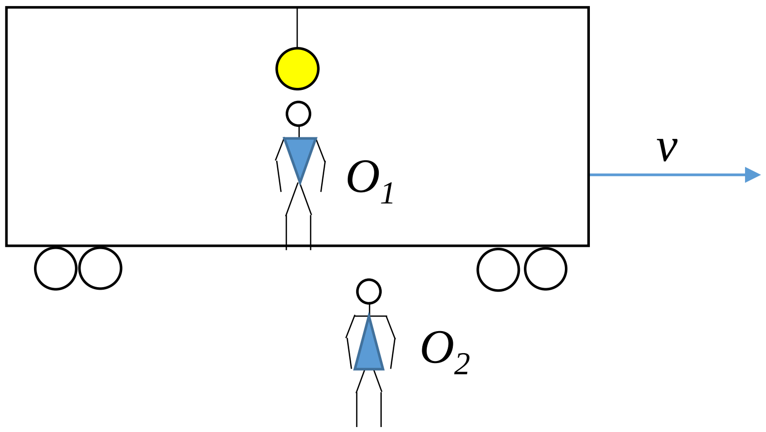
Section 2.2

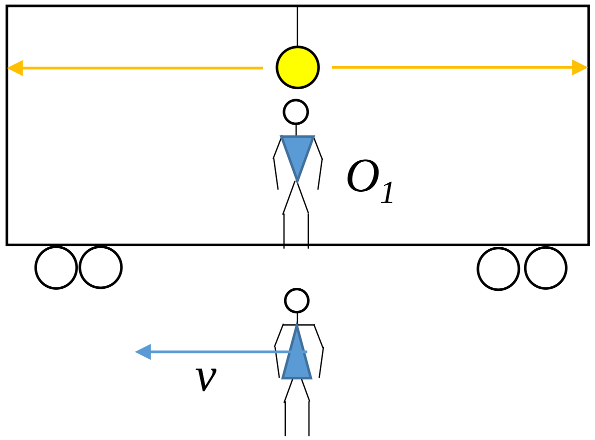
Implications of Einstein’s Postulates

The Relativity of Simultaneity

Consider two observers: O_1 on an open train car moving at high speed, and O_2 standing trackside watching the train pass.

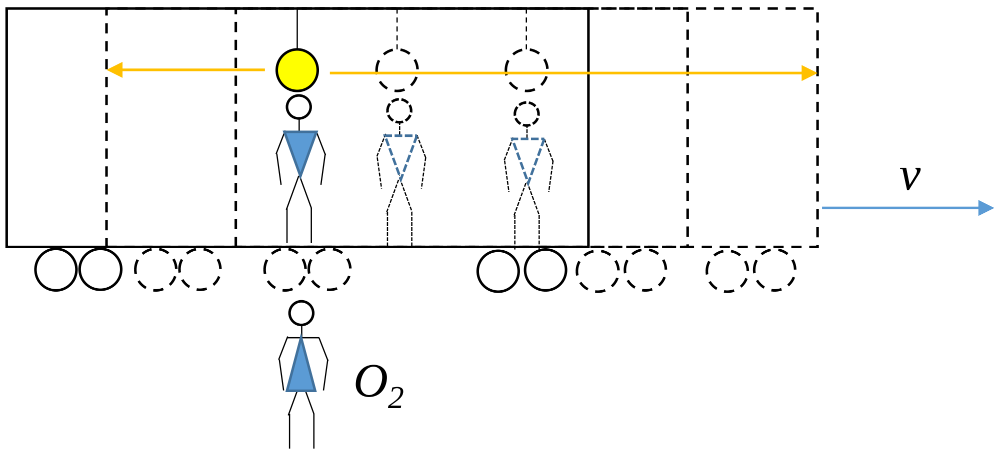


At the exact instant the O_1 and O_2 pass each other, a light flash is emitted from the center of the train car traveling in opposite directions. According to O_1, the two light beams hit the front and back of the train car simultaneously. Why? The light travels at the same speed an equal distance in each direction.



The Relativity of Simultaneity (cont'd)

According to O_2, the light hits the back of the train car first (as the back of the train car rushed forward to meet the backward traveling beam) and then later the forward traveling light beam hits the front of the train car (as the light has to catch up to the front that is moving away from it).

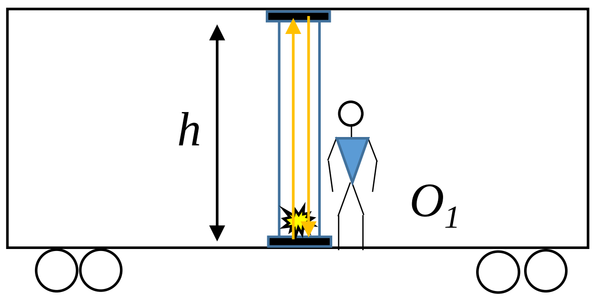


Who's right?

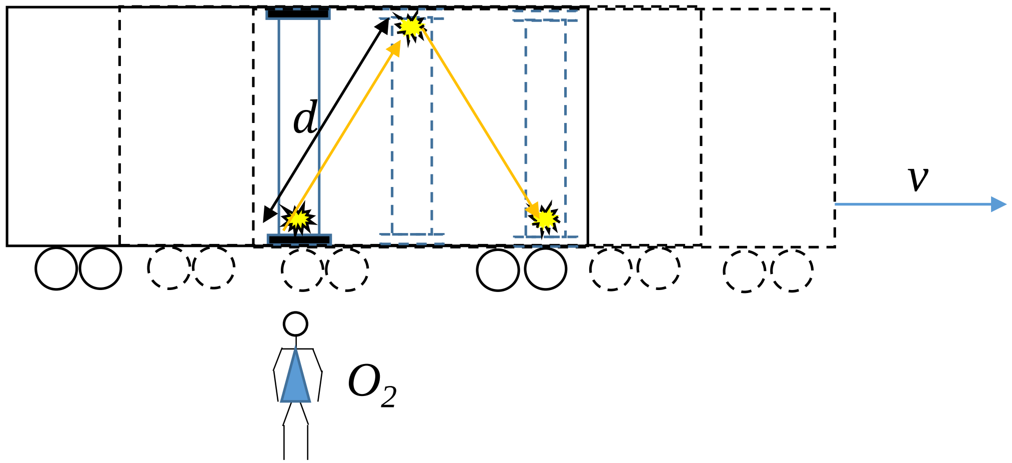
Both are! Simultaneity is NOT absolute.

The Relativity of Time (Time Dilation)

Consider the *light clock* in which the "ticks" and "tocks" are measured by a light pulse that bounces back and forth between two (perfectly reflecting) mirrors. We place the clock on the train car. According to O_1, the pulse travels the length of the clock in a time $\Delta t_1 = h/c$, where h is the distance between the two mirrors.



However, according to O_2, the pulse travels a greater distance d between the two mirrors each "tick" or "tock" because the clock moves forward between each reflection.



At this point, it is important to remind yourself of Einstein's second postulate that the speed of light is the same to BOTH observers…

Time Dilation

The relativity of time can be quantified by considering the right triangle formed by the distances involved from the perspectives of the two observers:

$h = c\Delta t_1$ is the distance the light pulse travels between the mirrors according to O_1.

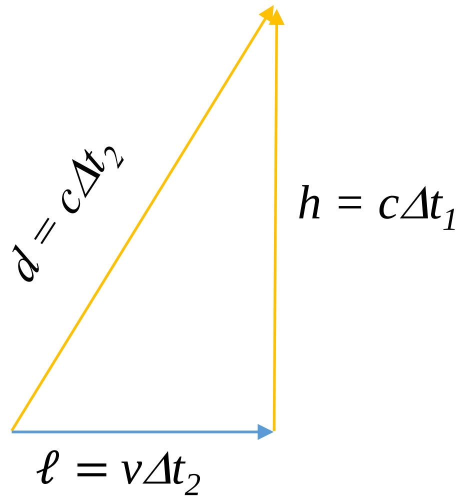
$d = c\Delta t_2$ is the distance the light pulse travels between the mirrors according to O_2.

$\ell = v\Delta t_2$ is the distance that the train car moves forward during the time that the light pulse between the mirrors according to O_2.

Using the Pythagorean Theorem: $d^2 = \ell^2 + h^2$

implies $(c\Delta t_2)^2 = (v\Delta t_2)^2 + (c\Delta t_1)^2$. After some

algebra and solving for Δt_2 in terms of Δt_1, we find: $\Delta t_2 = \frac{\Delta t_1}{\sqrt{1-\frac{v^2}{c^2}}}$.



For light covering the same distance, O_2 measures fewer “ticks” than O_1.

Conclusion: Moving clocks run slow!

The Lorentz factor and the Speed Parameter

The term $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ occurs so often in the theory of relativity that it is given its own symbol: γ.

The term γ is called the *Lorentz factor*.

The term $\frac{v}{c}$ also occurs often in the theory of relativity and is also given its own symbol: β.

The term β is called the *speed parameter*. So, the Lorentz factor can also be written $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

As will be shown later, the speed of light *c* is a "cosmic speed limit" that nothing can exceed.

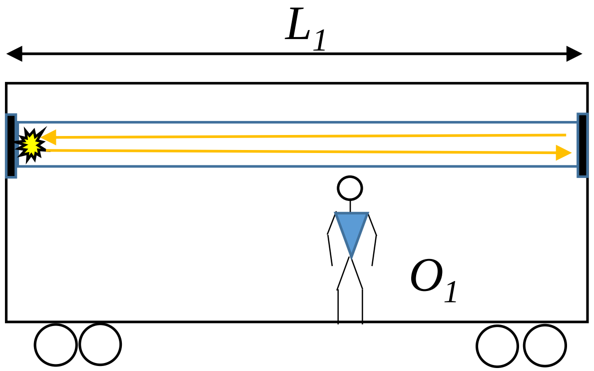
Therefore the speed parameter cannot exceed a value of 1. That is, $\beta < 1$.

However, as β approaches 1, the denominator of the Lorentz factor goes to zero, meaning that the Lorentz factor approaches infinity. For an object at rest, $\beta = 0$ so $\gamma = 1$. In summary, $\gamma \geq 1$.

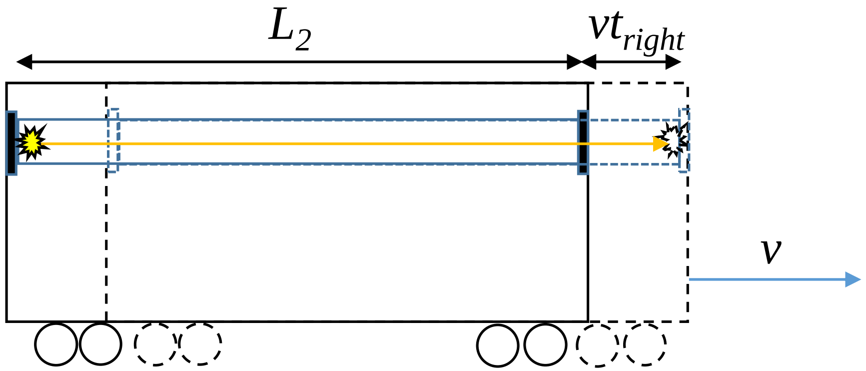
The previous slide shows $\Delta t_2 = \gamma \Delta t_1$. That is, moving clocks run slow by a factor of γ.

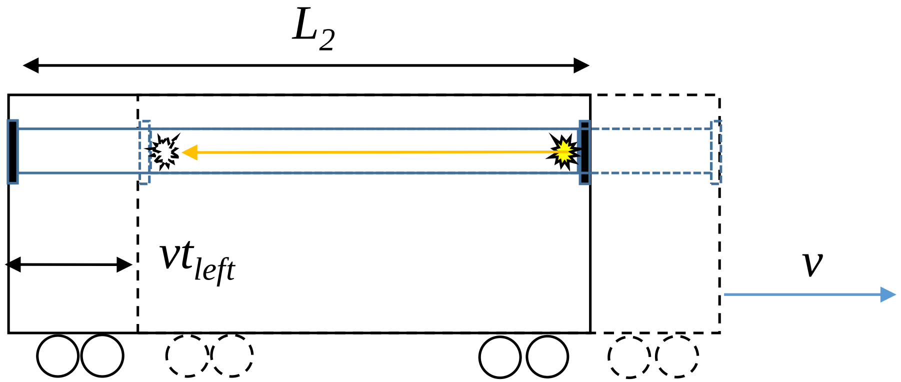
The Relativity of Length (Length Contraction)

Let us now orient the light clock on the train car horizontally and adjust its length to match that of the train car. According to O_1, the pulse travels a round-trip in a time $\Delta t_1 = 2L_1/c$, where L_1 is the length of the train car according to O_1.



However, according to O_2, the pulse travels a distance $ct_{right} = L_2 + vtr_{ight}$ and from the left mirror to the right mirror and a distance $ct_{left} = L_2 - vt_{left}$, where t_{right} and t_{left} are the times required for the light to move from the left to right, and right to left, respectively, and L_2 is the length of the train car according to O_2.





Length Contraction (cont'd)

The total time for the round-trip according to O_2 is $\Delta t_2 = t_{right} + t_{left}$,

where $t_{right} = \frac{L_2}{c-v}$ and $t_{left} = \frac{L_2}{c+v}$.

So, $\Delta t_2 = \frac{L_2}{c-v} + \frac{L_2}{c+v}$, which after some algebra gives $\Delta t_2 = \frac{2L_2}{c}\gamma^2$. (Try it!)

But recall that the round-trip time according to O_1 is $\Delta t_1 = \frac{2L_1}{c}$,

and that $\Delta t_2 = \gamma \Delta t_1$ (from the time dilation formula)

Equating the two expressions for Δt_2: $\frac{2L_2}{c}\gamma^2 = \gamma\frac{2L_1}{c}$.

Simplifying yields: $\gamma L_2 = L_1$ or $L_2 = \frac{L_1}{\gamma}$.

In other words, O_2 measures a shorter train car length than O_1.

Conclusion: Moving objects are shorter (by a factor of γ).

Example

While on a train that is moving at 90% the speed of light, Johnny is sitting in the dining car enjoying a "foot-long" hot dog (measured on the train). It takes Johnny 7 minutes to eat his hot dog (according to Johnny watch). Sally, is standing trackside watching Johnny through the dining car window as the train speeds past.

a) What are the speed parameter and the Lorentz factor?

$v = 0.90c$, so $\beta = 0.90$. $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.29415\ldots \approx 2.3$.

b) How long does it take Johnny to eat the hot dog according to Sally?

$\Delta t_{Sally} = \gamma \Delta t_{Johnny} = 16.0591\ldots \text{ min} \approx 16 \text{ min}$.

c) How long is the "foot long" hot dog from Sally's perspective?

$\Delta L_{Sally} = \frac{\Delta L_{Johnny}}{\gamma} = 0.43589\ldots \text{ ft} \approx 0.44 \text{ ft} = 5.2 \text{ inches}$.

And so…

How come we never noticed any of this before Einstein?

At the ordinary speeds of our everyday experiences, the Lorentz factor γ is so close to 1, that there was no reason for us to suspect that moving clocks run slow or that moving objects are shorter.

Example: 100 mph is about 45 m/s. What are the speed parameter and the Lorentz factor for this speed?

$$\beta = \frac{v}{c} = \frac{45\ \mathrm{m/s}}{3.0 \times 10^{8}\ \mathrm{m/s}} = 0.00000015 = 1.5 \text{ x } 10^{-7}.$$

$$\gamma = \frac{1}{\sqrt{1-\beta^{2}}} = 1.0000000000000001125\ldots \approx 1.0.$$