Momentum & Impulse

Analyzing Collisions

Defining *momentum*…

It is harder to stop a freight train than it is to stop a car going at the same speed. (Why?)

The train has more *momentum* than the car.

Conceptually, momentum is “inertia in motion.”

Mathematically, momentum = mass X velocity

$$\vec{p} = m\vec{v}$$

Newton's 2nd Law in terms of Momentum

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \ (m = \text{constant})$$

If $\sum \vec{F} = 0$, then $\vec{p} = \text{constant}$

For a system of particles: $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ ...$

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

If $\sum \vec{F}_{ext} = 0$, then $\vec{P} = \text{constant}$

Example #1

A stream of bullets each having a mass m and speed v are fired at a block of mass M that is initially at rest on a frictionless horizontal tabletop.

- Calculate the speed of the block after is has absorbed 8 bullets.

Example #2

Two masses (m_1 and m_2) on a frictionless horizontal tabletop are connected by a spring. The masses are pulled apart and then released.

- What fraction of the total kinetic energy will each block have?

What is a *Collision*?

An isolated event in which a relatively strong force acts on each particle for a relatively short time (e.g., two billiard balls).

Applying Newton's 2nd law, $\sum \vec{F} = \frac{d\vec{p}}{dt}$:

$$d\vec{p} = \vec{F}(t)dt$$

$$\Delta\vec{p} = \vec{F}\Delta t \text{ (constant force)}$$

$$\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt \text{ (variable force)}$$

Defining *Impulse*

$$\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{p}_f - \vec{p}_i \equiv \vec{J} \text{ (Impulse)}$$

or $\vec{J} = F_{ave}\Delta t$

That is, an impulse is a *change in* momentum.

Example #3

Example: A 140-gram ball hits a wall normal to its surface at a speed of 39 m/s and rebounds straight back with the same speed in a time of 1.2 ms.

- Calculate the impulse that the wall exerts on the ball to reverse its direction.
- Calculate the force and the average acceleration of the ball during the collision.
- Calculate the work done on the ball by the wall.

Types of Collisions

Elastic collision:

Particles rebound with no lasting deformation or generation of heat. ($\vec{p}$ and K are both conserved.)

Inelastic collision:

Particles permanently deform, generate heat, and possibly stick together. ($\vec{p}$ is conserved, but K is not conserved.)

Totally inelastic collision:

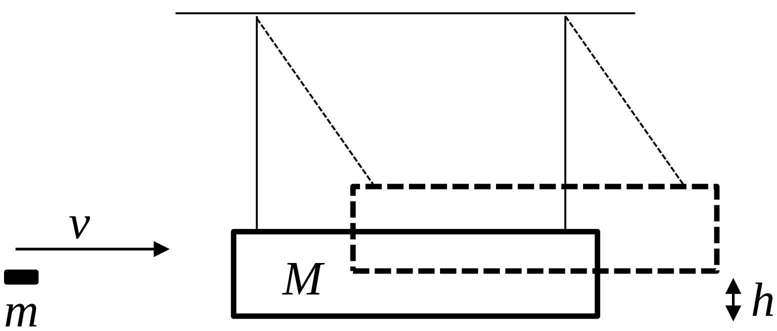
The particles stick together after the collision.

(Again, $\vec{p}$ is conserved, but K is not conserved.)

1-D Inelastic Collision Example

A ballistic pendulum was an old-fashioned way to measure the speed of bullets. A bullet having a mass of 9.5 grams hit a large block having a mass of 5.4 kg causing it to swing up to a height of 6.3 cm.

- Calculate the speed of the bullet.



1-D Elastic Collisions

Conservation of momentum and kinetic energy:

p: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

K: $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Given m_1, m_2, v_{1i} and v_{2i}, solve for v_{1f} and v_{2f} :

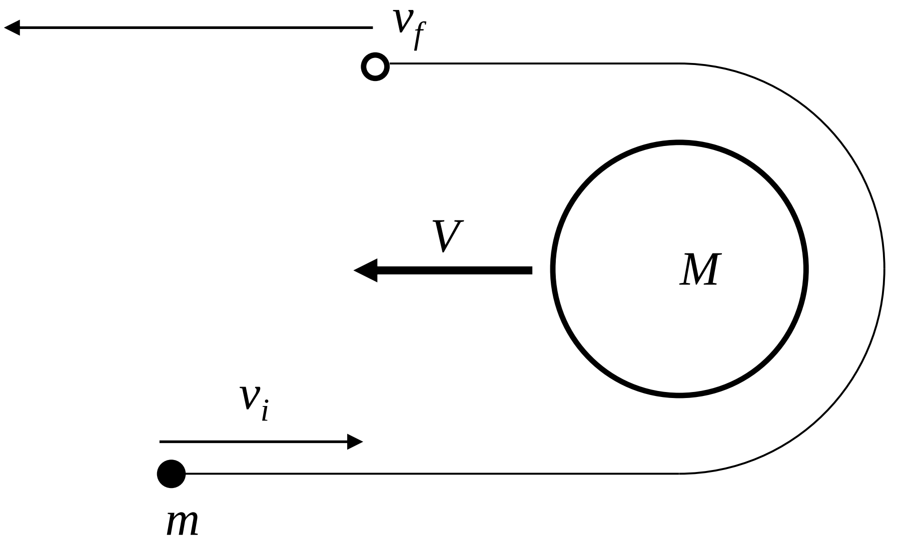
$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

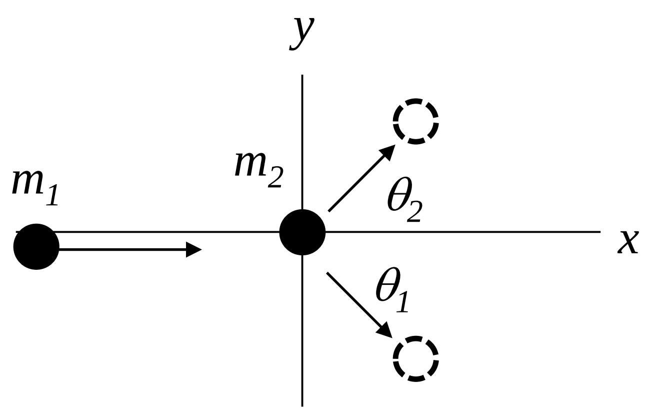
Example #4: Slingshot Effect

A space probe initially moving right at 14 km/s approaches a planet moving left at 13 km/s. The satellite orbits the planet for half an orbit and rebounds to the left.

- Determine the speed of the satellite after the encounter.



2-D Elastic Collisions



$$p_x: \quad m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cos\theta_2$$

$$p_y: \quad 0 = -m_1 v_{1f}\sin\theta_1 + m_2 v_{2f}\sin\theta_2$$

$$K: \quad \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

Example: #5 (2-D Totally Inelastic)

A 830-kg car traveling east and a 550-kg car traveling north collide at an intersection and stick together. V1 = 6.2 km/hr, V2 = 7.8 km/s.

- Determine the final velocity of the wreckage immediately after the accident.
- Calculate the fractional change in the kinetic energy that occurs.

Example #6

A prankster places a firecracker inside of a coconut of mass M that is initially at rest on a frictionless horizontal tabletop. The explosion blows that fruit apart into three pieces.

Given that $M_C = 0.30M$, $M_B = 0.20M$, $v_C = 5.0$ m/s, and the angles shown, calculate v_A and v_B.