Electric Potential
&
Electric Potential Energy

The force on a test charge q_0 in an **E**-field:

$$\vec{F}_{q_0,field} = q_0\vec{E}$$

To move a charge around in an electric field, work is by the field (and by an external agent).

$$W_{field} = -W_{ext.\,agent}$$

Recall: $$dW_{field} = \vec{F}_{q_0,field} \cdot d\vec{s}$$

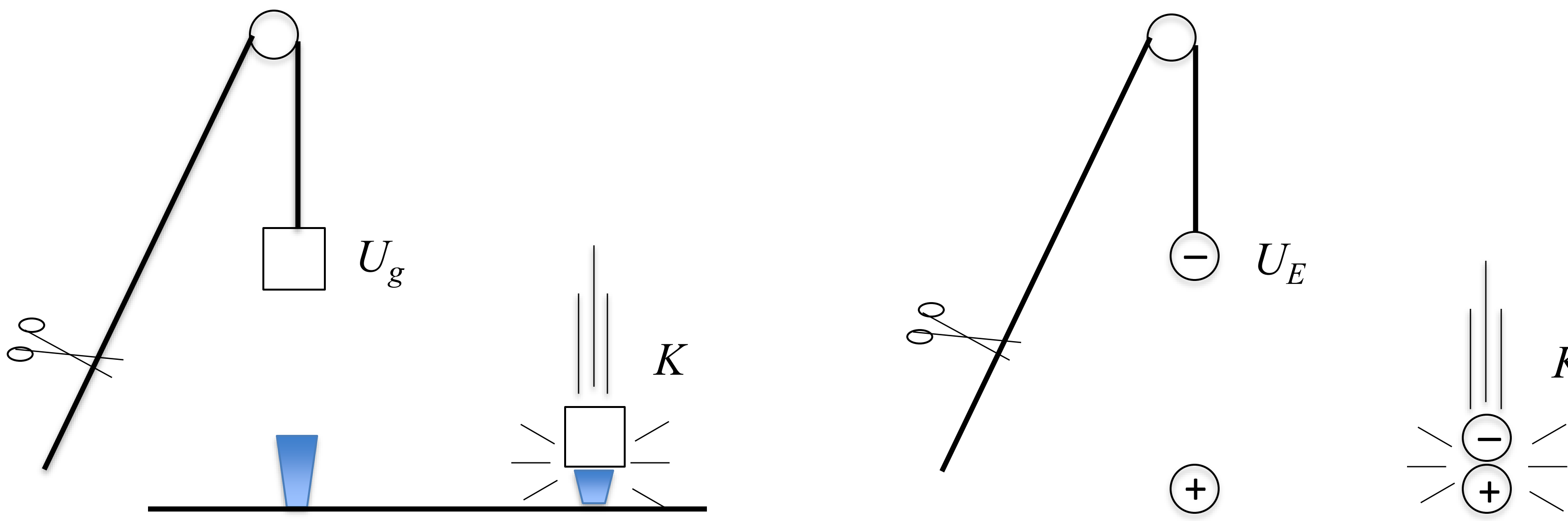
So… $$W_{field} = q_0 \int_A^B \vec{E} \cdot d\vec{s} = -\Delta U$$

where $d\vec{s}$ is a differential displacement along some path from Point A to Point B.

Since $\mathbf{F}_{\mathbf{field}}$ is a conservative force, what does this imply about how ΔU depends on the path taken from Point A to Point B?

No path dependence!

A mechanical analogy may by helpful…



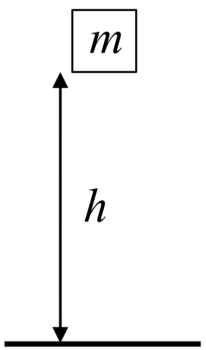
Because "opposites" attract and "likes" repel, assemblies of charge possess (electric) potential energy.

We'll see that like *universal gravitation*, it is often convenient to define $U = 0$ when the charges are infinitely far apart (at $r = \infty$).

Defining *Potential*

Again, consider a mechanical analogy: Gravitational Potential

We can define gravitational potential, "*GP*", as the "gravitational potential energy per unit mass."



$U_g = mgh$

That is, $GP = \frac{U_g}{m} = \frac{mgh}{m} = gh$

Note that *GP* depends only on location in space. There is no gravitational potential *energy* until a mass *m* is placed at a location where the gravitational potential is nonzero.

Twice the mass, then twice the U_g and twice the work to move it.

Ten times the mass, then ten times the U_g and ten times the work.

Defining *Electric Potential*

Similarly, moving a particle with twice the charge in an electric field requires twice as much work.

Moving a particle with ten times the charge means ten times the work.

We define Electric Potential, *V*, as the
the "electric potential energy per unit charge." That is,

$$V = \frac{U_E}{q_0}$$

where *V* is measured in (Joules/Coulomb) and 1 (J/C) ≡ 1 "Volt" and depends only on the location in space.

Potential Difference

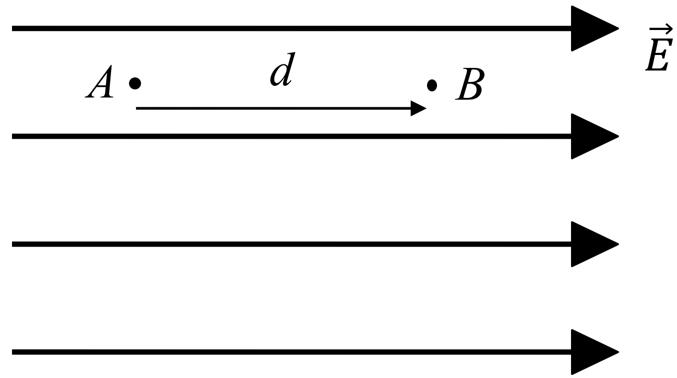
$$\Delta V = V_{final} - V_{initial} = (\Delta U_E/q_0) = (-W_{field}/q_0)$$

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

Again, since the $\mathbf{F}_{\mathbf{field}}$ is conservative,

ΔV is path independent.

Example: Uniform **E**-field

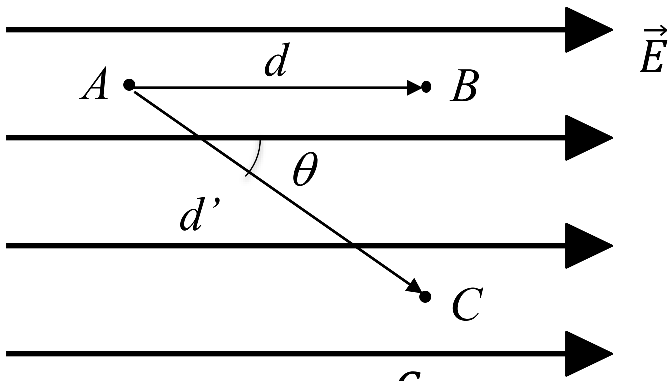


$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} = -Ed$$

$$\Delta U = q_0 \Delta V = -q_0 Ed$$

If $q_0 > 0$, then $\Delta U < 0$ (particle moves to a lower U.)

Uniform **E**-field (cont'd)



$$\Delta V = V_C - V_A = -\int_A^C \vec{E}\cdot d\vec{s} = -E\cdot d'$$

$$= -Ed'cos\theta = -Ed$$

For path A→B → C, $\Delta V = -\text{Ed} + \left(-\int_B^C \vec{E}\cdot d\vec{s}\right) = -\text{Ed}$

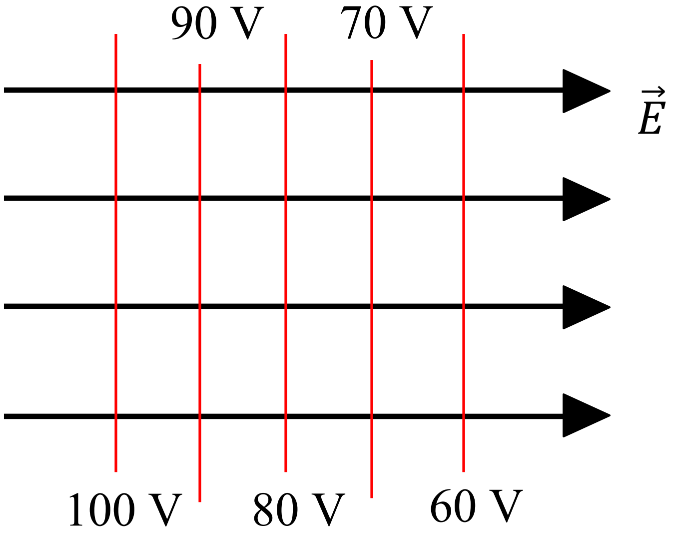
0, since $\vec{E} \perp d\vec{s}$ along path BC.

$\Delta V_{AC} = \Delta \text{V}_{\text{AB}}$ implies $V_C = V_B$.

Equipotential Contours/Surfaces

Paths along which the electric potential is constant are referred to as equipotential contours.

Question:
If **E** points to the right, is V higher or lower on the left side?



Answer: V is higher on the left side.

Question

To move an electron from 90 V to 70 V does an external agent do positive work, negative work or zero work?

A) $W_{ext} > 0$

B) $W_{ext} < 0$

C) $W_{ext} = 0$

D) Not enough information.

Question

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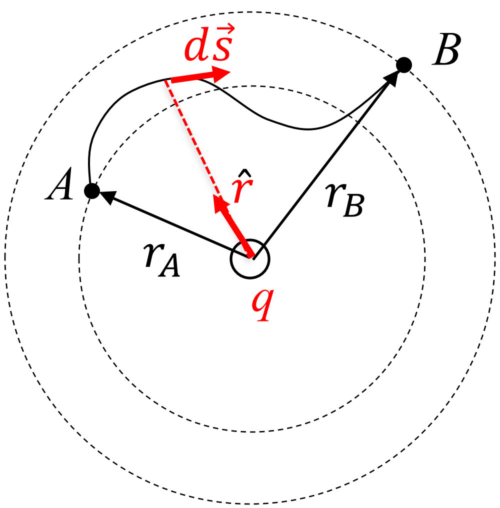
Electric Potential from a Point Charge

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}\,, \text{ where } \vec{E} = \frac{kq}{r^2}\hat{r}.$$

$$\text{Then, } \Delta V = -\int_A^B \frac{kq}{r^2}\hat{r} \cdot d\vec{s}$$

where $\hat{r} \cdot d\vec{s} = ds\cos\theta = dr$ (the projection of $d\vec{s}$ along $\hat{r}$).



$$\Delta V = -\int_A^B \frac{kq}{r^2}dr = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Letting $r_A = \infty$, then $V_A = 0$, and

$$V_B = \Delta V_B = \frac{kq}{r_B}$$

Some things to note:

- For a point charge, V at a location P in space depends only on radial the coordinate r from the charge.

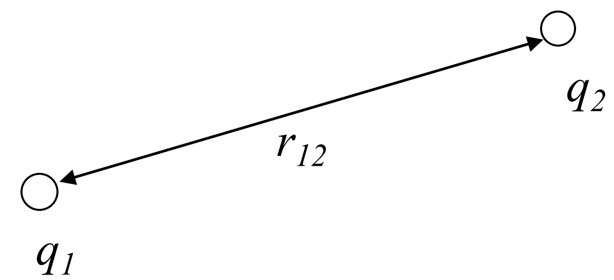
- Electric potential obeys superposition. So, for two or more points charges:

$$V_P = V_{1P} + V_{2P} + V_{3P} + \ldots + V_{NP} = \sum_{i=1}^{N} V_{iP} = k \sum_{i=1}^{N} \frac{q_i}{r_i}$$

- V_P is an algebraic sum rather than a vector sum. Thus, it is generally easier to calculate V_P than to calculate $\mathbf{E}_\mathbf{P}$.

Electric Potential Energy

Now bring a q_2 from ∞ and place it a distance r_{12} from q_1:

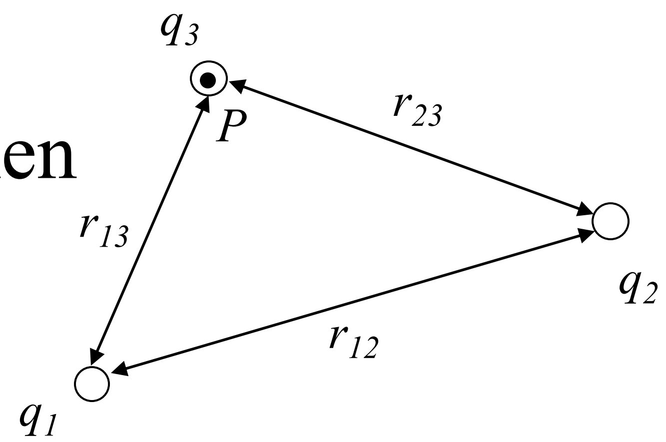


$$U_E = q_2 V_1 = \frac{k q_1 q_2}{r_{12}}$$ for the 2-particle system.

If instead you bring q_2 first, then bring q_1 from ∞ and place it a distance r_{12} from q_2, then…

$$U_E = q_1 V_2 = \frac{k q_1 q_2}{r_{12}}$$ (the same.)

Now, what happens when you then bring in a third point charge and place it at *P* near the first two?



Then $\Delta U = U_f - U_i = q_3 V_P = q_3(V_1 + V_2)$

The total electrical potential energy of the 3-particle system is $U_f = U_i + \Delta U = U_i + q_3 V_1 + q_3 V_2$ or

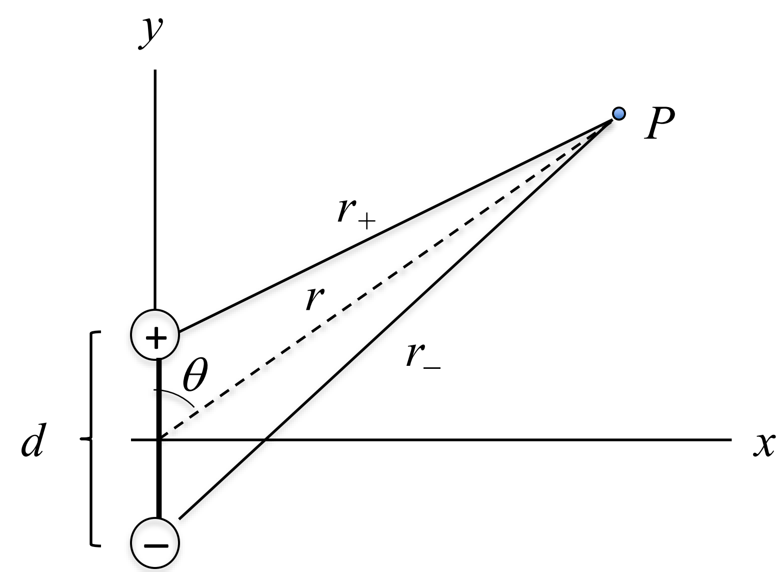
$$U_f = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

To calculate the total U_E, you must sum over every pair of charges. U_E does not obey superposition!

Electric Potential from a Dipole

At P: $V_P = \sum_{i=1}^{2} V_i = V_+ + V_- = k\left(\frac{q}{r_+} + \frac{-q}{r_-}\right) = kq\left(\frac{r_- - r_+}{r_+ r_-}\right)$

For $r \gg d$:
$r_- - r_+ \approx d\cos\theta$
and $r_+ r_- \approx r^2$.



Thus, $V = kq\frac{d\cos\theta}{r^2} = k\frac{p\cos\theta}{r^2}$,
since $p = qd$.

Where $r_+ = r_-$ (as in the case of any point in the xz-plane),
$V = 0$.