Linear Kinematics & Dynamics

(for rigid objects about a fixed axis)

Describing Rotational Motion

How far?

- Angular Position, θ

  Where something is relative to some defined origin.

- Path, Δs

  The total arc length a point on the object gors through.

- Angular Displacement, $\Delta\theta$

  The straight-line distance from start to end.

Describing Rotational Motion

- Average Angular Speed, $\omega_{ave} = \frac{Ang.\,Displacement}{Unit\ time} = \frac{\Delta\theta}{\Delta t}$
- Instantaneous Angular Speed, $\omega = = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \left|\frac{d\theta}{dt}\right|$
- Average Angular Acceleration, $\alpha_{ave} = \frac{\Delta\omega}{\Delta t}$
- Instantaneous Ang. Acceleration, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \left|\frac{d\omega}{dt}\right|$

Rate of Rotation with Constant Angular Acceleration

For the case when, $\alpha(t)$ = constant, then

$$\Delta\omega = \int_{t_0}^{t} \alpha(t)\,dt = \alpha\int_{t_0}^{t} dt = \alpha\Delta t$$

If we let $t_0 = 0$, then…

$\omega - \omega_0 = \alpha t$ (or $\omega = \omega_0 + \alpha t$)

Angular Displacement with Constant Angular Acceleration

Recall: $\omega(t) = \frac{d\theta}{dt}$

$$d\theta = \omega(t)dt$$

$$\Delta\theta = \int_{\theta_0}^{\theta} d\theta = \int_0^t \omega(t)dt$$

$$\Delta\theta = \int_0^t (\omega_0 + \alpha t)dt$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

The 5 kinematic equations for constant angular acceleration

$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$$

Example #1

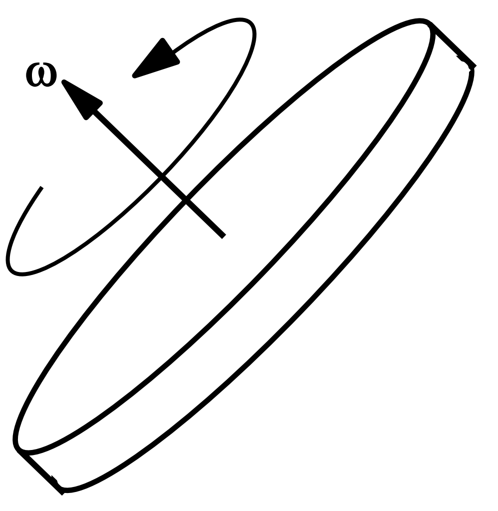
At a given instant, the angular velocity of a spinning disk is –4.6 rad/s. The disk is slowing with an angular acceleration od 0.35 rad/s^2.

- At what time will the disk come momentarily to rest before starting to spin the other way?
- At what time will the the angular displacement of the disk be 5.0 revolutions.
- Calculate the angular velocity at the time found in the previous part.

Angular Quantities as Vectors

The direction of angular <u>velocity</u> and acceleration is established by the "right-hand rule" (RHR).

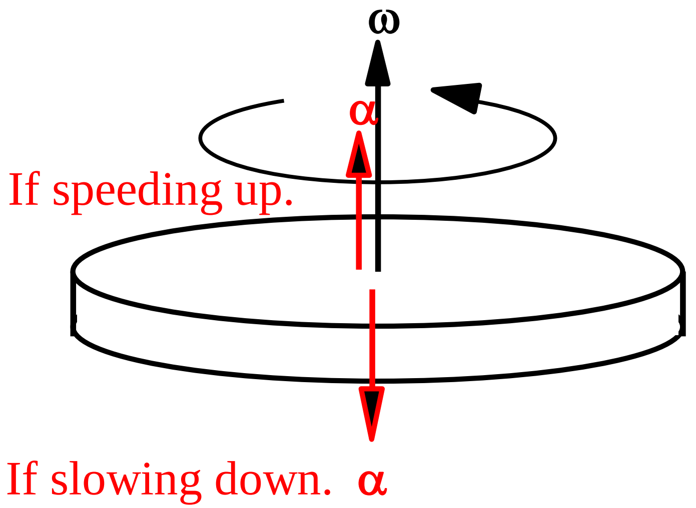
Curl your fingers in the direction of the rotation (using your right hand). Your thumb will point in the direction of ω.



Angular Velocity & Acceleration

If the rate of rotation is increasing, then the angular acceleration points in the same direction as the angular velocity.

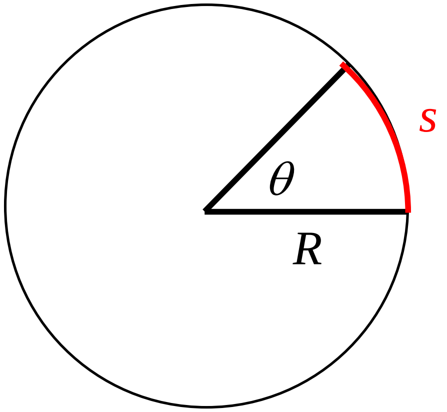
If the rate of rotation is decreasing, then the angular acceleration points in the direction opposite to the angular velocity.



Relationship between the linear variables and the rotational variables

Definition of radian measure:

$$\theta = \frac{s}{R}, \ (\theta \text{ in radians})$$



$$s = R\theta$$

For fixed R: $\frac{ds}{dt} = R\frac{d\theta}{dt} \Rightarrow v = R\omega$

Similarly: $\frac{dv}{dt} = R\frac{d\omega}{dt} \Rightarrow a = R\alpha$

Example #2

Astronaut training involves subjecting the trainees to extreme accelerations in a centrifuge. In this example, an astronaut is spun in a centrifuge having a radius of 15 m with a constant angular speed.

- Calculate the angular speed of the centrifuge if the astronaut is to experience 11*g* of acceleration.
- Calculate the linear speed of the asronaut.

Kinetic Energy of Rotation

Important Note about Acceleration

Recall that an object moving in a circle with constant speed (a.k.a., *uniform circular motion*) already has a *centripetal acceleration* that points *radially inward*, where $a_c = v^2/R = \omega^2 R$.

If the spinning object changes its rate of rotation (a.k.a., NON-uniform circular motion), the object will also have a *tangential acceleration* that points *tangent* to the circle, where $a_t = R\alpha$.

$$\vec{a} = -\omega^2 R\,\hat{r} + R\alpha\,\hat{t}$$