Work & Kinetic Energy

Introducing two new concepts that will provide insight into methods of solving Newtonian mechanics problems.

The Definition of *Work*

Merriam Webster's Encyclopedic Unabridged Dictionary of the English Language gives 53 definitions:

1. Exertion of effort directed to produce or accomplish something; Syn. LABOR, DRUDGERY, TOIL refer to exertion of body or mind in performing or accomplishing something.

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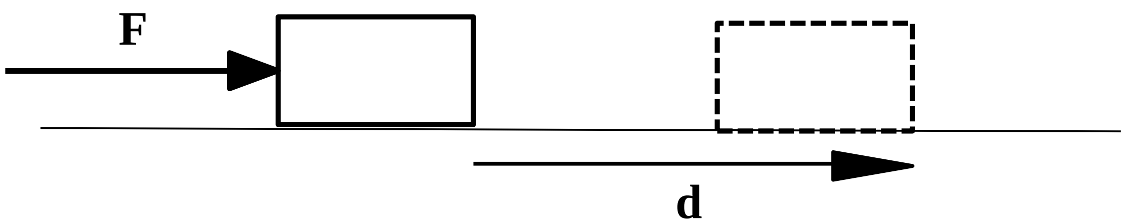
11. (Physics) Force times the distance through which it acts ; spec. the transference of energy equal to the product of the component of the force that acts in the direction of the motion of the point of application of the force and the distance through which the point of application moves.

1-Dimension, Constant Force

Consider a force applied to the crate shown below:



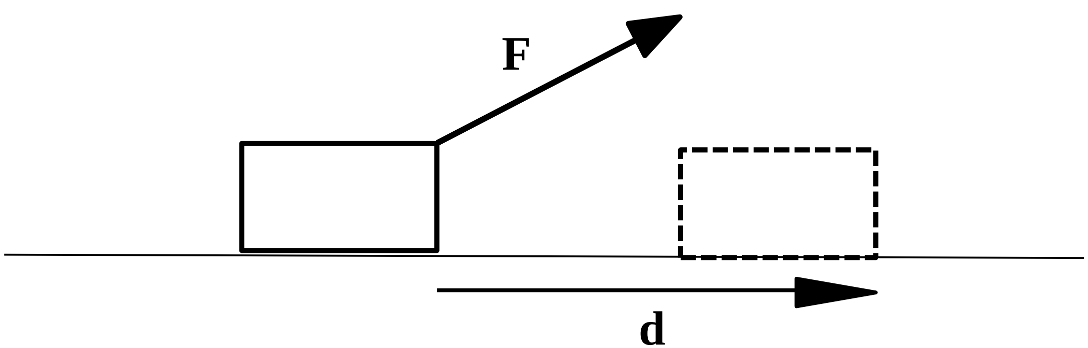
Work = Force x Displacement

$$W = Fd$$

Units: 1 Joule (J) = (1 N)(1 m) = 1 N-m

What if the $\vec{F}$ & $\vec{d}$ are not parallel?

To generalize the situation in which the there is some arbitrary angle between the force and the displacement (such as the situation shown below) we need to be familiar with how to multiply vectors.



Two ways to multiply vectors:

Scalar (or “Dot”) Product:

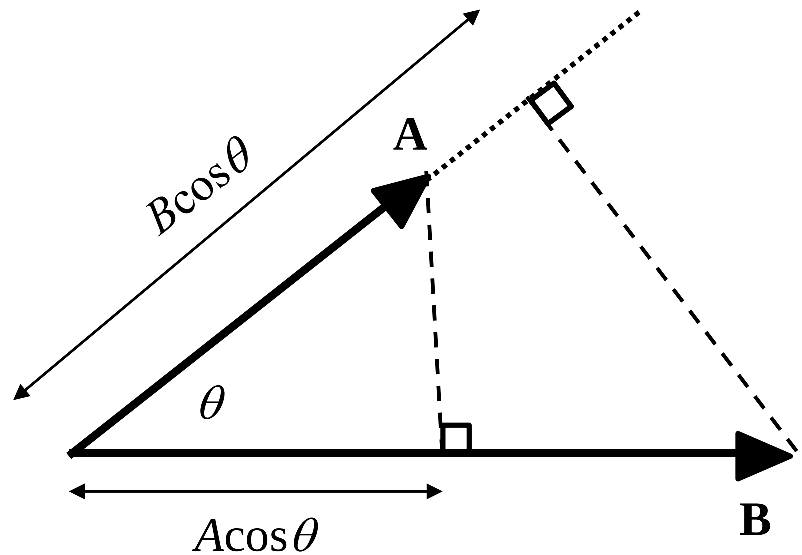
- Conceptually describes the tendency for two vectors to be parallel.
- Examples of quantities: Work, Power, Flux

Vector (or “Cross”) Product:

- Conceptually describes the tendency for two vectors to be parallel.
- Examples of quantities: Torque, Angular Momentum, Magnetic force

Scalar (or "Dot") Product

The scalar component of one vector along the direction of the other vector times the magnitude of the other vector.

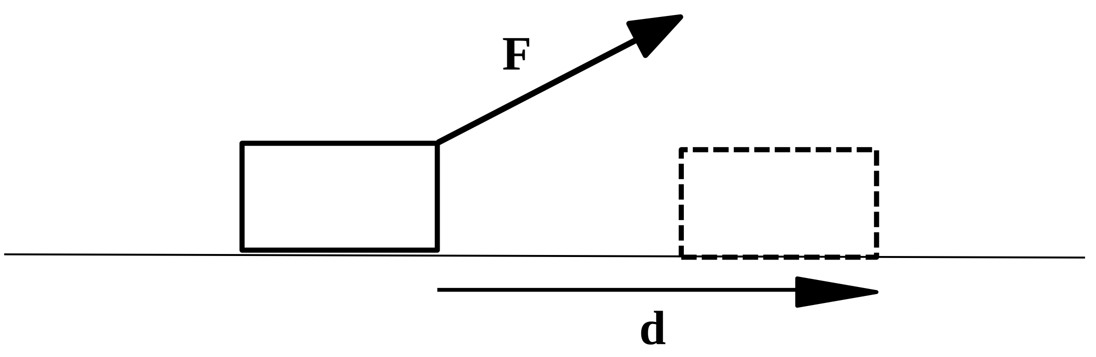


$$\vec{A} \cdot \vec{B} = AB\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

Constant Force, Arbitrary Direction

Work = Force "Dot" Displacement

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = F_x x + F_y y + F_z z$$



Example #1

A block of mass m = 25.0 kg is pushed up a ramp making an angle 25.0^{o} by an applied force F = 200 N parallel to the ramp. The frictional force is f_k = 80 N. The block move a distance d = 1.5 m up the ramp.

- Calculate the work done by the applied force, friction, the normal force and gravity.
- Calculate the total work done by the net force that acts on the block.

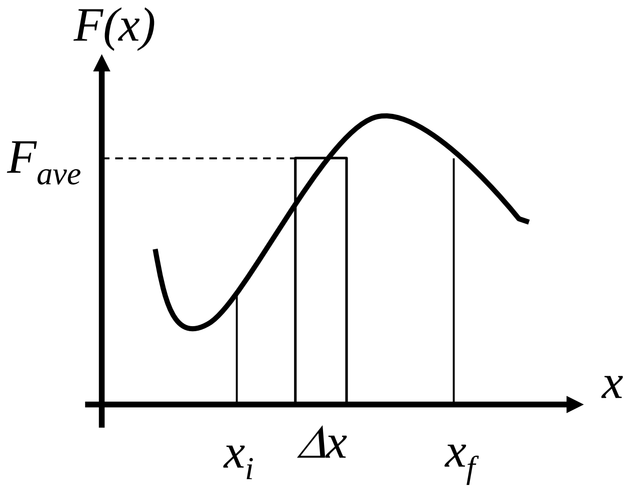
Example 2

An object moves through a displacement **d** and under the influence of a force **F** where

d = –3.0 m **i** and **F** = 2.0 N **i** – 6.0 N **j**.

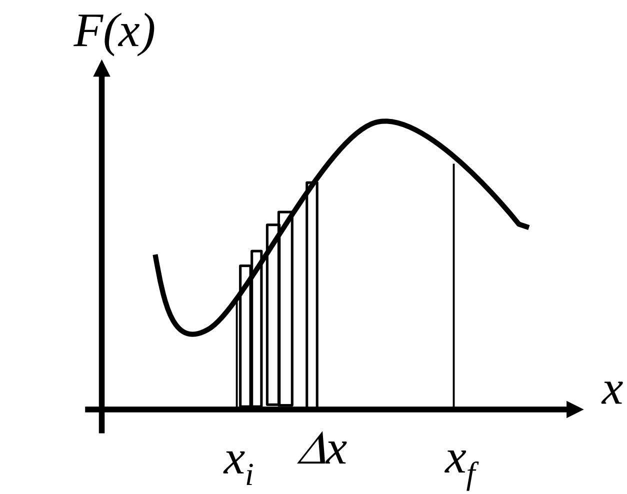
- Calculate the work done by this force.

Variable Force in 1 Dimension



$$\Delta W = F_{ave}\Delta x$$

$$W_{Total} = \sum W = \sum F\,\Delta x$$



For better accuracy, make reduce the strip width and *use more strips.*

Variable Force in 3 Dimensions

In the limit as $\Delta x \rightarrow 0, W = \int_{x_i}^{x_f} F(x)dx$ (1-D case)

In 3-dimensions:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(x) \cdot d\vec{r}$$

$$= \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

Example #3

An object at initial position $\mathbf{r_i}$ = 2.0 **i** + 3.0 **j** moves to final position $\mathbf{r_f}$ = 3.0 **i** while acted upon by a variable force **F** = 3.0*x* i + 4.0 j, where the **r**'s are in meters and **F** is in Newtons.

- Calculate the work done by the force.

A Special Variable Force

The relationship discovered by Robert Hooke (c. 1620) and known as Hooke's law for the force that a spring exerts on a particle when displaced a distance *x* from the relaxed state:

$$\vec{F} = -k\vec{x}$$

where k = "spring constant" or "spring stiffness."

Work by a Spring

In Hooke's law, the minus sign arises because the spring force is always directed oppositely to the displacement. That is, the spring tend to bring the particle back to the equilibrium position.

$$W_{sp} = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx$$

$$= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $x_i = 0$, then: $W_{sp} = -\frac{1}{2}kx_f^2$

Work-Energy Theorem

In instances where the forces involved are unknown or so complicated that it is impractical to use Newton's laws to solve, a new technique can be employed:

$$W = \Delta K$$

where $K = \frac{1}{2}mv^2$ and is defined as the "kinetic energy" of a mass m travelling with speed v.

Example #4

A block of mass m = 5.7 kg slides on a frictionless horizontal tabletop with an initial speed v_o = 1.2 m/s and hits a relaxed spring that has a spring constant k = 1500 N/m.

- Calculate the maximum compression of the spring.
- How much work was done by the spring to stop the object?

Example #5

A 500-kg mass is being lowered be a hoist at a costant speed of 4.0 m/s when suddenly the downward acceleration becomes g/5. The mass displaces 12 m downward before the acceleration returns to zero.

- Calculate the work done by gravity, the work done by the tension in the support cable, and the total work done on the mass.
- Calculate the kinetic energy and the speed of the mass after the acceleration ceases.

The *rate* at which work is done.

Consider these two situations:

1. Raise a mass to some height in a time of 10 s.
2. Raise the same mass to the same height in a time of 5 s.

In each case you do the same amount of work:

$$W = F\Delta x = Mgh$$

In case 2, you expend twice the *power*. That is, $P = \frac{dW}{dt}$

Power

Power is the rate at which work is done.

Average Power: $P_{ave} = \frac{\Delta W}{\Delta t}$

Instantaneous Power: $P = \frac{dW}{dt}$

Units: 1 Watt (W) = (1 J) per (1 s) = 1 J/s

Possibly Useful Relationships

1-D / Constant F: $P = \frac{dW}{dt} = \frac{Fdx}{dt} = F\frac{dx}{dt} = Fv$

More generally…

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta = F_x v_x + F_y v_y + F_z v_z$$

$$P = \int_{v_i}^{v_f} F(v)dv$$

$$P = \int_{\vec{v}_i}^{\vec{v}_f} \vec{F}(v) \cdot d\vec{v}$$

Example #6

A winch is to be used to hoist a 420-kg mass to a height of 120 m in a time of 5.0 min.

- Determine the minimum power that the winch must provide.