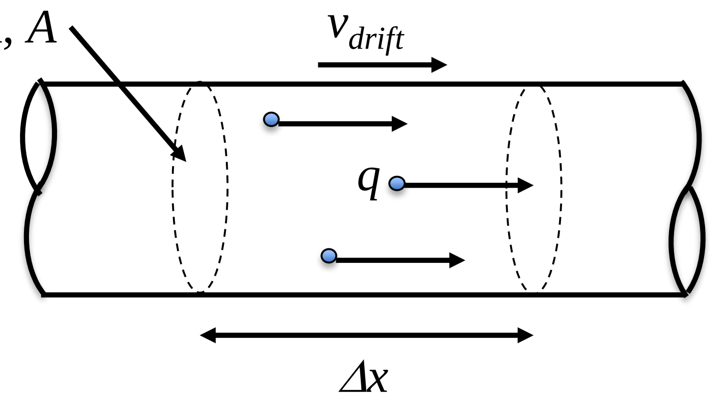
$$I \equiv \frac{dq}{dt}$$

Current describe the RATE at which charge flows.

Unit: Ampere, where 1 Ampere = $1\,\dfrac{\text{Coulomb}}{\text{second}}$

The direction of current is defined as the flow of positive charge in the applied electric field.

How much charge passes a particular point?



$$\Delta q = \int dq = \int_0^t I dt$$

The amount of charge Δq in the volume $A\Delta x$ is

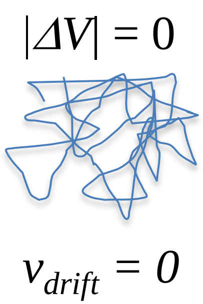
$$\Delta q = qn(A\Delta x) = qnAv_{drift}\Delta t,$$

where n = "# density" $\equiv \dfrac{\text{\# of mobile charge carriers}}{\text{unit volume}}$.

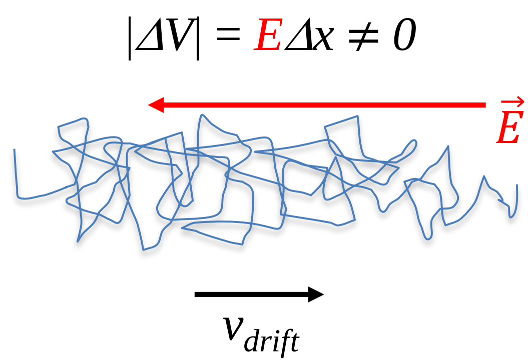
$$I = \frac{\Delta q}{\Delta t} = qnv_{drift}A$$

q = "elementary charge" = $-e$ = 1.602 x 10^{-19} C.

The charges move erratically with high speeds (~10^6 m/s). The motion after each collision with other charges, lattice ions, etc. are random.



No overall displacement of charge on average.



Note that v_{drift} is opposite to the electric field since the charge carriers (electrons) are negative ($q = -e$).

Current Density

Defining current density:

$$J \equiv \frac{I}{A} = nev_{drift}$$

If the current density is uniform across the cross-sectional area of the conductor, then the current in the wire is $I = JA = J\pi R^2$ for a circular wire.

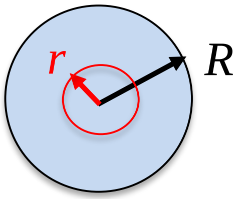
For non-uniform current density:

$$I = \int dI = \int \vec{J} \cdot d\vec{A} = JA$$

Example #1

Suppose in a wire of circular cross-section there is a *nonuniform* current density J(r) = J0(1 – r/R). Determine the total current flowing through the wire.

End view:



$$I = \int dI = \int \vec{J} \cdot d\vec{A} = \int J(r) dA = J_0 \int_0^R \left(1 - \frac{r}{R}\right)(2\pi r dr)$$

$$I = 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^R = \frac{\pi}{3} J_0 R^2$$

Ohm's Law

When a potential difference ΔV is maintained across a conductor, a current density $\vec{J}$ and an electric field $\vec{E}$ are established in that conductor.

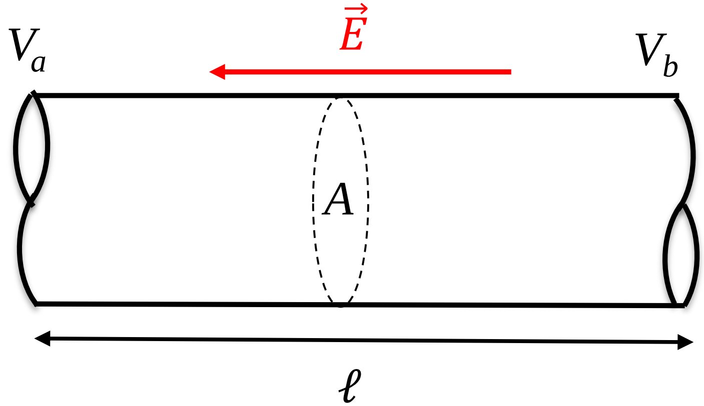
For isotropic materials, $\vec{J} \propto \vec{E} \quad \Rightarrow \quad \vec{J} = (constant)\vec{E}$,

where the proportionality constant called the conductivity of the material. In other words…

The ratio of the current density to the electric field is a constant (σ) which is independent of the electric field producing the current.

$\vec{J} = \sigma\vec{E}$ $\quad \Leftarrow$ This is Ohm's Law.

$$\Delta V = V_b - V_a = E\ell$$



$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$\Delta V = \frac{\ell}{\sigma} J = \frac{\ell I}{\sigma A}$. So, ΔV (or just V) $= \left(\frac{\ell A}{\sigma}\right) I$,

where $\left(\frac{\ell A}{\sigma}\right) \equiv$ Resistance, R. That is, $(\Delta)V = IR$.

Define resistivity, $\rho \equiv \frac{1}{\sigma}$. Then, $E = \rho J$ and $R = \frac{\rho \ell}{A}$.

Units

Unit of resistance $= \frac{Volt}{Ampere} \equiv$ the "Ohm" (Ω)

Unit of resistivity = "Ohm-meter" (Ω-m)

Define conductance, $C \equiv \frac{1}{R}$

Measured in units of Ω^{-1} called the "Mho"

Note: The resistivity (and therefore the conductivity) are intrinsic properties of the material, whereas the resistance (and therefore the conductance) depend on the dimensions of the material.

Special Note about Ohm's Law

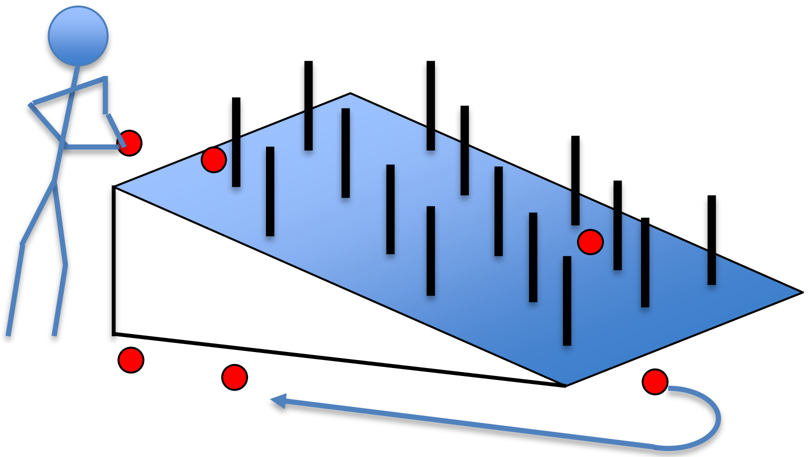
Ohm's Law is not a fundamental law of nature, but rather an empirical relationship that is valid only for certain materials.

A device "obeys" Ohm's law when the resistivity is independent of the magnitude and polarity of the applied electric field, $\vec{E}$. Such a device (for example, a resistor) is called an *ohmic* device.

Otherwise, the device is *non-ohmic*.

(For example, a diode.)

Model of Conduction



Consider a ramp with pegs. A person lifts balls to the top and releases them.

At the bottom, the balls are returned to the person.

Questions: What is acting as the battery? The charge? The current? The resistance?

Answers: The person. The balls. The balls rolling. The pegs.

Questions: What does the height of the ramp represent? What does the the steepness of the ramp represent?

Answers: The voltage, (Δ)*V*. The electric field strength, *E*.

An Electron in the Electric Field…

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$$

After each collision, the electron accelerates with a magnitude eE/m in a direction opposite to that of $\vec{E}$.

Let τ = mean free time (the average time between collisions).

Then $v_{drift} = a\tau = \frac{eE\tau}{m}$.

Recall: $\vec{J} = nqv_{drift} = ne\vec{v}_{drift}$. Then $v_{drift} = \frac{J}{ne}$.

Equate the v_{drift}'s: $\frac{J}{ne} = \frac{eE\tau}{m}$. Solve for J: $J = \left(\frac{ne^2\tau}{m}\right)E$.

Since $J = \sigma E$, we have that the conductivity $\sigma = \frac{ne^2\tau}{m}$.

Example #2

Determine the mean free time for electrons in copper at room temperature.

Use $\sigma = \frac{1}{\rho_{\mathrm{Cu}}} = \frac{ne^2\tau}{m}$, where $\rho_{\mathrm{Cu}} = 1.69 \times 10^{-8}$ Ω-m at 20 °C.

First, determine the # density, n:

$$n = \underbrace{\cancel{\left(\frac{\#\text{ conduction e}^{-\prime}\text{s}}{\text{atom}}\right)}}_{\substack{\text{Valence}\\ Z}} \underbrace{\cancel{\left(\frac{\#\text{ atoms}}{\text{mole}}\right)}}_{\substack{\text{Avogadro's \#}\\ N_A}} \underbrace{\cancel{\left(\frac{\text{mole}}{\text{mass}}\right)}}_{\substack{\text{(Molar Mass)}^{-1}\\ 1/M}} \underbrace{\cancel{\left(\frac{\text{mass}}{\text{unit volume}}\right)}}_{\substack{\text{Mass Density}\\ \rho}}$$

For copper, $Z = 1$ cond. e⁻/atom, $M = 0.06354$ kg/mol, $\rho = 8960$ kg/m^3.

This gives: $n = 8.49 \times 10^{28}$ e^-'s/m^3.

With, $m = 9.11 \times 10^{-31}$ kg and $e = 1.602 \times 10^{-19}$ C, the mean free time is:

$\tau = \frac{m}{\rho_{Cu} n e^2} = 2.47 \times 10^{-14}$ s (That about 40 trillion collisions / second!)

Example #3

Detemine the drift speed for electrons in a zinc wire of circular cross section with radius 0.900 mm and carrying a current of 0.017 A.

As in the previous example, determine the # density, n:

$$n = \frac{Z N_A \rho}{M}$$

For zinc, Z = 2 cond. e^{-}'s/atom, M = 0.06517 kg/mol, ρ = 7130 kg/m^3.

This gives: $n = 1.32 \times 10^{29}$ e^{-}'s/m^3.

Now use $J = nev_{drift}$, where $J = I/A$ and $A = \pi r^2$.

Solve for $v_{drift} = 3.17 \times 10^{-7}$ m/s (= 1.16 mm/hr!)

Power in Electric Circuits

Recall: $-dW_{field} = dW_{ext} = dU = (\Delta)Vdq = Vidt$

The rate of energy transfer (a.k.a. the POWER) is

$$\frac{dU}{dt} = P = VI.$$

A decrease in the electric potential energy by the motion of the charge becomes an increase in energy of another form.

Recall that $R = \frac{(\Delta)V}{I}$. Then $P = VI = I^2R = \frac{V^2}{R}$ which is the rate at which energy is dissipated as heat.

Example #4

Determine the filament resistance of a 40-Watt incandescent light bulb. Take ΔV to be the typical household voltage in the United States: 120 V. Repeat for a 60-W and an 100-W bulb. For much current flows through each filament?

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}. \text{ So, } R_{40} = \frac{(120\ V)^2}{40\ W} = 360\ \Omega$$

Similarly, $R_{60} = 240\ \Omega$ and $R_{100} = 144\ \Omega$.

$$V = IR \Rightarrow I = \frac{V}{R}. \text{ So, } I_{40} = \frac{120\ V}{360\ \Omega} = \frac{1}{3}\text{ A}$$

Similarly, $I_{60} = \frac{1}{2}$ A and $I_{100} = \frac{5}{6}$ A.