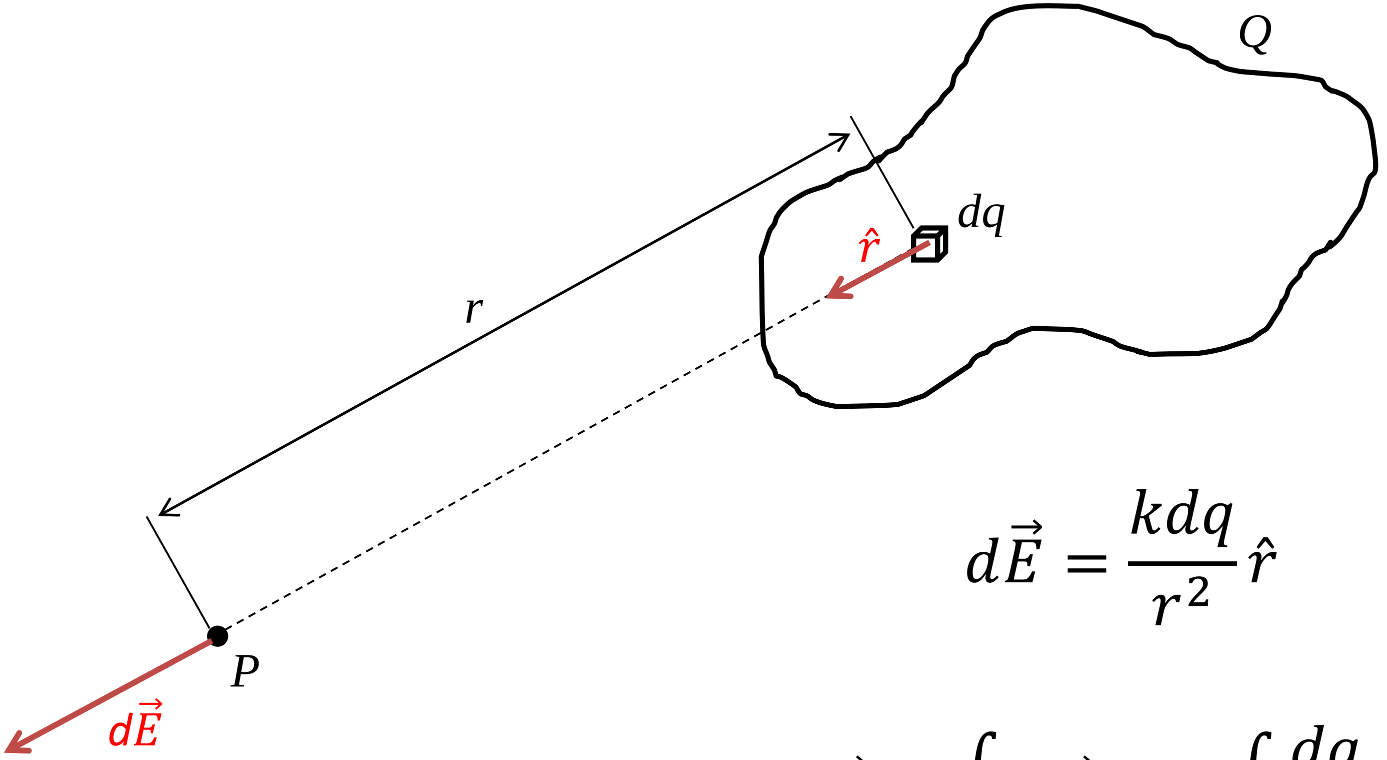
Continuous Distributions of Charge



$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

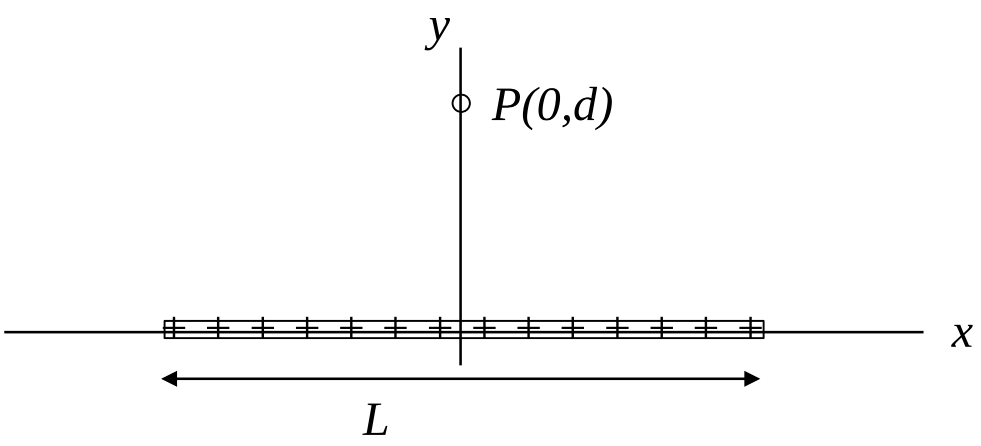
$$\vec{E} = \int d\vec{E} = k\int \frac{dq}{r^2}\hat{r}$$

Charge Density

Type	**Uniform**	**Nonuniform**
Linear	$\lambda = \frac{Q}{L}$	$\lambda = \frac{dq}{d\ell}$
Surface	$\sigma = \frac{Q}{A}$	$\sigma = \frac{dq}{dA}$
Volume	$\rho = \frac{Q}{V}$	$\rho = \frac{dq}{dV}$

Example #1

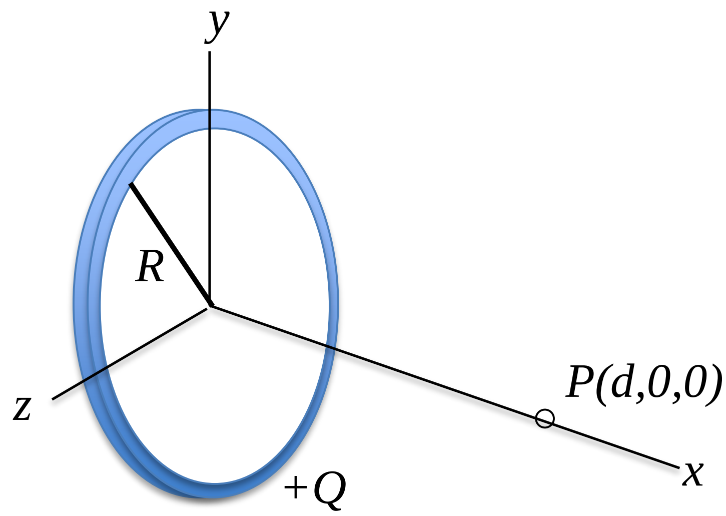
Uniform Finite Line of Charge (having total charge *Q* and radius *R* (centered about the y-axis).



Calculate the **E**-field for a point on the y-axis located a distance d from the line.

Example #2

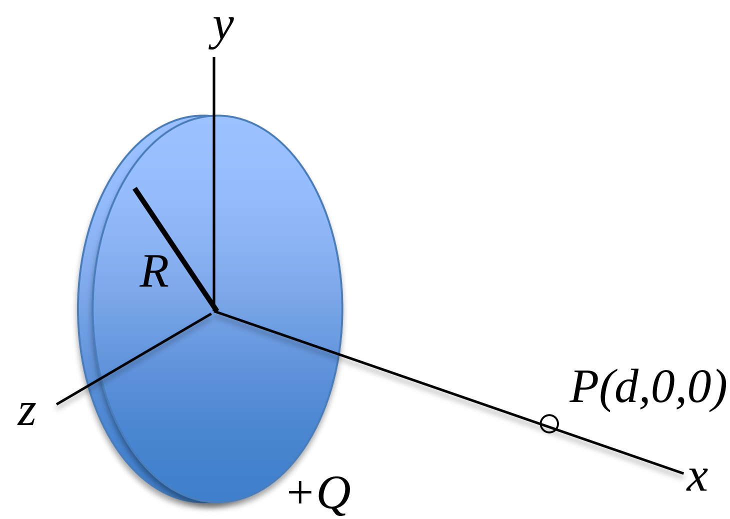
Uniform Ring of Charge (having total charge +*Q* and radius *R*).



Calculate the **E**-field for a point on the symmetry axis (taken to be the *x*-axis).

Example #3

Uniform Circular Disk of Charge (having total charge $+Q$ and radius R).



Calculate the **E**-field for a point on the symmetry axis (taken to be the *x*-axis).

Strategy for Calculating the Electric Field due to a Continuous Distribution of Charge

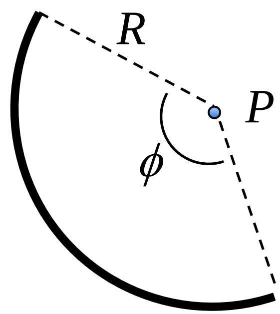
1. Divide the *Q* into *dq* elements.)
2. Look for and exploit symmetry.
3. Apply superposition:

$$\vec{E}_{net} = \sum_i \vec{E}_i \rightarrow \int d\vec{E} = k \int \frac{dq}{r^2}\hat{r}$$

   Replace *dq* with an equivalent expression involving charge density and a geometric quantity (e.g., λdx, σdA, ρdV). All angles and distribution elements must be in terms of the integration variable.
4. Evaluate the integral and check validity in extreme limits.

Example #4

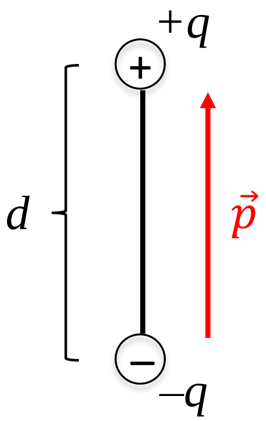
Uniform Circular Arc of Charge (of total angle ϕ and having total charge $+Q$ and radius R).



Calculate the **E**-field at the center of the circle generated by the arc.

Electric Dipole

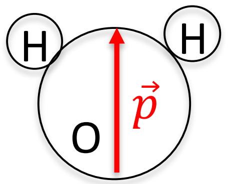
Recall an *electric dipole* consists of two equal but opposite points charges separated by a distance *d*:



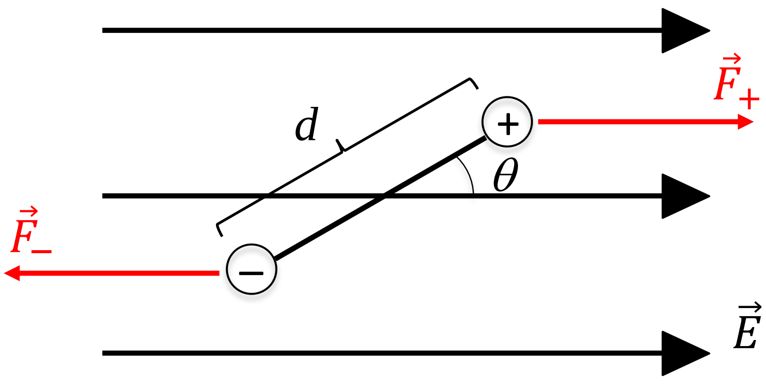
Define electric dipole moment:

$\vec{p} \equiv q\vec{d}$, (directed *from* –*q* *to* +*q*).

Example: Water molecule:



Electric Dipole in an (Uniform) **E**-field



$$|\vec{F}_+| = |\vec{F}_-| = F\ ,$$

$\therefore$ the net force = 0.

However, the net torque $\neq 0$ $\quad\quad \vec{\tau} = \vec{r} \times \vec{F}$

Arbitrarily choosing to compute $\vec{\tau}$ about $-q$,

$$\sum \vec{\tau}_{q\_} = 0 - Fd\sin\theta = -qEd\sin\theta = -pE\sin\theta$$

Since $\vec{p}$ and $\vec{E}$ are vectors: $\vec{\tau} = \vec{p} \times \vec{E}$

Calculating Potential Energy

$$dW_{field} = -\tau d\theta$$

$$dU = -dW_{field} = \tau d\theta = pE\sin\theta d\theta$$

$$\Delta U = \int_{\theta_i}^{\theta_f} pE\sin\theta = -pE\left(\cos\theta_f - \cos\theta_i\right)$$

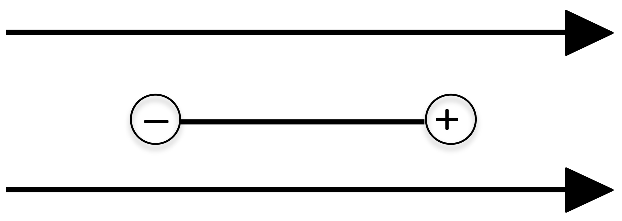
$$= -pE\cos\theta + U_0$$

Take $\theta_i = \frac{\pi}{2}$ rad. Then $U_0 = 0$ at $\theta = \frac{\pi}{2}$ rad.

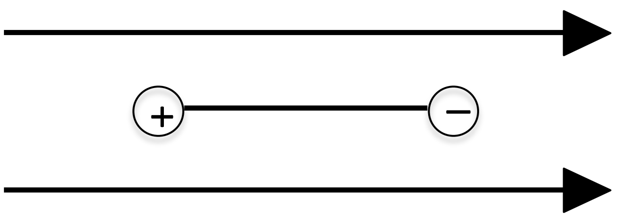
$$\Delta U = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$

Potential Energy of Electric Dipole in a Uniform E-field

U is least when:



U is most when:



U is zero when: