Systems of Particles

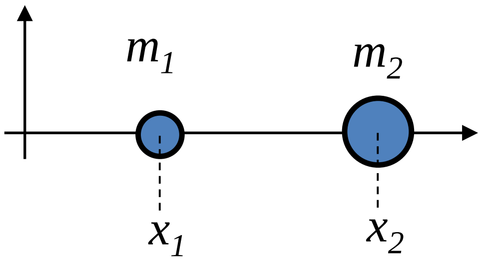
Center of Mass

Defining *Center of Mass*

Consider an extended object that is flipped across a room. Every part of the object moves in a very different way with respect to every other point.

Every object has a special point which does move as though it was a point particle with a total mass equal to that of the extended object.

Center of mass (in 1-D for a 2-particle system)



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Extend to the case of *n* particles:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ \ldots + m_n x_n}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M}$$

If the particles are distributed in 3 dimensions, we can generalize this to:

$$x_{cm} = \frac{\sum_i m_i x_i}{M} \qquad y_{cm} = \frac{\sum_i m_i y}{M} \qquad z_{cm} = \frac{\sum_i m_i z_i}{M}$$

where again, $M = \sum_i m_i$.

Example #1

Three masses:

m_1 = 1.00 kg located at (2.00 m, 0, 0),

m_2 = 2.00 kg located at (1.00 m, 2.00, 0),

m_3 = 3.00 kg located at (4.00 m, 3.00, 0).

- Locate the center of mass.

Continuous Distributions of Mass

What if the system gets so large that it has too many particles over which to sum?

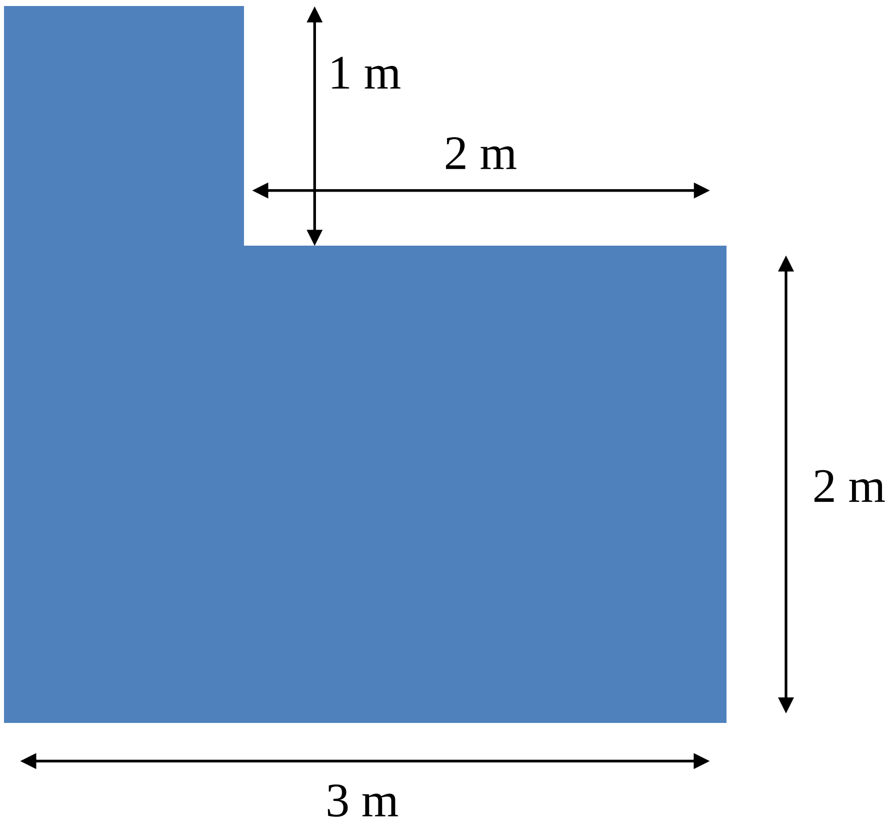
Then the particles are just infinitesimal mass elements *dm*. In that case, we replace the summation by an integration.

$$x_{cm} = \frac{\int x dm}{M} \qquad y_{cm} = \frac{\int y dm}{M} \qquad z_{cm} = \frac{\int z dm}{M}$$

Example #2

Consider the thin metal sheet shown.

- Locate the center of mass.



Example #3

Uniform rod of mass *M* and length *L*.

We know by symmetry that $x_{cm} = \frac{L}{2}$.

Let's find x_{cm} using integration. Mathematically, the rod has constant *linear mass density* $\mu(x) \equiv \frac{M}{L}$.

$$x_{cm} = \frac{\int_0^L x dm}{\int_0^L dm} = \frac{\int_0^L x\mu dx}{\int_0^L \mu dx} = \frac{L}{2}$$

Example #4

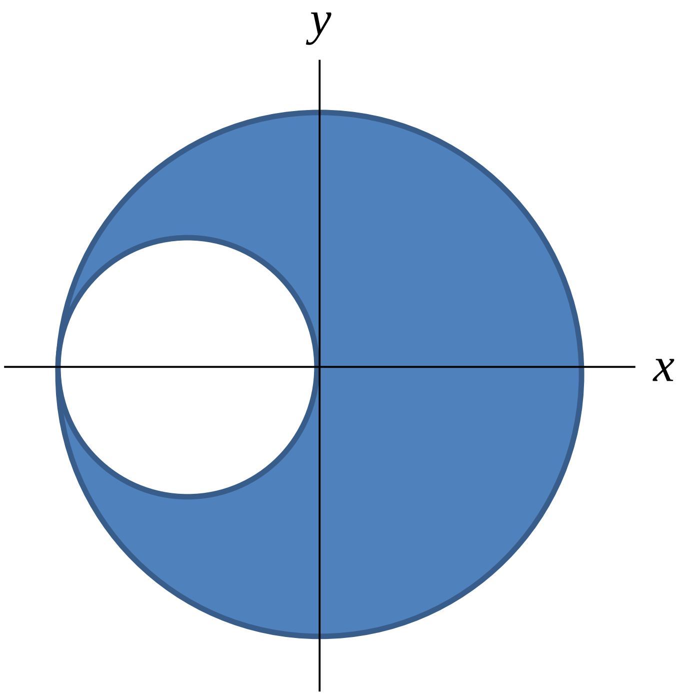
A non-uniform rod of length L and a linear mass density that varies according to $\mu(x) = Cx$, where C is a constant.

$$x_{cm} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x\mu(x)dx}{\int_0^L \mu(x)dx} = \frac{\int_0^L Cx^2dx}{\int_0^L Cxdx} = \frac{2L}{3}$$

Example #5

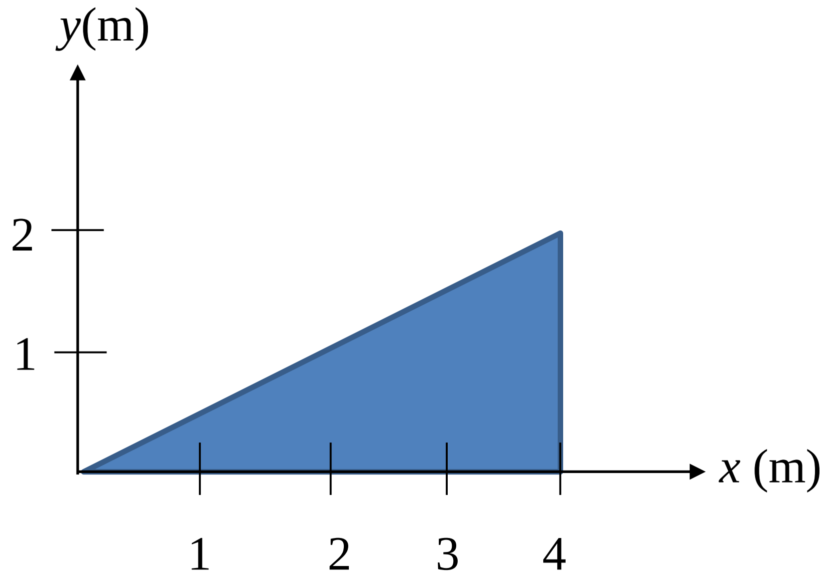
Uniform circular plate with a circular section removed as shown. The radius of the hole is R and the radius of the plate is $2R$.

- Locate the center of mass.



Example #6

Find the center of mass of the triangular figure shown below.



Newton's 2nd Law & Work-Energy

$$\sum \vec{F}_{ext} = M\vec{a}_{cm}$$

and

$$W_{ext} = \int_{\vec{r}_{cm,i}}^{\vec{r}_{cm,f}} \vec{F}_{ext} \cdot d\vec{r}_{cm} = K_{cm,f} - K_{cm,i}$$

where $K_{cm} = \frac{1}{2} M v_{cm}^2$

Example #7

Consider three masses:

m_1 = 4.00 kg located at (−2.00 m, 3.00, 0),

m_2 = 4.00 kg located at (1.00 m, −3.00, 0),

m_3 = 3.00 kg located at (4.00 m, 2.00, 0),

that experience the forces (respectively):

$\mathbf{F_1}$ = 6.00 N **i**,

$\mathbf{F_2}$ = 14.0 N **j**, and

$\mathbf{F_3}$ = 12.0 N directed 45o above the +x-axis.

- Calculate the magnitude and direction of acceleration of the center of mass.

Example #8

A person standing on a skateboard (combined mass of 55kg) initially at rest pushes away from a wall. When the person's hand breaks contact with the wall, the center of mass of the "person-skateboard" system has moved 18 cm and the velocity of the center of mass is 1.1 m/s.

- Calculate the work done by the wall on the person's hand while pushing away.
- Calculate the work done by the wall on the "person-skateboard" system.