Linear Kinematics

Kinematic Equations

How Fast?

Recall: $a(t) = \frac{dv}{dt}$

General Case: Variable Acceleration

$$dv = a(t)dt$$

$$\Delta v = \int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)dt$$

How fast?

Special case: Constant Acceleration

For constant acceleration, $a(t) = a$, so...

$$\Delta v = \int_{t_0}^{t} a(t)dt = a\int_{t_0}^{t} dt = a\Delta t$$

If we let $t_0 = 0$, then…

$$v - v_0 = at \quad \text{(or} \quad v = v_0 + at \text{)}$$

How far?

Recall: $v(t) = \frac{dx}{dt}$

$$dx = v(t)dt$$

$$\Delta x = \int_{x_0}^{x} dx = \int_0^t v(t)dt$$

$$\Delta x = \int_0^t (v_0 + at)dt$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

Deriving the 5 kinematic equations for constant acceleration

Displacement:	$x - x_0$
Initial velocity:	v_0
Final velocity:	v
Acceleration:	a
Time:	t

Each of the two equations derived thus far has four of these five variables. We just need any three of these five in order to find the other two.

If given $x - x_0$, v_0, v, find a and t.

You cannot use just Eqn. 1 or Eqn. 2 alone.

(Both have a and t in them.)

You can manipulate the equations to eliminate the unknowns. For example, eliminate t:

- Use Eqn. 1 to solve for $t = \frac{v - v_0}{a}$
- Substitute this expression in Eqn.2
- Do algebra… $v^2 - v_0^2 = 2a(x - x_0)$

Or, eliminate a:

- Use Eqn. 1 to solve for $a = \frac{v - v_0}{t}$
- Substitute this expression in Eqn.2
- Do algebra… $$x - x_0 = \frac{1}{2}(v + v_0)t$$

Or, eliminate v_0:

- Use Eqn. 1 to solve for $v_0 = v - at$
- Substitute this expression in Eqn.2
- Do algebra… $$x - x_0 = vt - \frac{1}{2}at^2$$

The 5 kinematic equations for constant acceleration.

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

$$x - x_0 = vt - \frac{1}{2}at^2$$

Example #1: Free Fall Ball

A pitcher throws a ball straight up. The ball leaves the pitcher’s hand at 25 m/s.

- How long until the ball reaches its highest point?
- How high does the ball go?
- How long until the ball reaches a height of 25 meters (above the release point)?

Example #2: Freely Falling Elevator

An elevator cab is 120 m above the ground level. The elevator cable breaks.

- What is the velocity of the cab when it gets to the ground level?
- How long does it take the elevator to fall?
- What is the speed of the elevator at the "half-way" point?

Example #3: Skydiving

A skydiver jumps out of a hovering helicopter and falls freely for 50 m before deploying her parachute. The skydiver then "decelerates" at 2.0 m/s^2 and reaches the ground at 3.0 m/s.

- How long was the skydiver in the air?
- At what height above the ground did the skydiver bail out of the helicopter?

Motion in 1-Dimension with Variable Acceleration

A particle moves according the equation

$$x(t) = 3.0t^2 - 1.0t^3$$

where x is in meters and t is in seconds.

- Find $v(t)$ and $a(t)$.
- Find the time when the particle reaches x_{max}.
- Find the *displacement* during the first 4.0 s.
- Calculate $v(t = 4.0\text{ s})$ and $a(t = 4.0\text{ s})$
- Calculate the *path* that the particle covers in the first 4.0 s.

Another Example

A particle moves according the equation

$$a(t) = At^3 - Be^{-Ct}$$

where x is in meters, t is in seconds, and A, B, and C are constants.

- Derive equation for $v(t)$ using definition of acceleration: $a = dv/dt$.

  (Hint: Multiply both side by dt. Then integrate.)

- Derive equation for $x(t)$ using definition of velocity: $v = dx/dt$.