Chapter 8

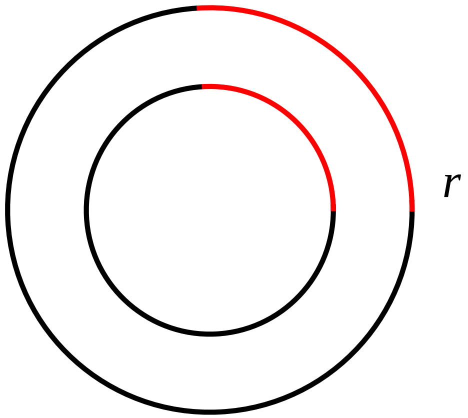
Rotational Motion

Tangential Speed vs. Rotational Speed

Consider a spinning disk. All parts of the disk complete a single rotation in the same time. However, a point on the rim has to travel a greater distance during that rotation than a point that is closer to the center.

Both points have the same *rotational speed*, but the point on the rim will have a greater *tangential speed* than the point closer to the center due to the larger circumference.

Greater distance covered at rim.



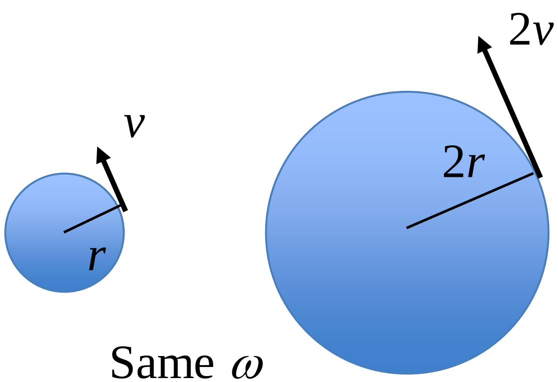
$v = r\omega$, where v is the tangential speed, r is the radial distance from the center, and ω is the rotational speed.

Example

Case 1:

Different sized wheels independently spinning with the same rotational speed, ω.

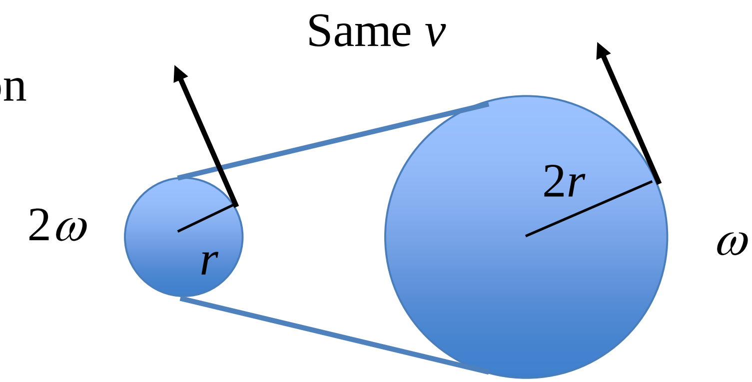
If the larger wheel has twice the radius, then the tangential speed on the rim of the larger wheel has twice the tangential speed compared to the smaller wheel.



Case 2:

Different sized wheels connected by a belt. Assuming the belt does not slip or stretch, points on the rims of both wheels have the same tangential speed, v.

If the larger wheel has twice the radius, then the rotational speed of the larger wheel has half the rotational speed compared to the smaller wheel.



Rotational Inertia…

…is the property of an object to resist changes in its state of rotational motion.

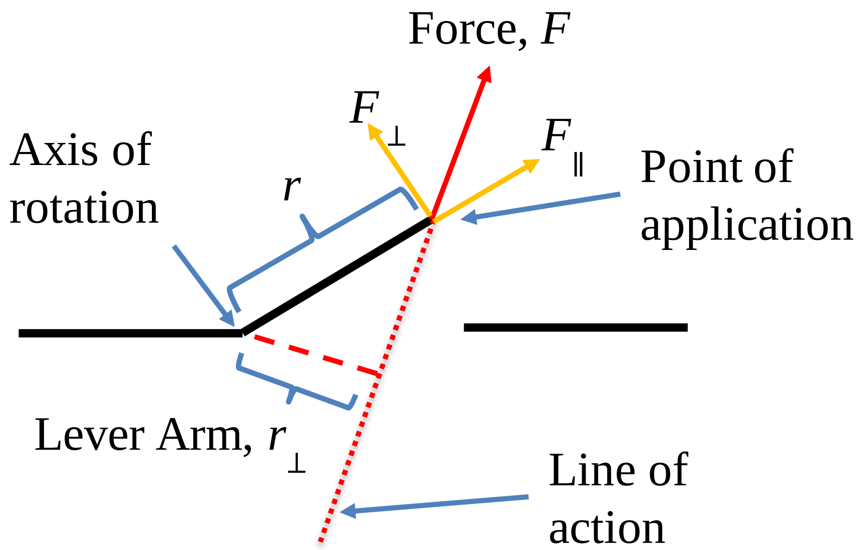
Just as an object at rest remains at rest and an object in motion tends to remain moving in a straight line with constant speed, an object rotating about an axis tends to remain rotating about that same axis unless acted upon by an external influence (called a *torque*).

In addition to mass, the rotational inertia also depends on how the mass is distributed about the axis of rotation.

Torque, τ = (Force) x (Lever Arm)

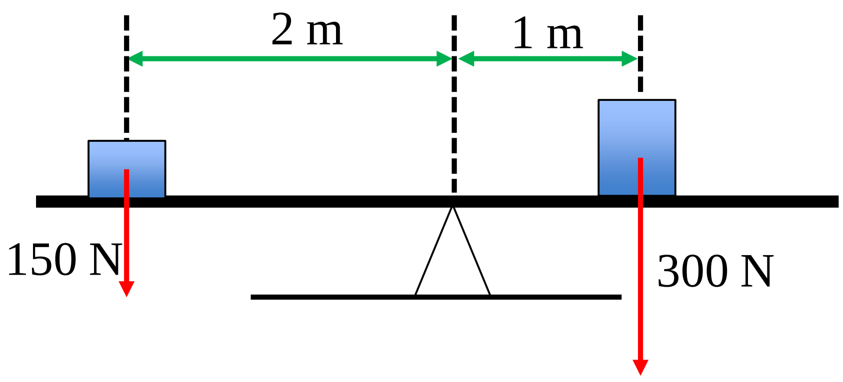
Consider the overhead view of a door jamb below. The door is partially open. A force *F* is applied at the end of the door which is a distance *r* from the axis of rotation. In the diagram, the force has broken into components that are perpendicular and parallel to the distance between the axis of rotation and the point of application of the force.

The "twisting effect" or *torque* is: $\tau = F_{\perp} r$ and or $Fr_{\perp}$, where $r_{\perp}$ is the perpendicular distance from the rotation axis to the line of action of the force known as the *lever arm*.



Example

Although the forces pushing down on each side of the see-saw are different, the torques on of side ARE the same. The smaller weight is compensated by the larger lever arm distance.



The see-saw is balanced since

$$\tau_{left} = \tau_{right}$$

$$F_{left} \times r_{\perp, left} = F_{right} \times r_{\perp, right}$$

$$(150\ \text{N})(2\ \text{m}) = (300\ \text{N})(1\ \text{m})$$

$$300\ \text{N}\cdot\text{m} = 300\ \text{N}\cdot\text{m}$$

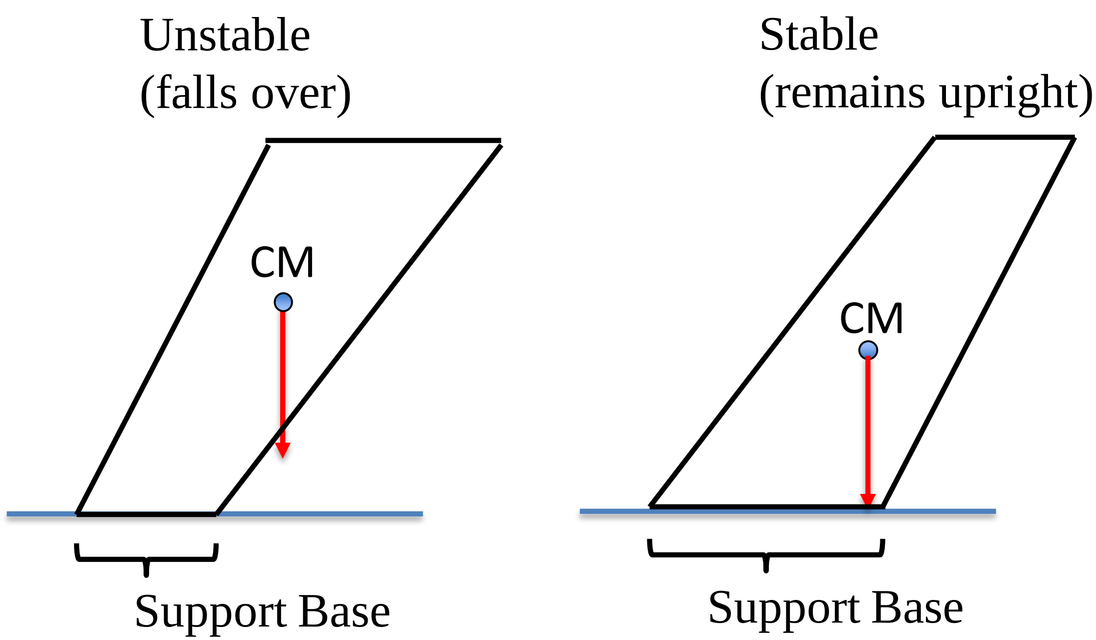
Center of Mass

The center of mass (CM) of an object is the average position of all the mass that makes up the object. For a symmetric object (like a ball or a rectangular plate), the CM is at the geometric center of the object. When an irregularly shaped object is tossed in the air, it wobbles about the CM, while the CM follows the smooth arc like that of a tossed ball.

Stability

What makes a resting object stable?

As long as the CM is over the support base, the object will not topple over.



Centripetal Force

Recall from Newton's 1st law that a moving object will tend to move with constant velocity (straight line with constant speed) unless acted upon by a force.

Consider an object moving in a circle at constant speed. Since the velocity is continuously changing (because the direction is continuously changing), the object must be undergoing a continuous acceleration. Since the speed remains constant, this acceleration (and therefore the net force) must be perpendicular to the velocity at every instant.

This force, called the *centripetal force* has a magnitude

$$F_{centripetal} = \frac{mv^2}{r}$$

and points toward the center of the circle.

Angular Momentum

The rotational counterpart to linear momentum, angular momentum describes the "amount of rotation" or how difficult it is to stop something that is rotating.

The symbol for angular momentum is L. (I don't know why.)

Recall that linear momentum is given by $p = mv$, where m is a measure of the (translational) inertia, a.k.a., the mass, and v is the (linear) speed.

Analogously, the angular momentum is given by $L = I\omega$, where I is the rotational inertia and ω is the rotational speed.

For a mass twirling on the end of a string, the angular momentum is:

$$L = mvr$$

Units of angular momentum: (kg)(m/s)(m) = kg·m^2/s = J·s.

Conservation of Angular Momentum

Recall that the linear momentum of a system is conserved when there is no external net force acting on the system.

Similarly, the angular momentum of a system is conserved when there is no external net torque acting on the system.

The most common example is the spinning ice skater:

The skater goes into a slow spin with arms and a leg extended, the skater's rotational inertia is large. When the skater pulls on their arms and leg close to their body (thus reducing their rotational inertia), the rotational speed increases. No external torque acts on the skater. So…

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$