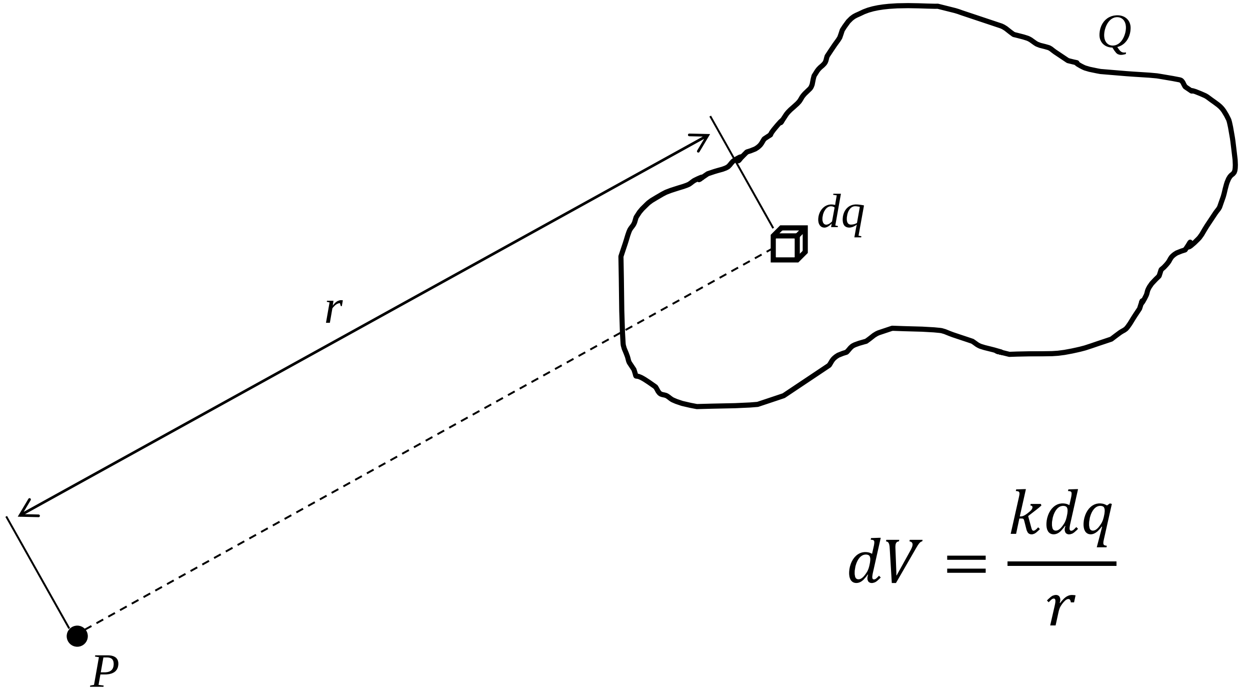
# Electric Potential
# &
# Continuous Distributions

# Electric Potential from Continuous Distributions of Charge

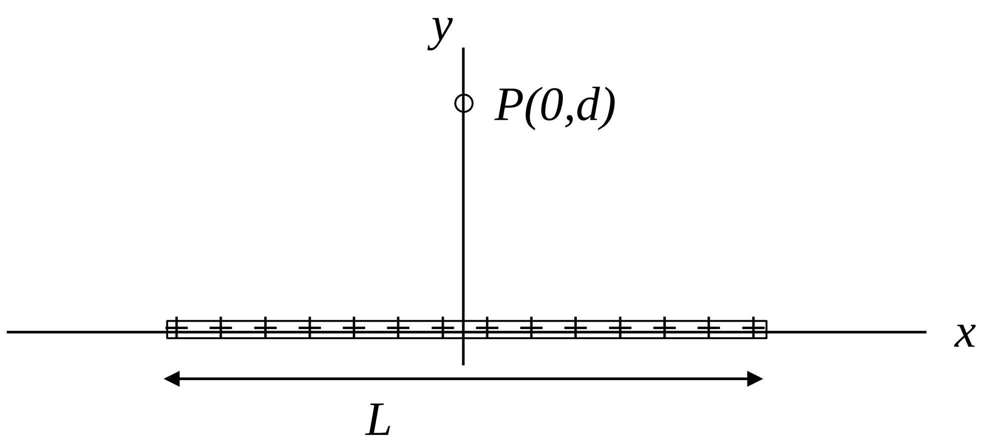



$$dV = \frac{k\,dq}{r}$$

$$V_P = \int dV = k \int \frac{dq}{r}$$

# Example 1:

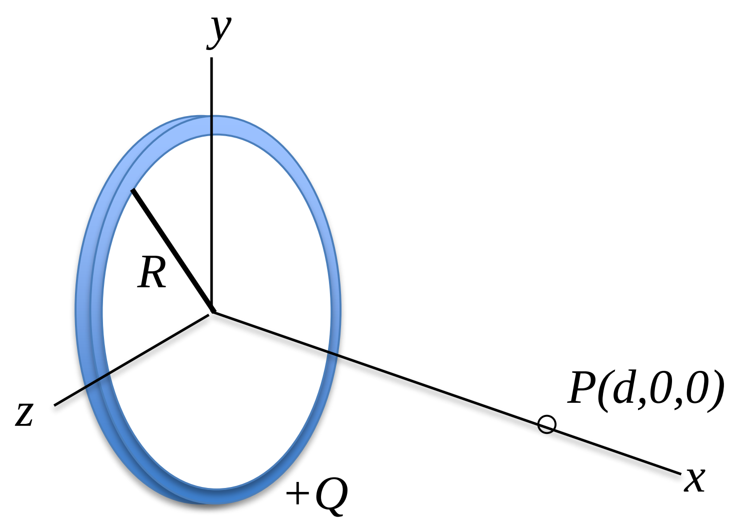
Uniform Finite Line of Charge (having total charge $Q$ and radius $R$ (centered about the y-axis).




Calculate the electric potential for a point on the $y$-axis located a distance $d$ from the line.

# Example 2:

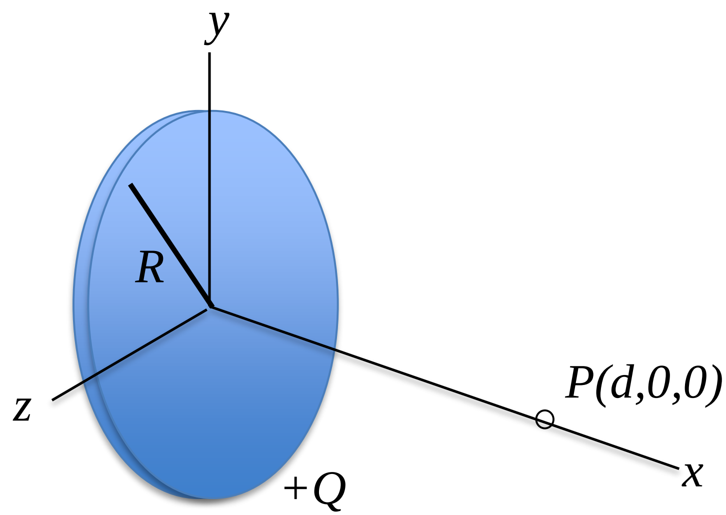
Uniform Ring of Charge (having total charge $+Q$ and radius $R$).




Calculate the electric potential for a point on the symmetry axis (taken to be the $x$-axis).

# Example 3:

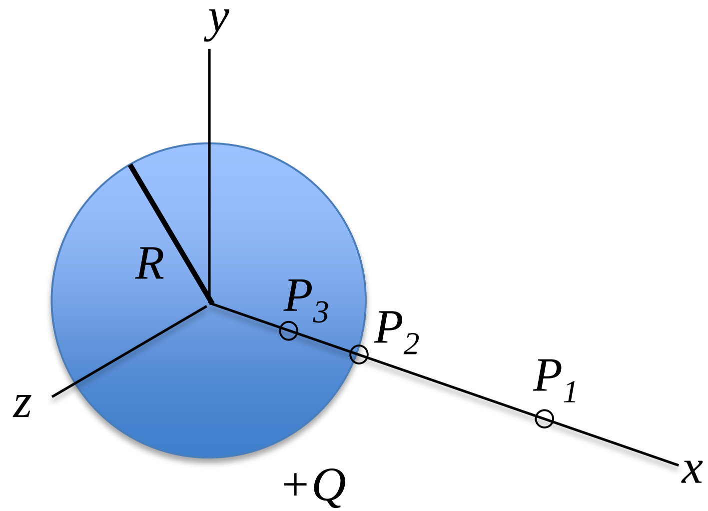
Uniform Circular Disk of Charge (having total charge $+Q$ and radius $R$).




Calculate the electric potential for a point on the symmetry axis (taken to be the $x$-axis).

# Example 4:

Uniformly Charged Insulating Sphere of Charge (having total charge $+Q$ and radius $R$).



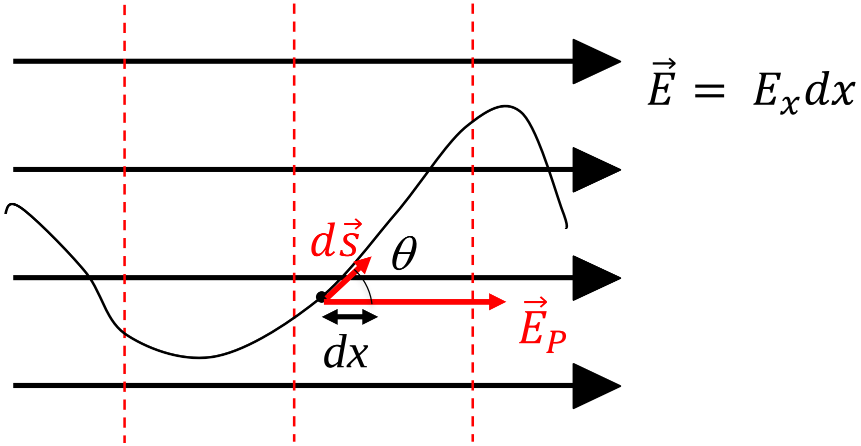

Calculate the electric potential outside the sphere, on the surface of the sphere, and inside the sphere.

# Obtaining **E** from *V*

If **E** has only 1 component,

then $dV = -\vec{E} \cdot d\vec{s} = -E_x dx$,

where $dx = ds\cos\theta$.




The component of $\vec{E}$ along $d\vec{s}$:

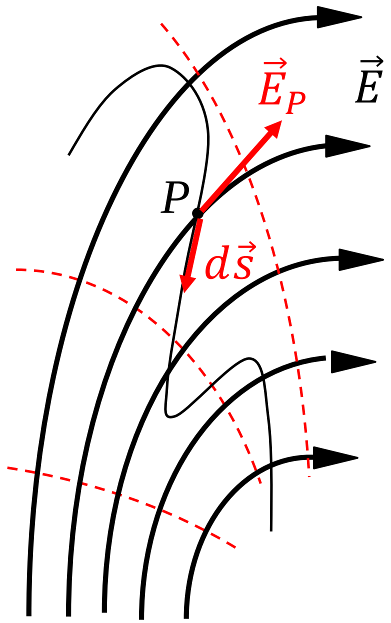
$E_s = E cos\theta = -\frac{dV}{ds}$ which implies $E_x = -\frac{dV}{dx}$

Since E can be chosen to lie along any direction, we can in a similar fashion say that, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

# Obtaining **E** from *V*

The component of **E** along any direction is just the negative rate of change of *V* with respect to that direction:

$$E_x = -\frac{dV}{dx}, \qquad E_y = -\frac{dV}{dy}, \qquad \text{and } E_z = -\frac{dV}{dz}$$




In general,

$$\vec{E}_P = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

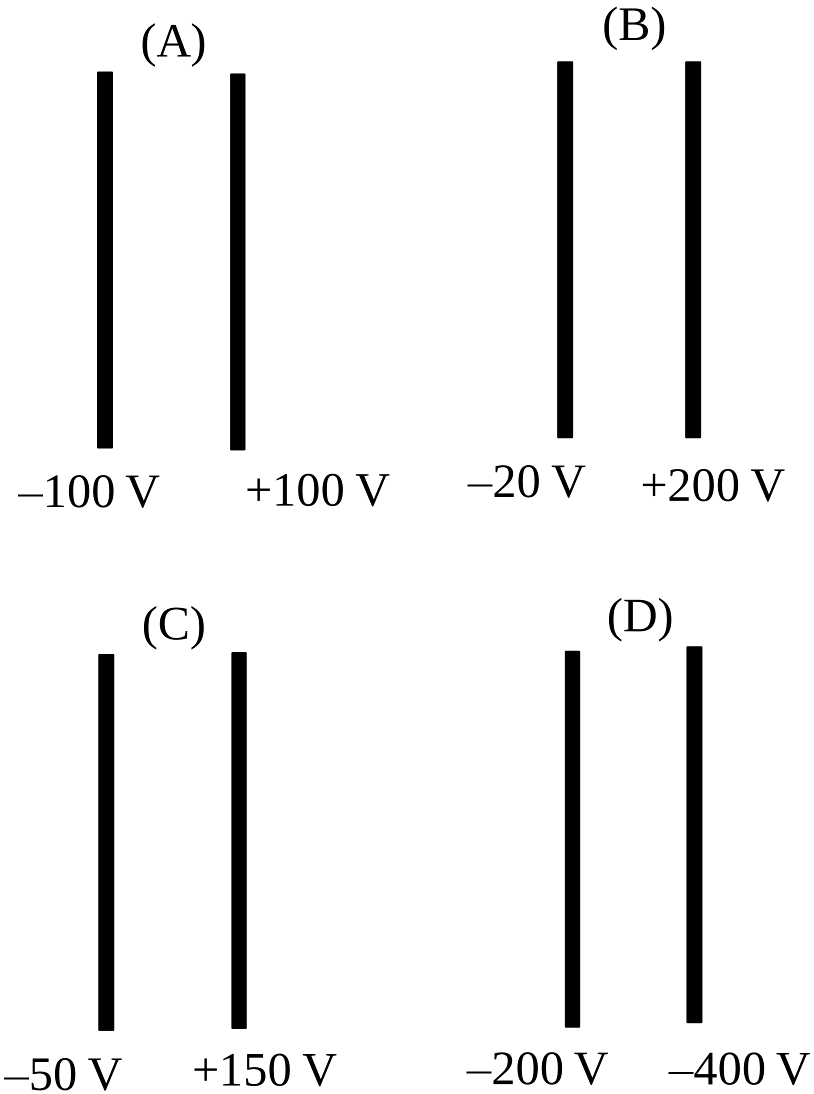
$$\vec{E}_P = -\nabla V = -\text{grad(V)}$$

# Questions…

Shown are four pairs of charged plates with the potentials shown.

- Rank the pairs according to their greatest **E**-field strength between the plates.

- For which pair(s) (if any) does the E-field point left?



# Questions…

The electric potential in a certain region of space is given by $V(x,y,z) = 2xy - 5xz + 3y^2$, where $x$, $y$, and $z$ are in meters and $V$ is in Volts.

- Calculate the electric potential at position (1, 1, 1)?

$$V(1, 1, 1) = 0 \text{ V}$$

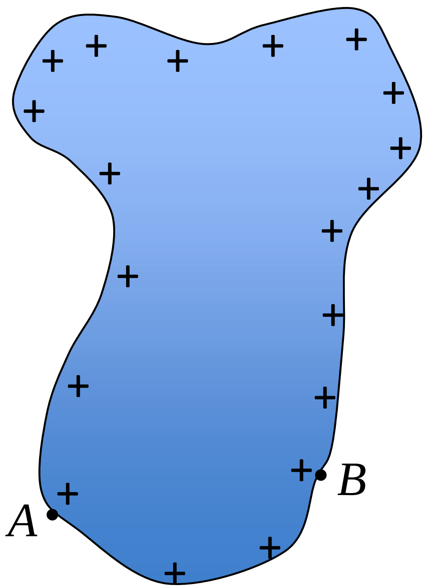
- Determine the electric field at position (1, 1, 1). Write in unit vector notation.

$$\vec{E}(1,1,1) = \left(3\hat{\imath} - 8\hat{\jmath} + 5\hat{k}\right)\frac{\text{V}}{\text{m}}$$

- Calculate the magnitude of the electric field at (1, 1, 1).

$$E(1,1,1) = 9.90 \text{ V/m}$$

# Electric Potential of a Charged Isolated Conductor




$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

For the path along the surface that connects points $A$ and $B$, $\vec{E} \perp d\vec{s}$ at all points on that path.

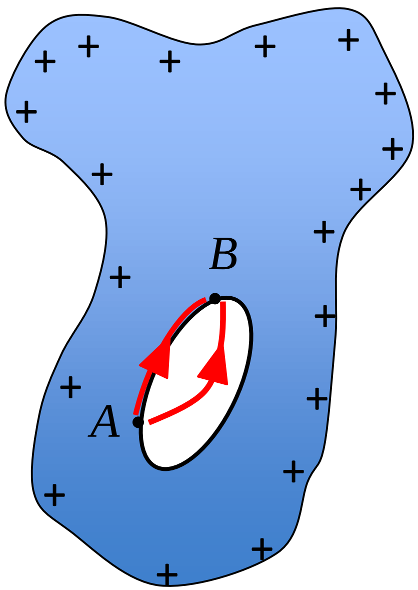
Thus, $V_B - V_A = 0$

This is true for any path along the surface.

$\therefore V$ = constant everywhere on that surface.

Furthermore, since $\vec{E} = 0$ inside the conductor, $V_A = V_B$ for ANY pair of points everywhere in the conductor including the surface for a conductor in electrostatic equilibrium.

# Cavity in the Conductor?




$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

For a conductor with a cavity (such as a spherical shell)…

If $\vec{E} \neq 0$, then there could always be some path found for which $\vec{E} \cdot d\vec{s} \neq 0$. But $V_A = V_B$.

$\therefore$ $\vec{E} = 0$ inside the cavity

(as long as there are no charges placed in the cavity).