Capacitance
&
Capactiors

What do the following all have in common?

- A compressed gas. Internal Energy
- A stretched spring. Elastic PE
- A raised mass. Gravitational PE
- Glucose molecule. Chemical PE

They all have (potential) energy stored in them.

Storing Electrical Potential Energy

A device that stores EPE is called a capacitor.

It consists of two isolated conductors with equal but opposite charges on them.

Capacitance defined

Each conductor is an equipotential surface. Thus, there exists some potential difference between them. This ΔV is found to be proportional to the magnitude of the charge on either conductor. That is,

$$|q| = C|\Delta V|$$

The proportionality constant, C, is defined as the "capacitance."

$$C \equiv \left|\frac{charge\ on\ either\ conductor}{potential\ difference\ between\ the\ conductors}\right| = \left|\frac{q}{\Delta V}\right|$$

Note that C is ALWAYS a positive quantity.

Capacitance

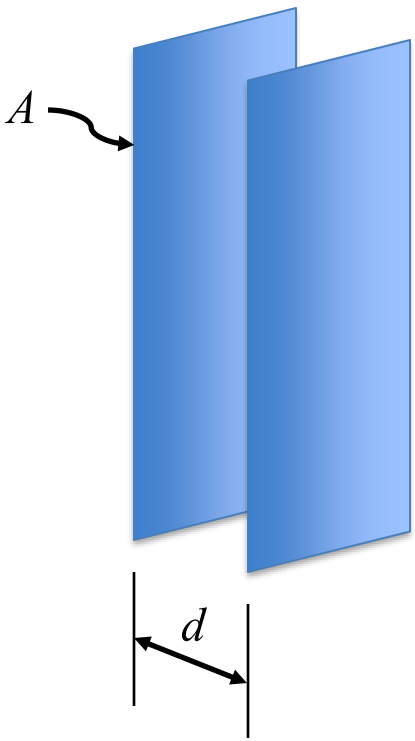
1. C describes how much (equal, but opposite) charge must be placed on the two conductors in order to produce a certain ΔV between them.

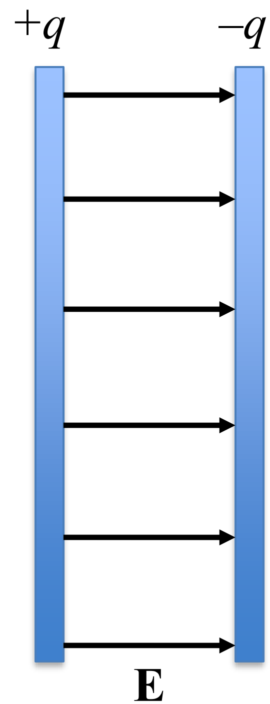
2. C depends on NEITHER the charge q NOR the potential difference ΔV, but rather only on the geometry of the conductors.

3. C has units of $\left(\frac{Coulombs}{Volt}\right)$ defined as "Farads"

That is, 1 F ≡ 1 C/V.

Example 1: Parallel Plates





From Gauss law:

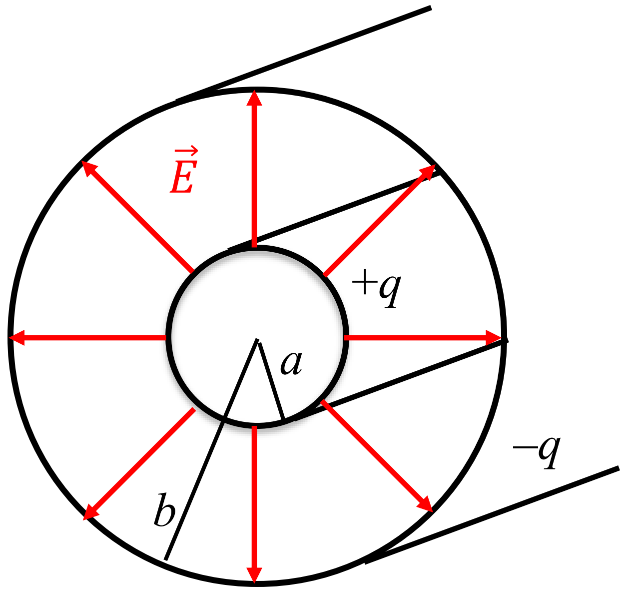
$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

Using $\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$:

$$\Delta V = -Ed = -\frac{q}{\varepsilon_0 A} d$$

So then, $C \equiv \left|\frac{q}{\Delta V}\right| = \frac{q}{\left(\frac{qd}{\varepsilon_0 A}\right)} = \frac{\varepsilon_0 A}{d}$

Example 2: Concentric Cylinders



$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$$

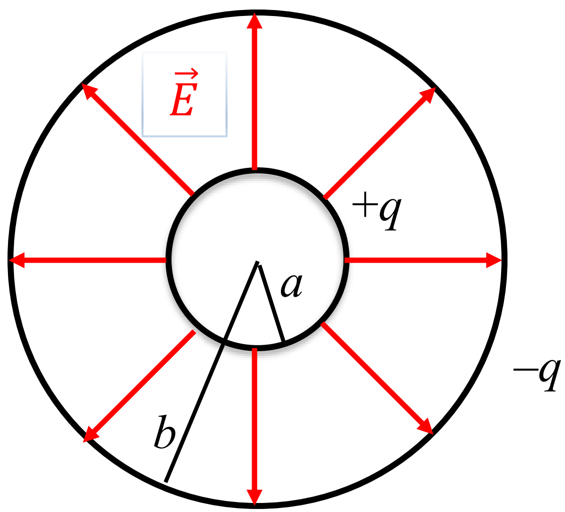
$$= -\int_a^b E dr$$

$$= -\int_a^b \frac{q}{2\pi\varepsilon_0 L r} dr$$

$$= -\frac{q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$\therefore C \equiv \left|\frac{q}{\Delta V}\right| = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Example 3: Concentric Spheres



$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{s}$$
$$= -\int_a^b E\,dr$$
$$= -\int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\,dr$$
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$\therefore C \equiv \left|\frac{q}{\Delta V}\right| = \frac{4\pi\varepsilon_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} = \frac{4\pi\varepsilon_0 ab}{(b-a)}$$

Questions…

What is the capacitance of an isolated conducting sphere?

Answer: This is the just the previous example while letting $b \to \infty$. That is…

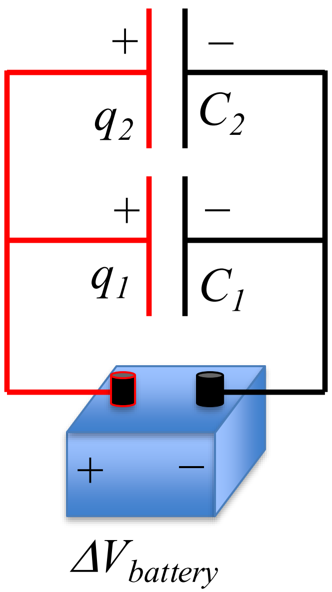
$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} \to 4\pi\varepsilon_0 a$$

0

Calculate the capacitance of a sonducting sphere that is the size of the Earth (R_{Earth} = 6370 km).

$$C = 708\ \mu\text{F}$$

Parallel Combinations



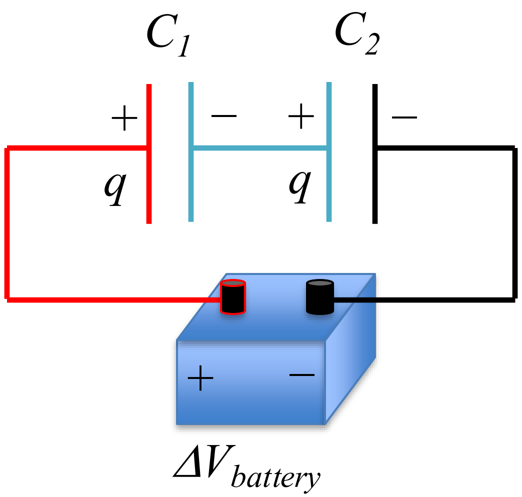
The battery transfers electrons from the left plates to the right plates. Note that each capacitor has the same $\Delta V_{battery}$ across them.

The total charge separated on the capacitors is $q = q_1 + q_2$.

$$C_{equiv} = \frac{q}{\Delta V} = \frac{q_1 + q_2}{\Delta V} = \frac{q_1}{\Delta V} + \frac{q_2}{\Delta V} = C_1 + C2$$

Extending to N capacitors: $C_{equiv} = \sum_{i=1}^{N} C_i$

Series Combinations



The battery transfers electrons from the left plate of C_1 and the right plate of C_2.

No charge is transferred to the inner plates.

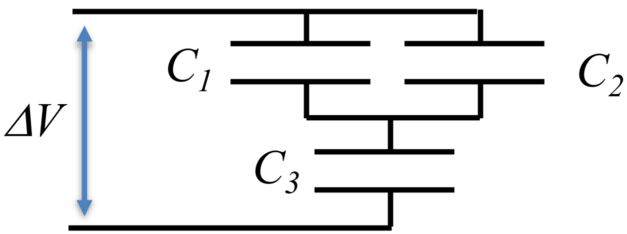
Note that each capacitor has the same charge q on each plate $\Delta v_{battery}$.

$\Delta V_{battery} = \frac{q}{C_{equiv}}$ across the combination is $\Delta V_1 + \Delta V_2 = \frac{q}{C_1} + \frac{q}{C_1}$

Then $\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2}$

For N capacitors: $\frac{1}{C_{equiv}} = \sum_{i=1}^{N} \frac{1}{C_i}$

Example #1

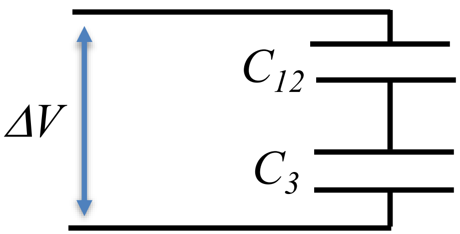


$C_1 = 12.0\ \mu F$ $\quad$ $C_2 = 5.30\ \mu F$

$C_3 = 4.50\ \mu F$ $\quad$ $\Delta V = 12.5\ V$

Determine C_{equiv}

Solution:

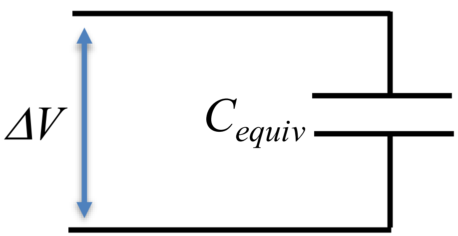


C_1 and C_2 are in parallel:

$C_{12} = C_1 + C_2 = 12.0\ \mu F + 5.30\ \mu F = 17.3\ \mu F.$

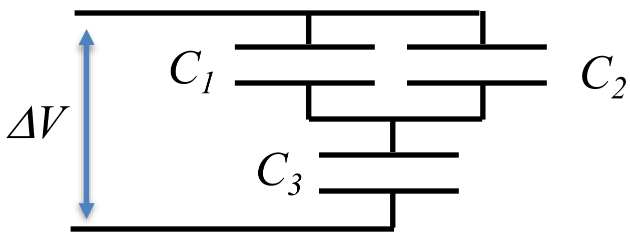
C_{12} and C_3 are in series:

$$\frac{1}{C_{equiv}} = \frac{1}{C_{12}} + \frac{1}{c_{32}} = \frac{1}{17.3\ \mu F} + \frac{1}{4.50\ \mu F} = 0.280\ \mu F^{-1}$$



$C_{equiv} = 3.57\ \mu F$

Example #1 cont'd



$C_1 = 12.0\ \mu F$ $C_2 = 5.30\ \mu F$

$C_3 = 4.50\ \mu F$ $\Delta V = 12.5\ V$

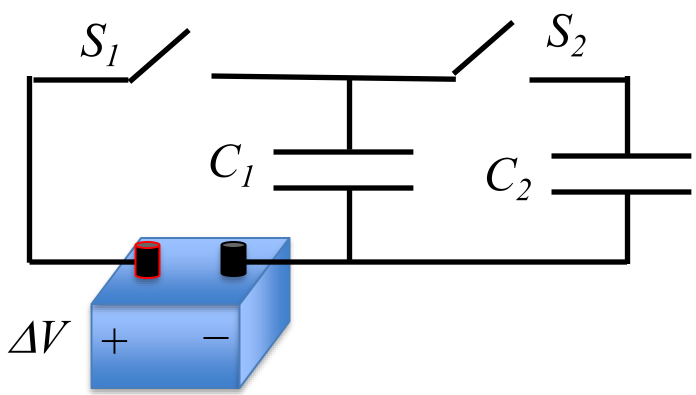
Determine the q_1 on C_1.

Solution: Knowing C_{equiv} has same ΔV as the battery, the total charge on the network is $q_{total} = C_{equiv}\,\Delta V = (3.57\ \mu F)(12.5\ V) = 44.6\ \mu F$. Working backwards, this is the charge on C_3 and C_{12} since they are in series. The ΔV across C_1 and C_2 is the same as C_{12} since C_1 and C_2 are in parallel. $\Delta V_{12} = C_{12}/q_{total} = 17.3\ \mu F / 44.6\ \mu F = 2.58\ V$.

So, $q_1 = C_1 \Delta V_1 = (12\ .0\ \mu F)(2.58\ V)$

$q_1 = 31.0\ \mu C$

Example #2



$C_1 = 3.55\ \mu F$ $C_2 = 8.95\ \mu F$

$\Delta V = 6.30\ V$

Close S_1 to fully charge C_1.

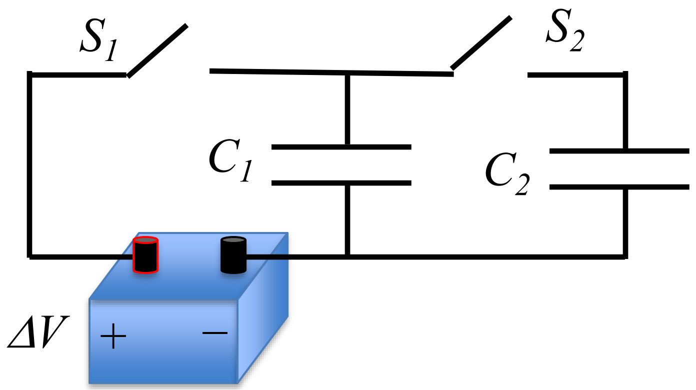
Open S_1, then close S_2.

Wait until $\Delta V_1 = \Delta V_2 = \Delta V_f$.

Determine ΔV_f.

$\Delta V_f = 1.79\ V$

Solution to Example #2



When S_1 is closed, C_1 acquires a charge $q_{1i} = C_1 \Delta V$.

When S_1 is opened, the charge remains on C_1.

When S_2 is closed, some of the $q1i$ transfer to C_2 until

$$\Delta V_1 = \Delta V_2 = \Delta V_f.$$

Conservation of charge required that $q_{1f} + q_{2f} = q_{1i}$.

$$C_1 \Delta V_f + C_2 \Delta V_f = C_1 \Delta V$$

Solve for $\Delta V_f = \frac{C_1 \Delta V}{C_1 + C_2} = \frac{(3.55\ \mu F)(6.30\ V)}{3.55\ \mu F + 8.95\ \mu F} = 1.79\ \text{V}$

Energy Stored in the **E**-field

In order to move charge from one conductor to the other, you need to do WORK.

If q already on plates, giving a potential difference ΔV at that instant,

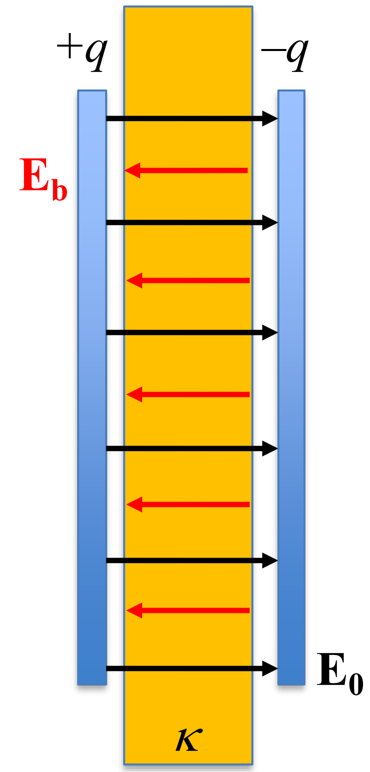
$$\Delta V = -\frac{W_{field}}{q} = \frac{W_{ext}}{q}$$

$$dW_{ext} = \Delta V dq = \frac{q}{C} dq$$

$$W_{ext} = \int dW_{ext} = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C} = \Delta U_E$$

Dielectrics

Dielectric: An insulating material placed between the oppositely charged conductors that alters the capacitance as it becomes polarized upon insertion.

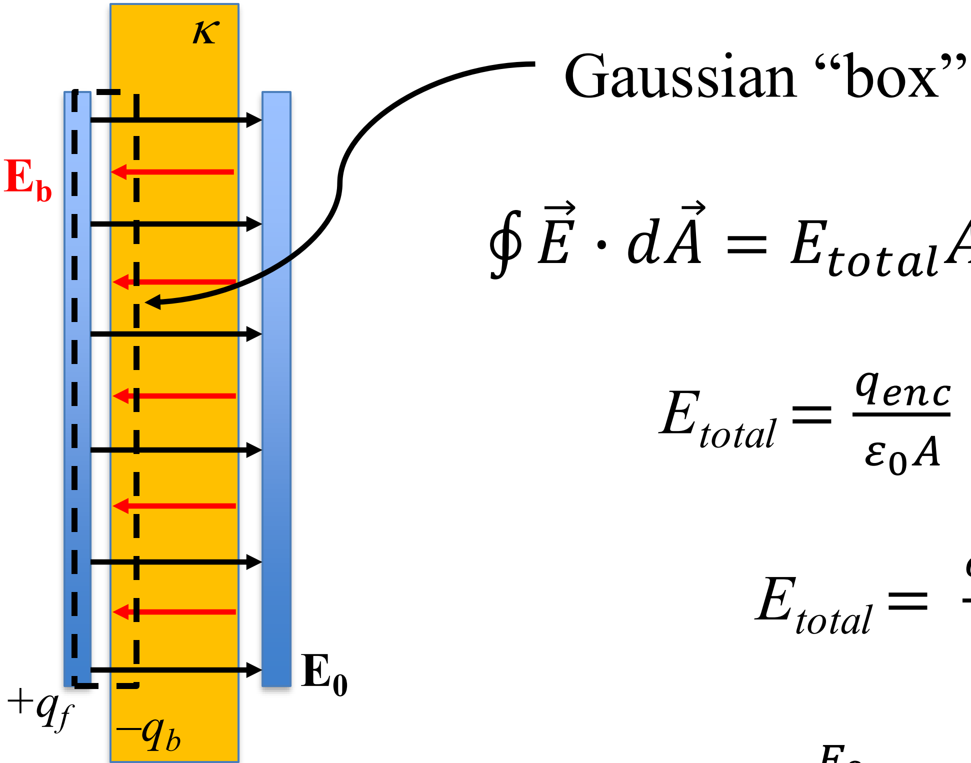


The new capacitance, $C' = \kappa C$,

where $\kappa \equiv$ dielectric constant.

$\kappa \geq 1.$ $(\kappa_{vacuum} = 1)$

$$E_{total} = E_0 - E_b = \frac{E_0}{\kappa}, \text{ since } E_{total} = \Delta V_{new} = \frac{\Delta V}{\kappa} d = \frac{E_0}{\kappa}$$



$$\oint \vec{E} \cdot d\vec{A} = E_{total}A = (E_0 - E_b)A$$

$$E_{total} = \frac{q_{enc}}{\varepsilon_0 A} = \frac{q_f - q_b}{\varepsilon_0 A}$$

$$E_{total} = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$$

$$\therefore E_b = E_0 - \frac{E_0}{\kappa} = E_0\left(1 - \frac{1}{\kappa}\right).$$

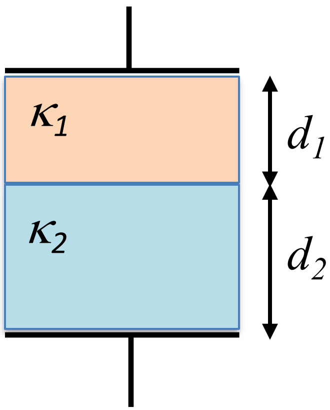
Recall that $1 < \kappa < \infty$. So, E_b is <u>always</u> less than E_0.

Also, since $E \propto \sigma$, $\sigma_b = \sigma_f\left(1 - \frac{1}{\kappa}\right)$ σ_b is <u>always</u> less than σ_f.

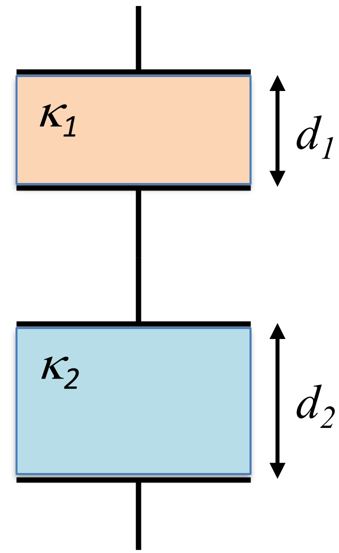
For $\kappa = 1$: $\sigma_b = 0$ (No dielectric inserted.)

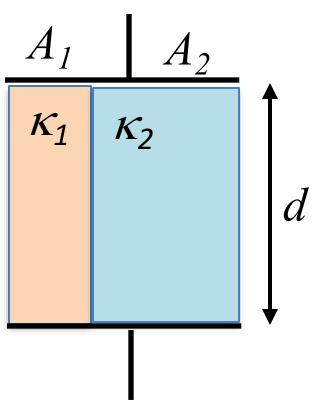
For $\kappa = \infty$: $\sigma_b = \sigma_f$ (Conducting slab inserted.)

Composite Capacitors

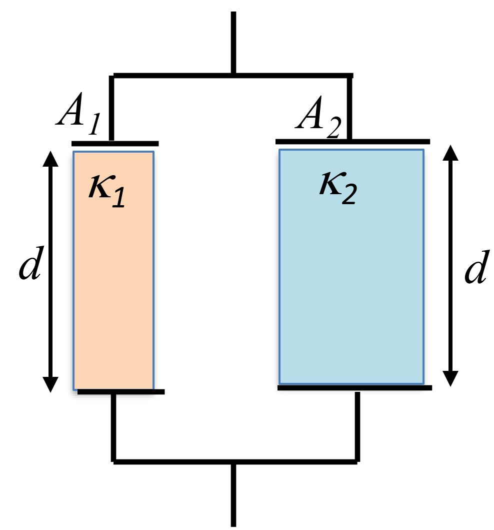


Treat as capacitors
in series:





Treat as capacitors
in parallel.



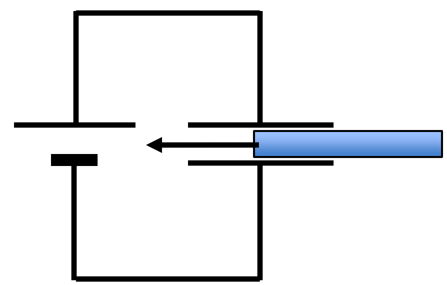
Two cases:

1) Inserting the dielectric with the battery attached:

ΔV = constant

$C' = \kappa C$

$q' = \kappa q$



2) Inserting the dielectric with the battery disconnected:

q = constant

$C' = \kappa C$

$\Delta V' = \Delta V/\kappa$