Electric Flux
&
Gauss' Law

Calculating Electric Fields

Previously, we learned to calculate **E**-fields of a given charge distribution using Coulomb's Law:

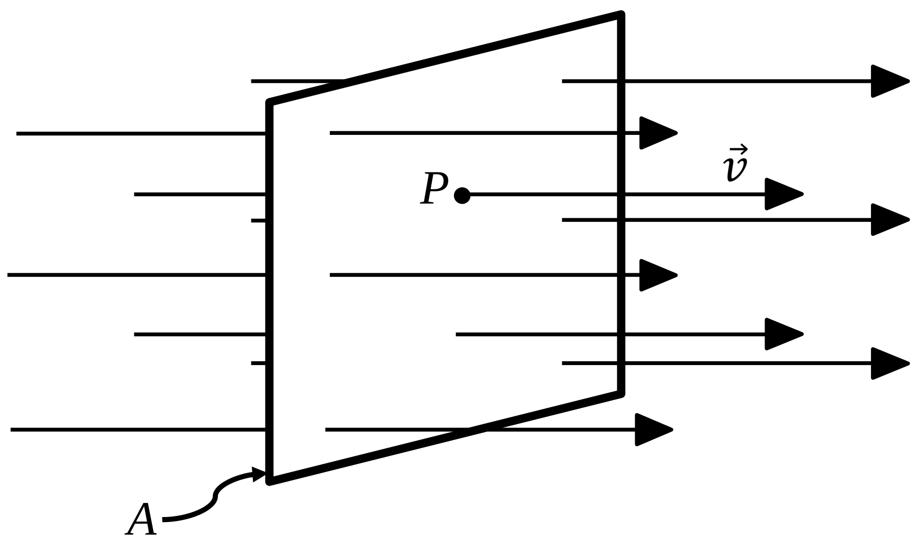
$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

We now introduce an alternative method that is MUCH easier to use when calculating **E**-fields of highly symmetric charge distributions:

Gauss' Law

But first: the concept of FLUX

Consider a stream of water (say in a river or a pipe) moving uniformly with velocity **v**, through a rectangular loop of area *A* and perpendicular to that area.



Let Φ represent the "volume flow rate" through the loop. That is,

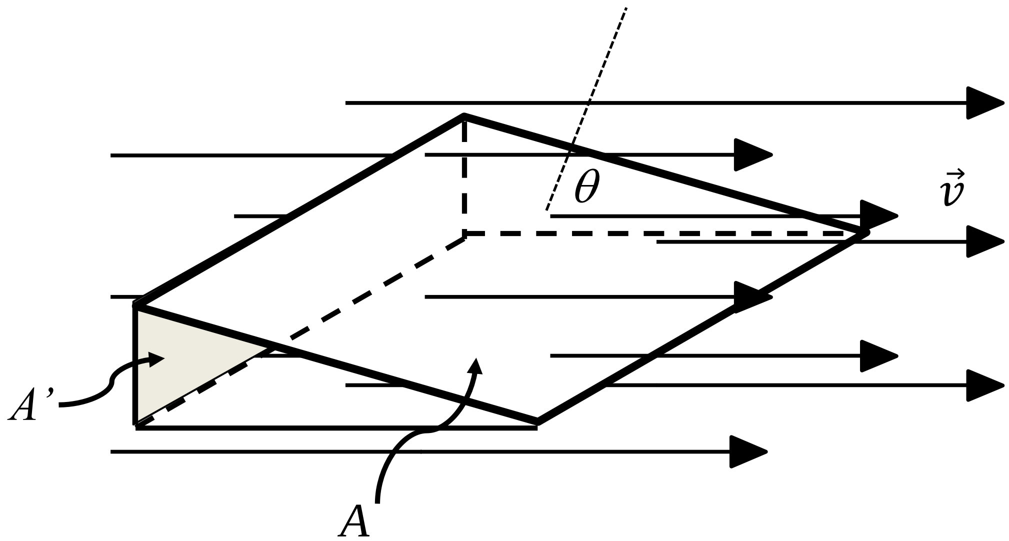
$$\Phi = \frac{\text{volume}}{\text{unit time}}$$

Note that Φ depends on v, A and the angle θ that the loop makes with the direction of flow.

If the area A is not perpendicular to the flow the number of "velocity lines" passing through the loop is less.

However, the # of lines passing through A is equal to the # that passes through the projected rectangular area $A' = A\cos\theta$ which IS perpendicular to the flow.

Note: θ is the angle between **v** and the dashed line which is perpendicular to the plane of the loop.



Since the volume flow rate depends on the component of **v** that is normal (⊥) to the plane of the loop ($v\cos\theta$) we can write the flow rate or volume flux (in this example) as

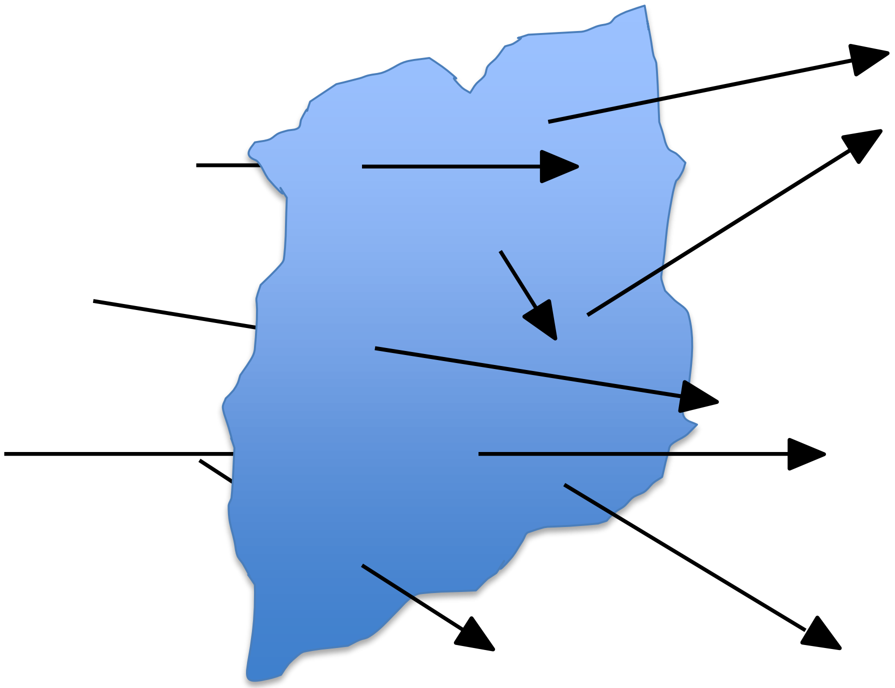
$$\Phi = vA' = vA\cos\theta = \vec{v}\cdot\vec{A}$$

Now consider this:

What if the velocity varied (in magnitude and direction) from point to point?

and/or

What is the area was not flat? (Imagine a crinkled wire mesh so that each little opening of the mesh had a different orientation relative to the flow.)



Then, $\Phi = \int \vec{v} \cdot d\vec{A}$

Electric Flux

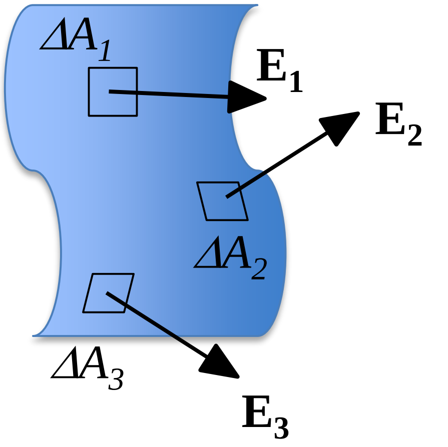
Just as we can define a volume flux of a velocity field of a fluid through an area, we define electric flux (flux of an electric field) as

$\Phi = EA$ for a uniform field passing $\perp$ through a (flat) area A.

If A makes an angle θ with the (uniform) field, then

$$\Phi = EA\cos\theta$$

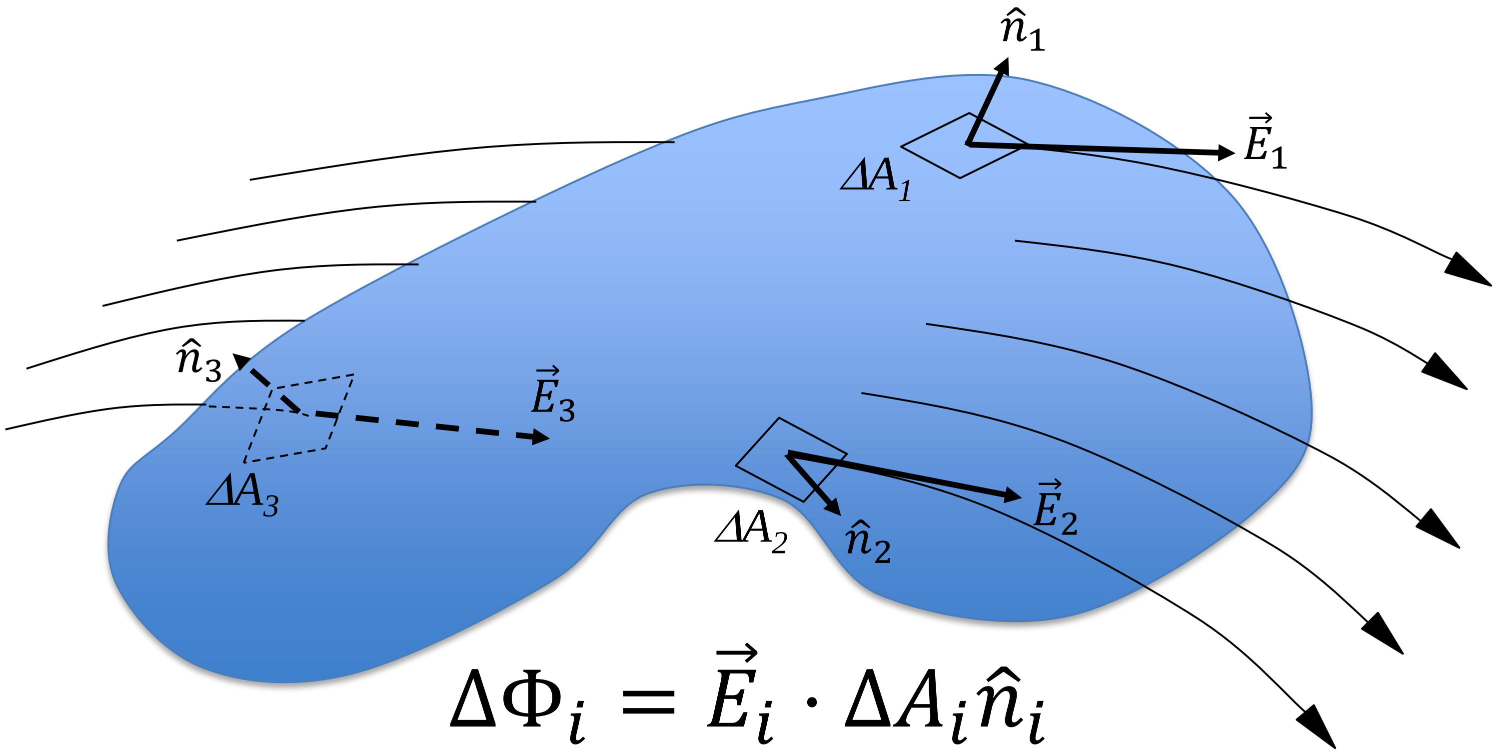
In general, the electric field may vary (in both magnitude and direction) across an arbitrary asymmetric surface:



In this case, the electric flux through the surface by summing over all the *ΔA*'s where each *ΔA* is small enough to consider the **E**-field through that *ΔA* to be constant. Then,

$$\Phi = \lim_{\Delta A_i \to 0} \sum \vec{E}_i \cdot \Delta \vec{A}_i = \int_{surf} \vec{E} \cdot d\vec{A}$$

Flux Through a Closed Surface



$$\Delta\Phi_i = \vec{E}_i \cdot \Delta A_i \hat{n}_i$$

By convention, choose the direction of the various surface elements to be the outward normal, $\hat{n}_i$.

Then $\Delta\Phi_1$, $\Delta\Phi$ are > 0 , and $\Delta\Phi_3 < 0$.

Summarizing…

- Field lines that *leave* the (closed) surface produce a *positive* flux.
- Field lines that *enter* the surface produce a *negative* flux.

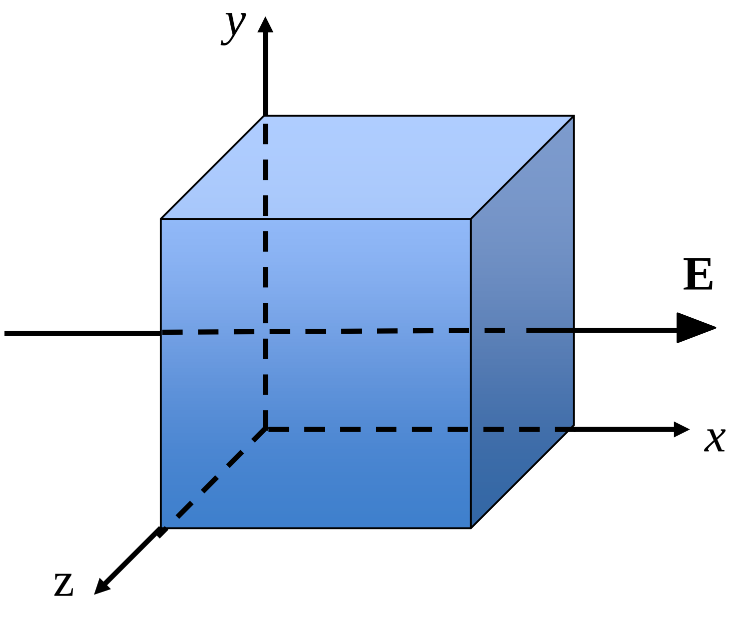
The net flux is $\propto$ (# lines leaving – # lines entering) the closed surface.

So the net electric flux through a closed surface is:

$$\Phi_{net} = \oint \vec{E} \cdot d\vec{A}$$

Example

A uniform electric field points in the +*x*-direction. A cube is situated with a corner at the origin and three edges along the +*x*-, +*y*-, and +*z*-axes. Calculate the electric flux through each face of the cube and the net electric flux through the cube.



Gauss' Law

- A much simpler way to calculate electric fields for highly symmetric charge distributions.
- Relies on the inverse-square relationship of the electric field of point charges.
- Relates the net electric flux through a closed surface (which is called a Gaussian surface) to the net electric charge enclosed by that surface.

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

…where *A* is <u>any</u> surface that encloses a net amount of charge, q_{enc}.

Gauss' law states that Φ_E is independent of the location of the charge within the surface.

This result applies to included several charges or even continuous distributions of charge within the surface. (That is, $q_{enc} = \Sigma_i q_i$ within the surface.)

Continuous Distribution in Surface?

If there is a charge density ρ within the volume V enclosed by the surface, then the total charge enclosed is:

$$q_{enc} = \int \rho dV$$

So, now Gauss' law is now in general form is:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int \rho dV$$

Examples…

- Charge uniformly spread on the surface of a sphere (σ = constant).
- Charge uniformly spread throughout the volume of a sphere (ρ = constant).
- An infinitely long wire with a uniform (linear) charge density, λ.
- An infinite planar sheet with a uniform (surface) charge density, σ.

Conductors in Electrostatic Equilibrium

- Due to electrical repulsion, ALL excess charge must reside on the surface of the conductor.
- No currents inside (electrostatic equilibrium).
- $\Phi_E = 0$, so $q_{enc} = 0$. Thus, **E** = 0 inside.
- **E**-field just outside the surface of a conductor is σ/ε_0, not $\sigma/2\varepsilon_0$. (Why?)