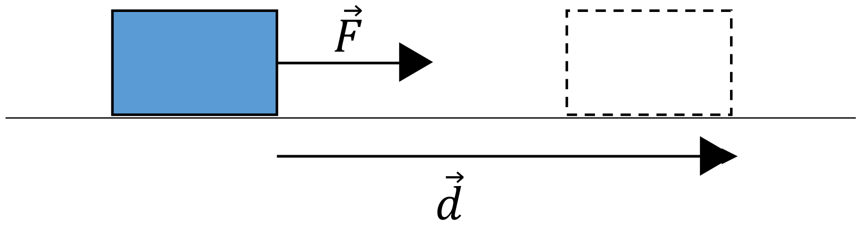
Work & Energy

Defining WORK



Work ≡ Force x Distance. That is, $W = Fd$

Standard units of work:

Force Units x Distance Units = (Newtons) x (meters) = N·m

where 1 Newton·meter ≡ 1 "Joule." (That is, 1 J = 1 N·m.)

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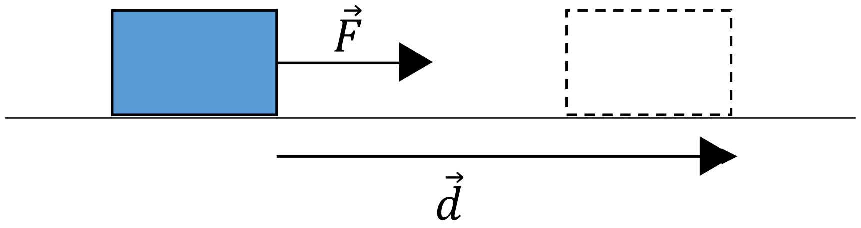

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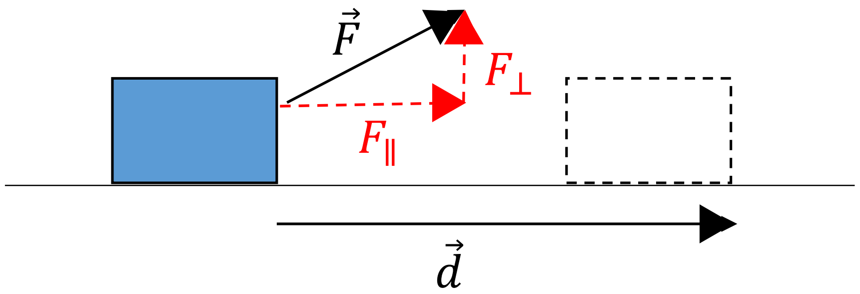
Defining WORK (cont'd)

What if the force is not pointing along the direction of the displacement?



Work ≡ Parallel component of the Force$_{||}$ x Distance.

That is, $W = F_{||}d$

Any force that is perpedicular to the displacement does ZERO work.

If $F_{||}$ points opposite of the displacement then W < 0.

Defining POWER

Power is the RATE at which work is done: $P = \frac{W}{t}$

Standard units of power:

Work Units / Time Units = (Joules) / (second) = J/s, where 1 J/s ≡ 1 "Watt."

Question: Which expends more power?

A) Doing 240 J of work in 6 s, or

B) doing 250 J of work in 5 s?

Answer: (B) since 250 J /5 s = 50 Watts (whereas 240 J / 6s = 40 Watts).

Definition of ENERGY

Formally, energy is defined as "the ability to do work."

You can think of energy as the "stuff that makes things go." Energy is an abstract concept. It is not concrete nor does it describe any mechanism.

What make energy a useful concept?

Energy cannot be created or destroyed, so the total energy of the universe is constant. However, energy comes in many different forms and energy can transfer from one form to another. So the energy of the universe is constant.

This is the law of *conservation of energy*.

Different Forms of ENERGY

What does a rolling bowling ball have that a stationary one doesn't?

Answer: Movement. Energy of motion is called *kinetic energy, KE*.

What does a raised mass have that a mass on the ground doesn't?

Answer: Elevation. Energy of position (or configuration) is called *potential energy, PE.*

In this case, we say that the raised mass has "gravitational potential energy" or *GPE.*

What do the following all have in common with the raised mass?

A) A compressed spring;

B) A glucose molecule;

C) A plutonium-239 atom;

D) A pair of electrically charged plates.

Answer: Energy of configuration or *potential energy*.

A) Elastic PE

B) Chemical PE

C) Nuclear PE

D) Electrical PE

Formulas

Energy of motion (kinetic energy): $KE = \frac{1}{2}mv^2$

Energy of elevation (gravitational potential energy): $GPE = mgh$

While there are formulas for other forms of potential energy, we shall only concern ourselves with gravitational potential energy.

Mechanical energy, $ME = KE + GPE = \frac{1}{2}mv^2 + mgh$

The Work-Energy Theorem

Work, $W = Fd = mad = m\frac{\Delta v}{\Delta t}d$.

Recall that $\Delta v = (v_f - v_i)$ and $d = v_{ave}\Delta t$, where $v_{ave} = \frac{1}{2}(v_f + v_i)$.

Then $W = m(v_f - v_i)\frac{1}{2}(v_f + v_i) = \frac{1}{2}m(v_f^2 + v_f v_i - v_i^2 - v_i v_f)$

$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

The punchline: $W = \Delta KE$

And so…

When you or some object “does work” on another object (by applying a force as the object displaces), then you are:

you are giving that other object energy when $W > 0$,

and you are taking energy away when $W < 0$.

The amount of energy you give (or take away) is just the amount of work done. Its that simple!

Conservation of Energy

Mechanical Energy (ME) = KE + PE

If thermal energy can be neglected then mechanical energy (ME) is conserved. That is, ME = KE + PE = constant. Then…

$$ME_i = ME_f$$

$$KE_i + GPE_i = KE_f + GPE_f$$

$$\frac{1}{2}m{v_i}^2 + mgh_i = \frac{1}{2}m{v_f}^2 + mgh_f$$

Example

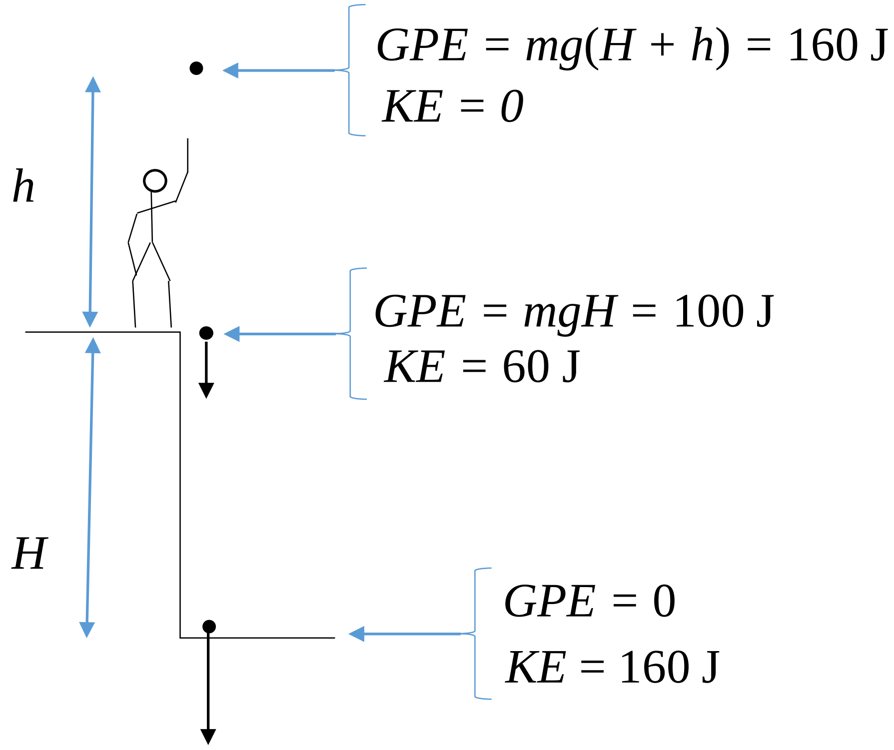
A 2-kg ball is tossed vertically upward by a person standing on top of a wall that is 5 m high. The ball reaches its maximum height 3 m above the top of the wall.

What are the gravitational potential energy and kinetic energy of the ball…

a) at the top of its flight,

b) at the top of the wall,

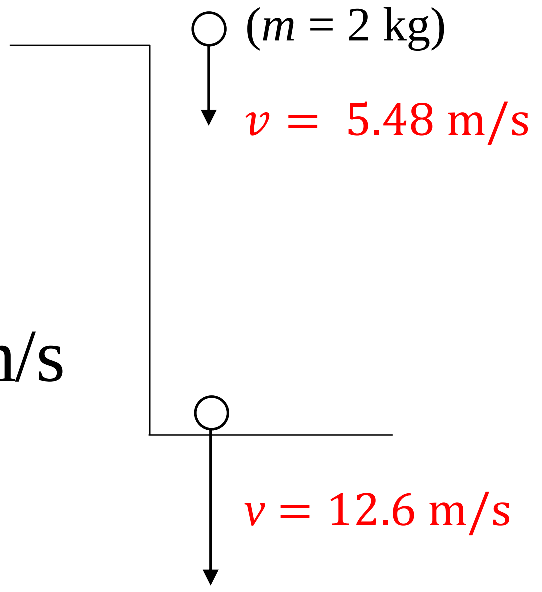
c) and the instant just before it hits the ground?



Follow-up to the previous example:

What is the speed of the ball as it passes the top of the wall in its way back down?

$$KE = \frac{1}{2}mv^2.\ \text{Solve for } v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(60\ \text{J})}{(2\ \text{kg})}} = 5.48\ \text{m/s}$$



What is the speed of the ball the instant just before it hits the ground?

$$KE = \frac{1}{2}mv^2.\ \text{Solve for } v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(160\ \text{J})}{(2\ \text{kg})}} = 12.6\ \text{m/s}$$

Simple Machines

A simple machine is any device that multiplies force and/or changes the direction of a force. Since energy cannot be created or destroyed (by conservation of energy), the work output by the machine is equal to the work input into the machine. If frictional effects can be ignored then…

$$W_{out} = W_{in}, \text{ (ideally).}$$

That is, $F_{out}\, d_{out} = F_{in}\, d_{in}$

Since "you cannot get something for nothing," there is a trade-off. If the machine increases the force output by some factor, the you machine must compensate by decreasing the output distance by the same factor.

Simple Machines (cont'd)

Mechanical advantage describe how much the force output is compared to the force input:

$$M.A. = \frac{F_{out}}{F_{in}}$$

That is, how much "easier" it is to do the work.

Efficiency describe how well the machine converts the work input into useful work output:

$$e = \frac{W_{out}}{W_{in}}$$

Ideally, $W_{out} = W_{in}$, so…

$e_{ideal} = 1$ (or 100%).

Again, due to frictional effects, $W_{out} < W_{in}$, so…

$e_{real} < 100\%$.