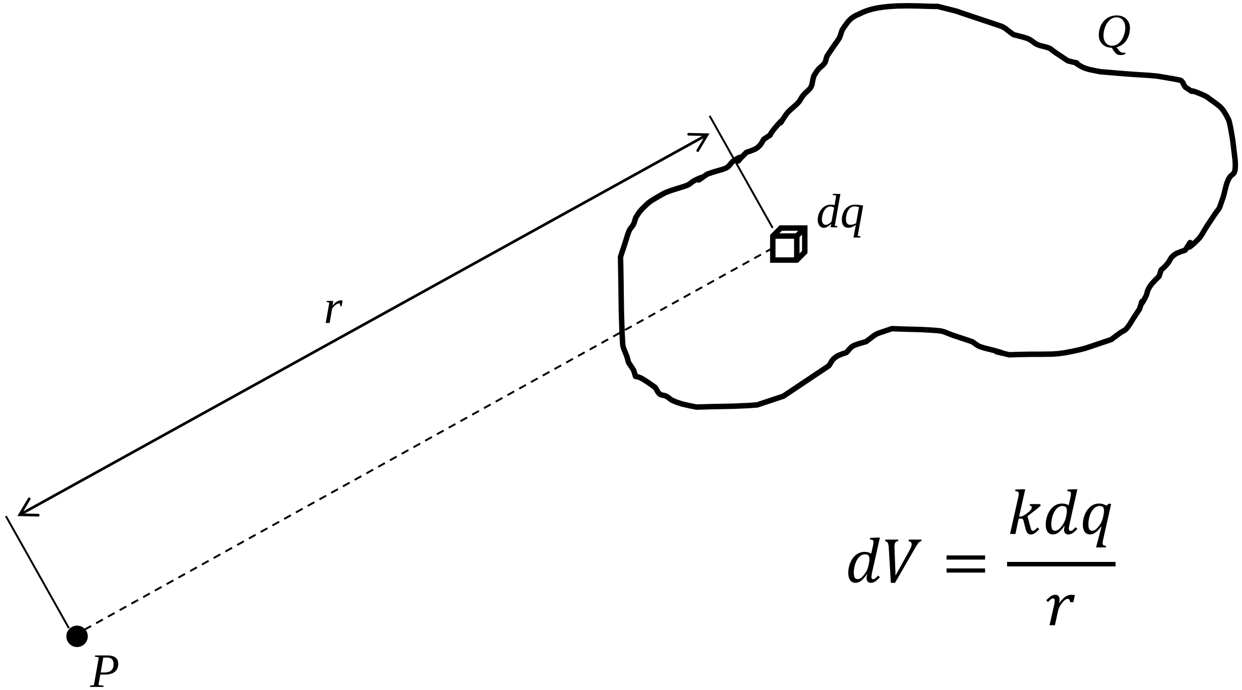
Electric Potential
&
Continuous Distributions

Electric Potential from Continuous Distributions of Charge

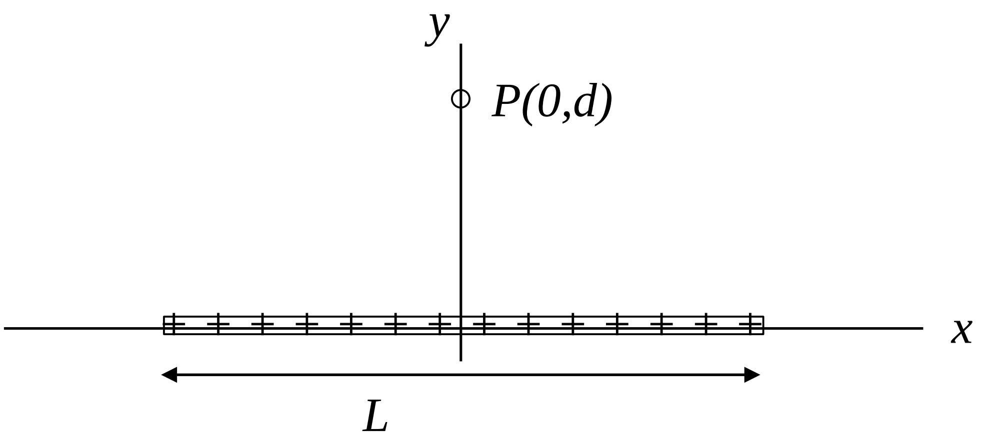


$$dV = \frac{k\,dq}{r}$$

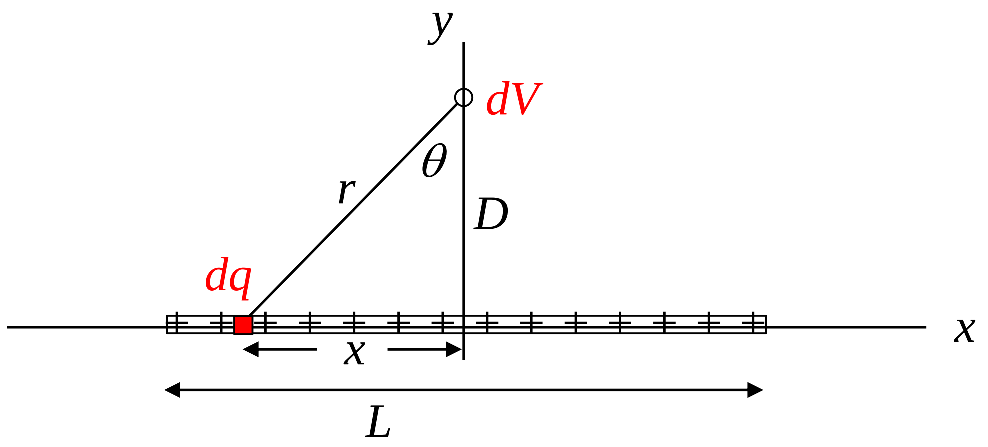
$$V_P = \int dV = k \int \frac{dq}{r}$$

Example 1:

Uniform Finite Line of Charge (having total charge *Q* and radius *R* (centered about the y-axis).



Calculate the electric potential for a point on the *y*-axis located a distance *d* from the line.



$$dV = \frac{kdq}{r} = \frac{k\lambda dx}{(x^2 + D^2)^{½}}$$

Integrate from –L/2 to +L/2:

$$V_P = \int_{-L/2}^{+L/2} dV = \int_{-L/2}^{+L/2} \frac{\lambda dx}{(x^2+D^2)^{½}} = k\lambda \ln\left[x + \left((x^2 + D^2)^{½}\right)\right]_{-L/2}^{+L/2}$$

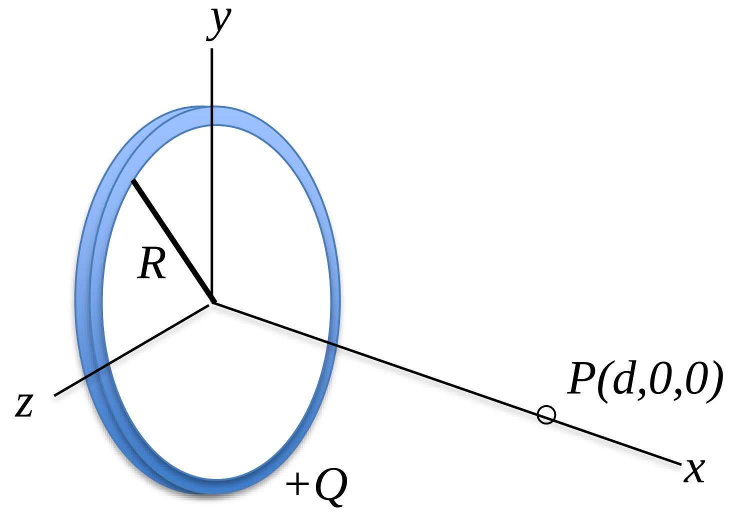
$$V_P = k\lambda \left[\frac{\frac{L}{2} + \left(\frac{L^2}{4} + D^2\right)^{1/2}}{\frac{L}{2} + \left(\frac{L^2}{4} + D^2\right)^{1/2} - \frac{L}{2}}\right]$$

If Point P was (say) over the left end, you would do the same integral, but integrate from $x = 0$ to $x = L$.

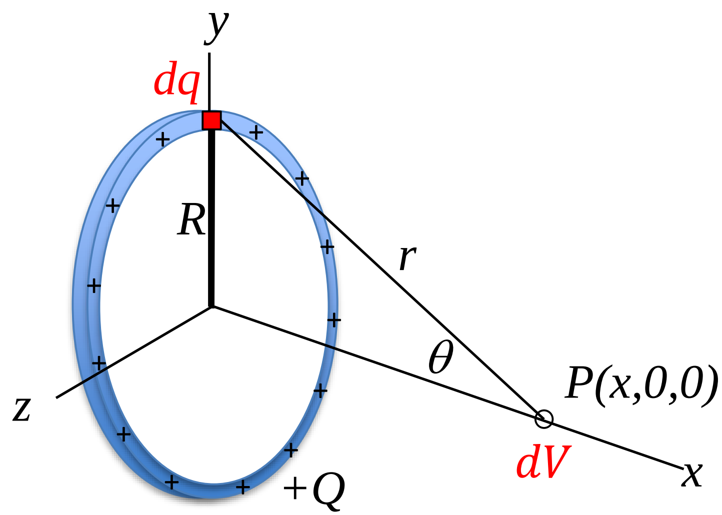
Right end? Then integrate from $x = –L$ to $x = 0$.

Example 2:

Uniform Ring of Charge (having total charge $+Q$ and radius R).



Calculate the electric potential for a point on the symmetry axis (taken to be the x-axis).



$$dV = \frac{kdq}{r} = \frac{kdq}{(x^2+R^2)^{1/2}}$$

Note that r is constant.

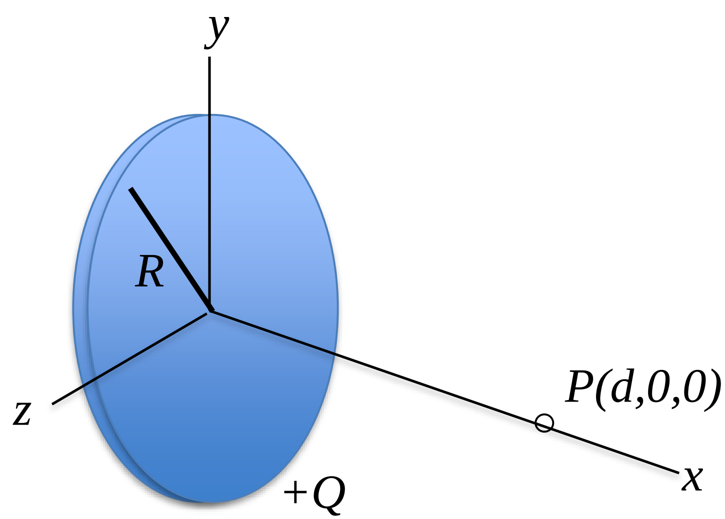
So, $V = \int dV = \int \frac{kdq}{r} = \frac{k}{(x^2+R^2)^{1/2}} \int dq.$

Integrate over Q:

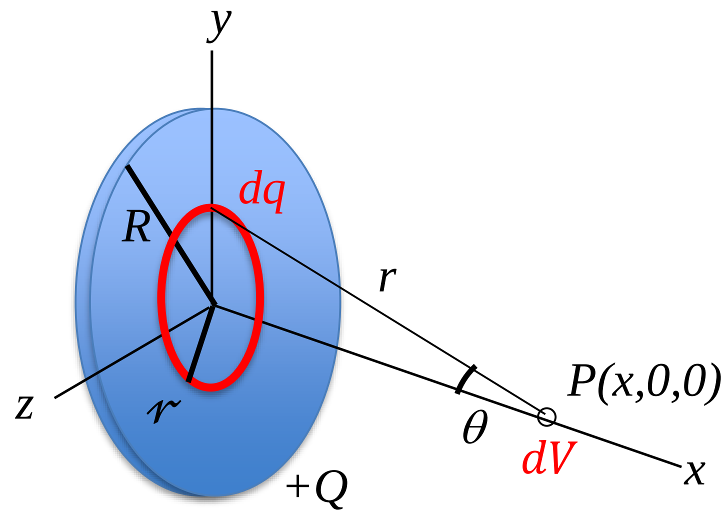
$$V_P = \frac{kQ}{(x^2+R^2)^{1/2}}$$

Example 3:

Uniform Circular Disk of Charge (having total charge $+Q$ and radius R).



Calculate the electric potential for a point on the symmetry axis (taken to be the x-axis).



$$dV = \frac{k\,dq}{r} = \frac{k\sigma\,dA}{r} = \frac{k\sigma 2\pi r\,dr}{(x^2+r^2)^{1/2}}$$

Note that r is NOT constant.

Areal charge density: $\sigma = \dfrac{Q}{\pi R^2}$

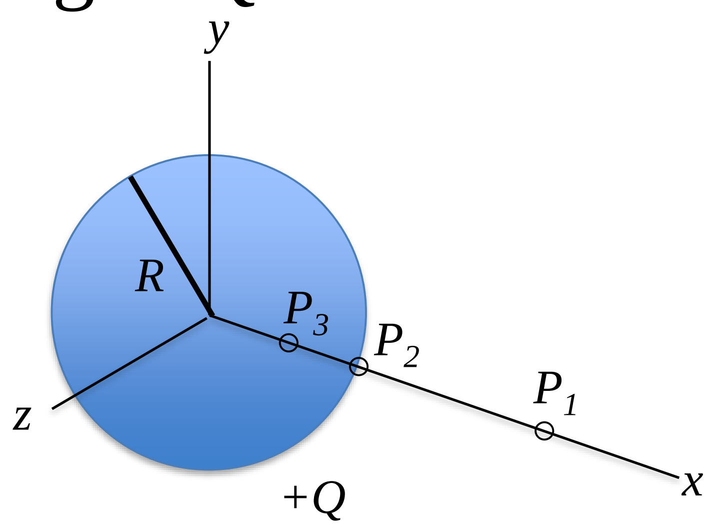
So, $V_P = \int dV = \pi k\sigma \int_0^R \frac{2r\,dr}{(x^2+r^2)^{1/2}}$

Integrate over r: $V_P = \pi k\sigma \left.\dfrac{(x^2+r^2)^{1/2}}{(1/2)}\right|_0^R$

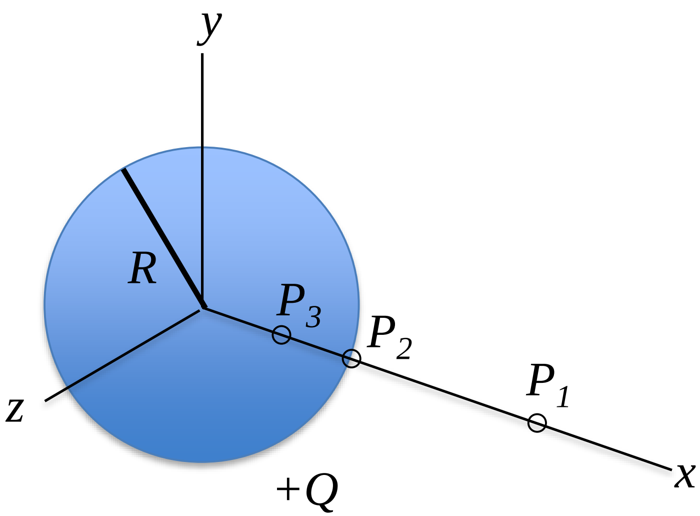
$V_P = 2\pi k\sigma[(x^2+R^2)^{1/2} - x]$ or $V_P = \dfrac{2kQ}{R^2}[(x^2+R^2)^{1/2} - x]$

Example 4:

Uniformly Charged Insulating Sphere of Charge (having total charge $+Q$ and radius R).



Calculate the electric potential outside the sphere, on the surface of the sphere, and inside the sphere.



At Point P_1 $(r > R)$:

$$(\Delta)V_{r>R} = -\int_{\infty}^{r} \vec{E} \cdot d\vec{s} = -\int_{\infty}^{r} \frac{kQ}{r^2} dr$$

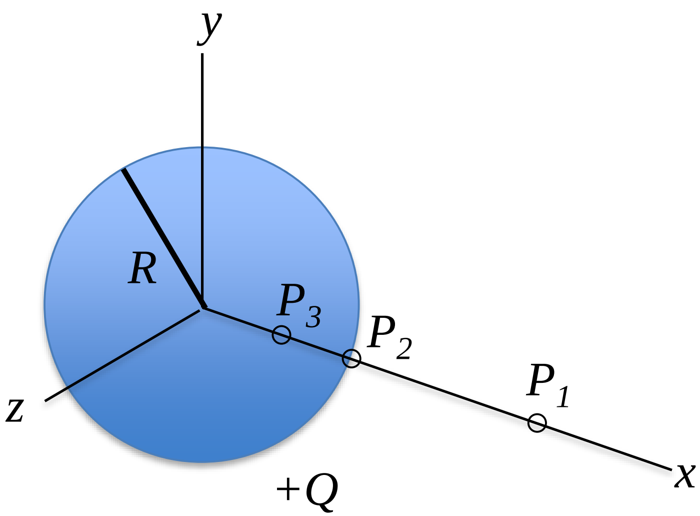
$$V_{r>R} = \frac{kQ}{r}$$

Note that this value for $V_{(r>R)}$ implicitly assumes $V = 0$ at $r = \infty$.

On the surface (P_2) of the insulating sphere, $r = R$, so $V_{r=R} = \frac{kQ}{R}$.

Recall that from Gauss' Law, $\vec{E} = \frac{kQr}{R^3}\hat{r}$ inside a uniformly charged insulating sphere.

$$(\Delta)V_{r<R} = -\int_{R}^{r<R} \vec{E} \cdot d\vec{s} = -\frac{kQ}{R^3}\int_{R}^{r<R} r\,dr = \frac{kQ}{2R^3}(R^2 - r^2)$$



At Point P_3 ($r < R$):

$$(\Delta)V_{r<R} = \frac{kQ}{2R^3}(R^2 - r^2)$$

$$(\Delta)V_{r<R} = \frac{kQ}{2R} - \frac{kQr^2}{2R^3}$$

Note that this integral is the *change in V* between the surface and the interior point. Since the potential at the surface was NOT zero, the value of the potential at P_3 is equal to the ΔV above PLUS the potential on the surface.

That is, $V_{r<R} = (\Delta)V_{r<R} + V_{r=R} = \left(\frac{kQ}{2R} - \frac{kQr^2}{2R^3}\right) + \frac{kQ}{R} = \frac{3kQ}{2R} - \frac{kQr^2}{2R^3}$.

At the center of the sphere, the potential is $V_{r=0} = \frac{3kQ}{2R}$.

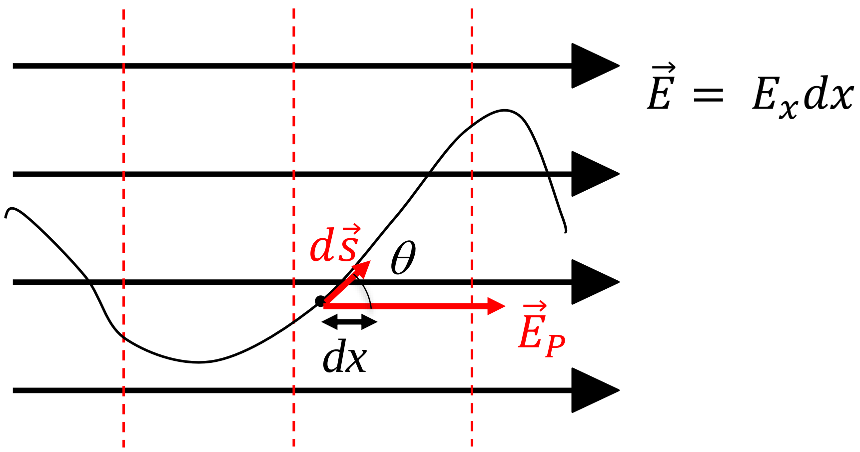
(Note that the potential at the center is NOT zero. This should make sense since the potential is a scalar and adds algebraically rather than as a vector.)

Obtaining **E** from V

If **E** has only 1 component,

then $dV = -\vec{E} \cdot d\vec{s} = -E_x dx$,

where $dx = ds\cos\theta$.



The component of $\vec{E}$ along $d\vec{s}$:

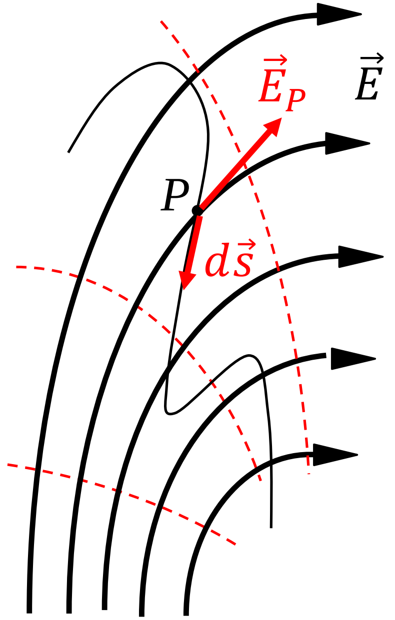
$E_s = E cos\theta = -\frac{dV}{ds}$ which implies $E_x = -\frac{dV}{dx}$

Since E can be chosen to lie along any direction, we can in a similar fashion say that, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

Obtaining **E** from *V*

The component of **E** along any direction is just the negative rate of change of *V* with respect to that direction:

$$E_x = -\frac{dV}{dx}, \qquad E_y = -\frac{dV}{dy}, \qquad \text{and } E_z = -\frac{dV}{dz}$$



In general,

$$\vec{E}_P = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

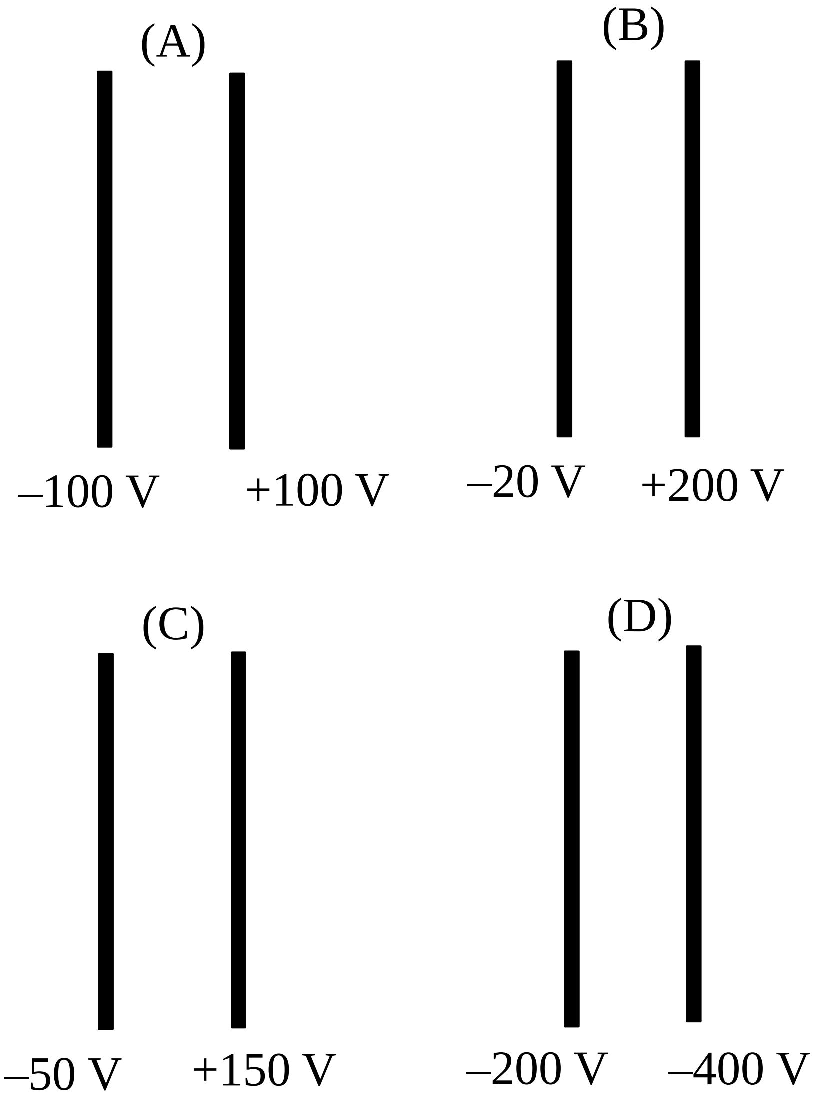
$$\vec{E}_P = -\vec{\nabla}V = -\text{grad(V)}$$

Questions…

Shown are four pairs of charged plates with the potentials shown.

- Rank the pairs according to their greatest **E**-field strength between the plates.

- For which pair(s) (if any) does the E-field point left?



Questions…

The electric potential in a certain region of space is given by $V(x,y,z) = 2xy - 5xz + 3y^2$, where x, y, and z are in meters and V is in Volts.

- Calculate the electric potential at position (1, 1, 1)?

$$V(1, 1, 1) = 0 \text{ V}$$

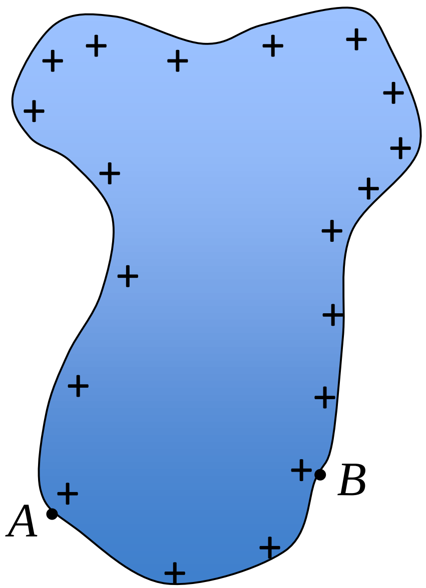
- Determine the electric field at position (1, 1, 1). Write in unit vector notation.

$$\vec{E}(1,1,1) = \left(3\hat{\imath} - 8\hat{\jmath} + 5\hat{k}\right)\frac{\text{V}}{\text{m}}$$

- Calculate the magnitude of the electric field at (1, 1, 1).

$$E(1,1,1) = 9.90 \text{ V/m}$$

Electric Potential of a Charged Isolated Conductor



$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

For the path along the surface that connects points A and B, $\vec{E} \perp d\vec{s}$ at all points on that path.

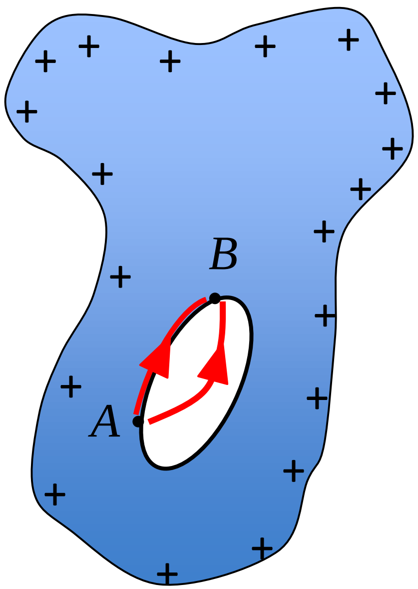
Thus, $V_B - V_A = 0$

This is true for any path along the surface.

$\therefore V$ = constant everywhere on that surface.

Furthermore, since $\vec{E}$ = 0 inside the conductor, $V_A = V_B$ for ANY pair of points everywhere in the conductor including the surface for a conductor in electrostatic equilibrium.

Cavity in the Conductor?



$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

For a conductor with a cavity (such as a spherical shell)…

If $\vec{E} \neq 0$, then there could always be some path found for which $\vec{E} \cdot d\vec{s} \neq 0$. But $V_A = V_B$.

$\therefore$ $\vec{E} = 0$ inside the cavity

(as long as there are no charges placed in the cavity).