Conservation of Energy

Putting things into perspective…

1. Define position, velocity, acceleration…
2. Derive kinematic equations.
3. Apply Newton's Laws (esp. $\sum \vec{F} = m\vec{a}$)
   a. Draw (Extended) Free-body Diagram.
   b. Resolve forces along coordinate axes
   c. Do algebra

But…what if we don't know the forces involved (or even we do, they are too complicated)?

Basic methods for analyzing introductory mechanics problems

1. Newton’s Laws (esp. $\sum \vec{F} = m\vec{a}$)

2. Conservation Laws (Energy & Momentum)

3. Relationship between a conservative force and a potential energy.

What is *Energy*?

There is a particular fact about nature that (with no observed exceptions thus far) governs all natural phenomena: Conservation of Energy

There is a particular quantity discovered by physicists (called *energy*) that remains constant in the grand changes that the universe undergoes.

It is an abstract idea, for it does not describe any mechanism. It just an odd fact that we can calculate a number, and after nature is finished dong “its thing,” we can recalculate that number and find that it is the same.

Types of Energy

- Kinetic energy (energy of motion)
- Potential energy (energy of position or configuration) such as:

Gravitational	Elastic
Electrical	Magnetic
Chemical	Nuclear
Mass	

- Thermal energy (heat)
- Radiant energy (light)

Mechanical Energy

Total *mechanical energy*, $E = K + U$, where
K = kinetic energy and U = potential energy.

In the absence of dissipative effects (e.g, friction),
E = constant. That is $\Delta E = 0$. So, $\Delta K + \Delta U = 0$.

If $W = \Delta K$ (Work-Energy Theorem) then, $W = -\Delta U$.

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx \quad \text{(1-D motion)}$$

Apply to Two Particular Cases

- Gravity: $F = -mg$

$$\Delta U = U(y_f) - U(y_i) = -\int_{y_i}^{y_f} -mg\,dy$$

$$\Delta U = mgy_f - mgy_i = mg\Delta y$$

- Spring: $F = -kx$

$$\Delta U = U(x_f) - U(x_i) = -\int_{x_i}^{x_f} -kx\,dx$$

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

How to go the other way?

As just seen, if we know *F(x)*, we can get *U(x)*:

$$\Delta U = U(x_f) - U(x_i) = -\int_{x_i}^{x_f} F(x)dx$$

But what if we know *U(x)* and want to find *F(x)*?

$$F(x) = -\frac{dU}{dx} \quad \text{(1-D Motion)}$$

Energy Conservation Examples

Particle near Earth's surface in free fall:

$$E = K + U = \frac{1}{2}mv^2 + mgy \ \ (= \text{constant})$$

Mass attached to a spring:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 (= \text{constant})$$

What about Friction?

Consider a object of mass *m* having an initial speed *v* sliding to rest on a rough tabletop.

We cannot get the lost *K* back. If stored, it is inaccessible. As the block slides, the surface heats up as the organized *K* of the block is converted into the disorganized random motions of the atoms in the tabletop and the block. (That is, they get warmer.)

We cannot reverse the process. Therefore, we cannot associate a potential energy with friction.

Example #1

A mass *m* initially at rest is dropped from a height *h* above the floor.

- Calculate the speed of the mass just as it hits the floor.

Example #2

Starting with an initial speed of v_o, a mass m slides down a curved track and arrives at the bottom with a speed v.

- Calculate the height h above the floor where the mass started.

Example #3

A block of mass m = 1.7 kg slides on a frictionless horizontal tabletop with an initial speed v_o = 2.3 m/s and hits a relaxed spring that has a spring constant k = 320 N/m.

- Calculate the maximum compression of the spring.
- For what value of x will the energy be equally divided between kinetic and potential.
- Calculate the speed of mass at this position.

Example #4

A bungee jumper of mass 61.0 kg leaps from a bridge attached to the a bungee cord that has an unstretched length of 25.0 m. The bridge is 45.0 m above the surface of the water. The bungee cord has a spring constant of 160 N/m.

- Calculate the height above the water that the feet of the jumper reach when the cord is stretched to its maximum.

Conservative Forces

Conservative Force:

- The work done on a particle that moves through a roundtrip is zero.
- The work done on a particle that moves between two points is the same for any path connecting the two points.

Examples:

gravity, ideal spring, electrostatic

Nonconservative Forces

Nonconservative Force:

- The work done by the force depend on the path.

Examples:

kinetic friction, drag, a non-ideal spring

If friction is present…

$$W_{Total} = \sum W_c + W_f = \Delta K$$

$$W_f = \Delta K - W_c = \Delta K + \sum \Delta U$$

Change in mechanical energy:

$$\Delta E = \Delta K + \sum \Delta U$$

(That is, if E changes, W_f is equal to that change.)

Where is the "missing" energy?

If nonconservative forces act, mechanical energy is no longer conserved. The "missing" energy goes into the random disordered motion of the atoms. It is disorganized kinetic energy that is referred to as *internal energy* (or thermal energy).

Define: $\Delta U_{int} = -W_f$

Conservation of energy becomes:

$$K + \sum U + U_{int} = 0$$

Example #5

A roller coaster with its passengers has a mass of 1000 kg and an initial speed of 4.0 m/s at the top of a hill that is 10 m above the ground.

- Calculate the speed of the of the roller coaster at the bottom of the hill assuming no friction.

Suppose that because of friction, the speed at the bottom is 13.8 m/s.

- Calculate the thermal energy developed during the descent.

Example #6

The potential energy of a 2.0-kg object as a function of position is given by:

$$U(x) = (6.5\ \text{J/m})|x|.$$

The total mechanical energy of the particle is 10 J.

- Determine how far the object can move along the x-axis in either direction.
- Calculate the maximum speed of the object.