Physics 2111
Unit 15

Today's Concepts:

a) Parallel Axis Theorem

b) Rotations as vector (RHL)

c) Torque & Angular Acceleration

Your Comments

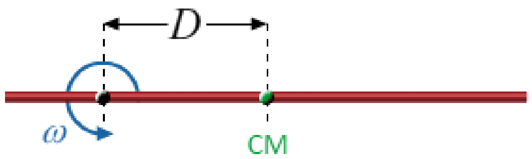
- Probably the parallel axis theorem, it is simple now but when we introduce different moment of inertia's and more objects it can get confusing.
- Seeing concrete formulas and how they are used.
- I confused about moment of inertia of Dumbell and right hand rule.
- how to find the moment of inertia of any system. using parallel axis theorem and what the actual number represents and at what point.
- I found very confusing the parallel axis theorem and also the example and checkpoint of the dumbbell.
- More examples like the dumbbell one and cross product
- I think I understand this section, I think I just need to see some examples of the types of problems. question 1 tripped me up in the Pre-lecture though.
- more confusing than the last one, I want to go over the parallel axis theorem a bit more The Torque cross product was a little hard to follow.
- The kinetic energy of a rotating system was confusing.

Parallel Axis Theorem

KEY POINT:

We can think of all motion of extended objects as:

1) Motion *of* the center of mass.
2) Motion *around* the center of mass



Parallel Axis Theorem

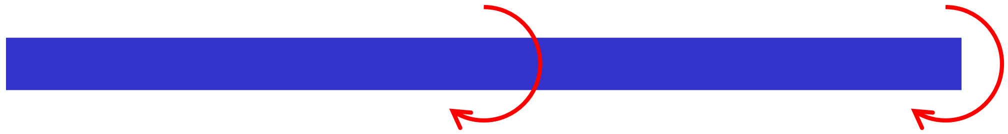
$$I_{Total} = MD^2 + I_{CM}$$

MOI *of* the center of mass

MOI *around* the center of mass

Example 15.1 (Moment of Inertia of Rod)

What is the moment of inertia of a uniform 10kg rod, 0.5 meter in length about its center?



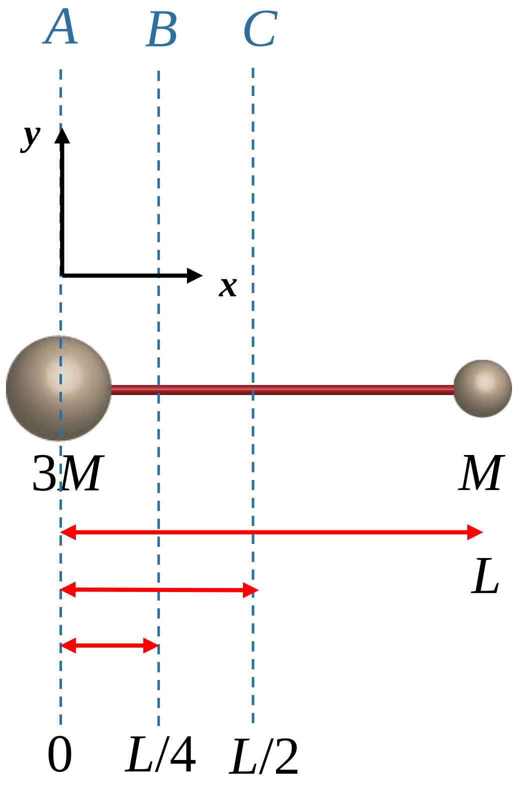
What is the moment of inertia of a uniform 10kg rod, 0.5 meter in length about one end?

CheckPoint

A ball of mass $3M$ at $x = 0$ is connected to a ball of mass M at $x = L$ by a massless rod. Consider the three rotation axes A, B, and C as shown, all parallel to the y axis.

For which rotation axis is the moment of inertia of the object smallest? (It may help you to figure out where the center of mass of the object is.)

$$I = \Sigma m_i r_i^2$$
$$I_{parallel} = I_{CM} + MD^2$$
$$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$$
$$I_{hoop} = MR^2$$
$$I_{solid\text{-}sphere} = {}^2/_5 MR^2$$
$$I_{spherical\ shell} = {}^2/_3 MR^2$$
$$I_{rod\text{-}cm} = {}^1/_{12} ML^2$$
$$I_{rod\text{-}end} = {}^1/_3 ML^2$$



Example 15.2 (two solids spheres)

What is the moment of inertia of these two solids sphere with radius $R = 10cm$ and mass $M = 10kg$, attached to a rod of mass m = 2kg and length $d = 25cm$?

$$I = \Sigma m_i r_i^2$$
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$$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$$
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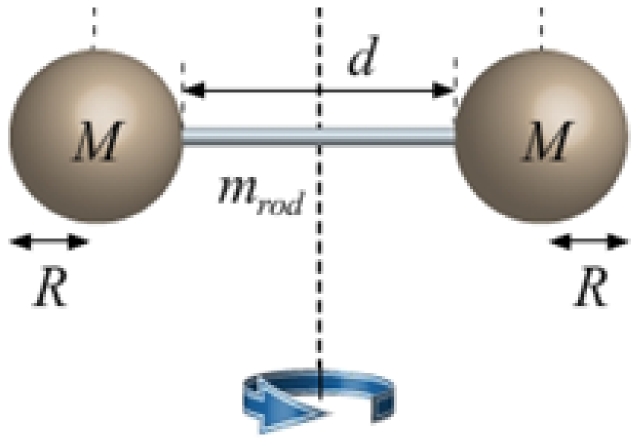
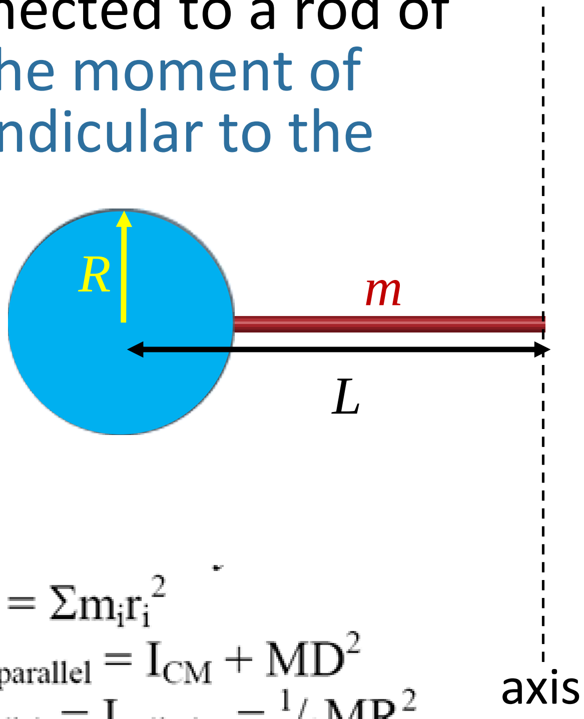


$$= (I_{COM\text{-}ROD} + M_{ROD}D^2) + (I_{COM\text{-}SPHERE} + M_{SPHERE}D^2)$$

$$=(1/12\ md^2 + 0) + 2*(2/5MR^2 + M(d/2+R)^2)$$

Question

A solid ball of mass M and radius R is connected to a rod of mass m and length L as shown. What is the moment of inertia of this system about an axis perpendicular to the other end of the rod?



A) $I = \frac{2}{5}MR^2 + ML^2 + \frac{1}{3}m(L-R)^2$

B) $I = \frac{2}{5}MR^2 + M(L+R)^2 + \frac{1}{3}mL^2$

C) $I = \frac{2}{5}MR^2 + m(L+R)^2 + \frac{1}{3}mL^2$

D) $I = \frac{2}{5}MR^2 + \frac{1}{3}mL^2$

$I = \Sigma m_i r_i^2$

$I_{parallel} = I_{CM} + MD^2$

$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$

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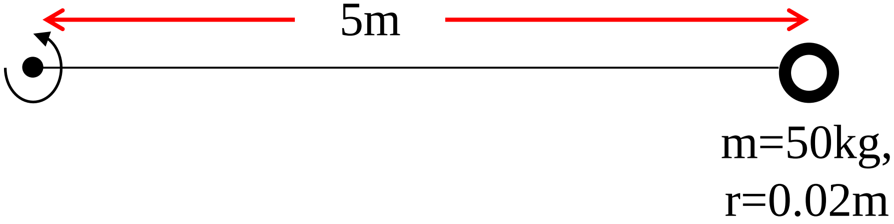
Question



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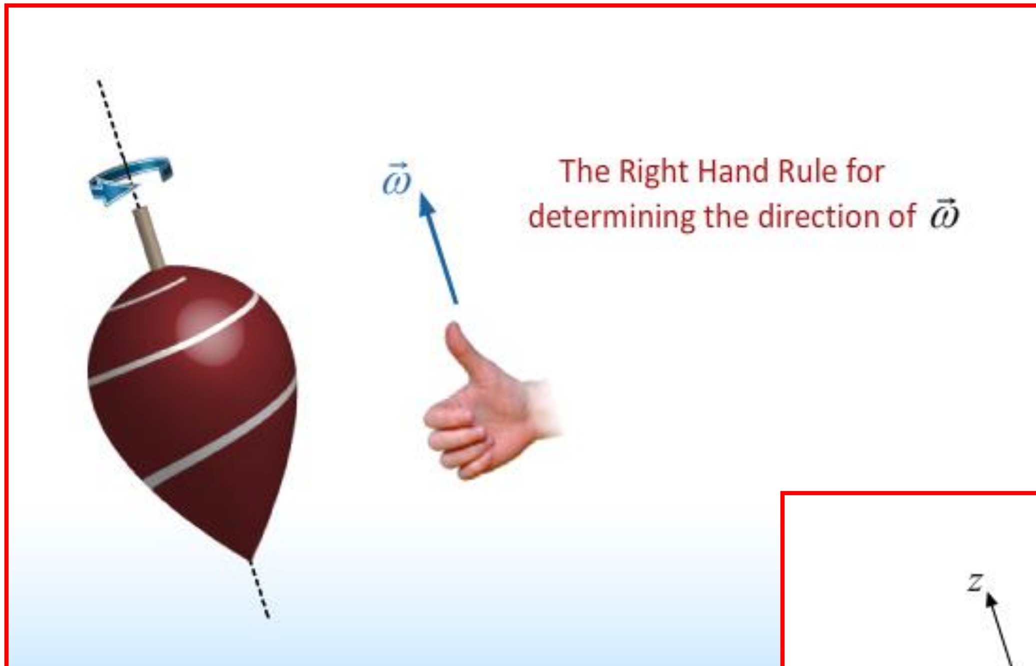
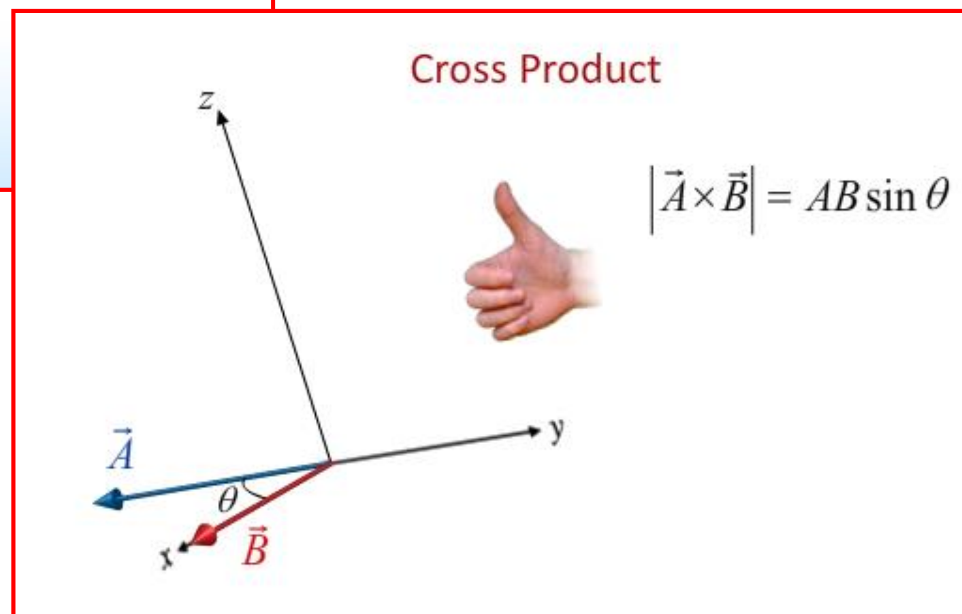
A thin walled hollow sphere of mass 50kg and radius 2cm is attached to one end of a massless, rigid rod of length 5m. If the rod and sphere are rotated about the opposite end of the rod, which answer below is roughly the moment of inertia of the system.

A. 2/5 (50kg)(0.02m)^2
B. 2/3 (50kg)(0.02m)^2
C. 2/3 (50kg)(5m)^2
D. (50kg)(5m)^2
E. 7/5(50kg)(5m)^2

Right Hand Rule for finding Directions





Why do the angular velocity and acceleration point perpendicular to the plane of rotation?

Question

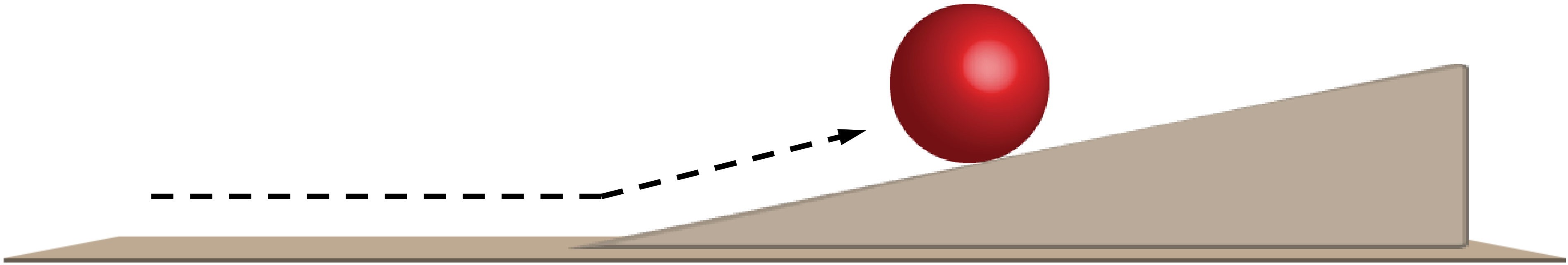
A ball rolls across the floor, and then starts up a ramp as shown below. In what direction does the angular velocity vector point when the ball is rolling up the ramp?

A) Into the page

B) Out of the page

C) Up

D) Down



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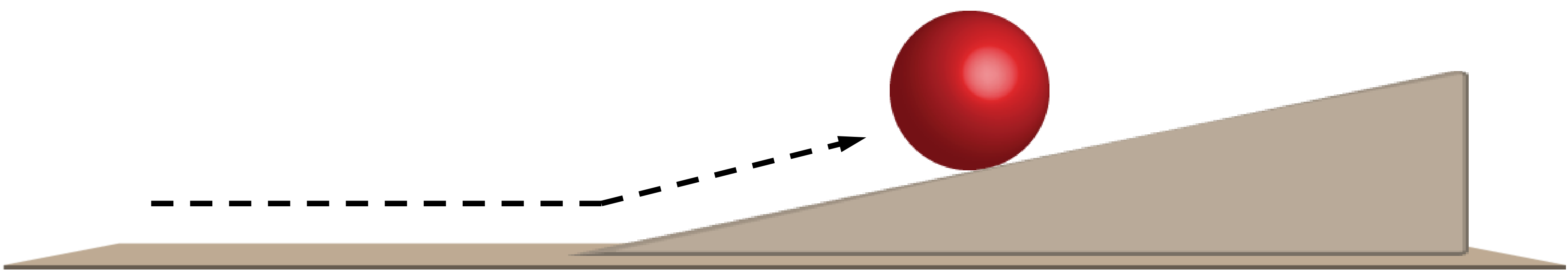
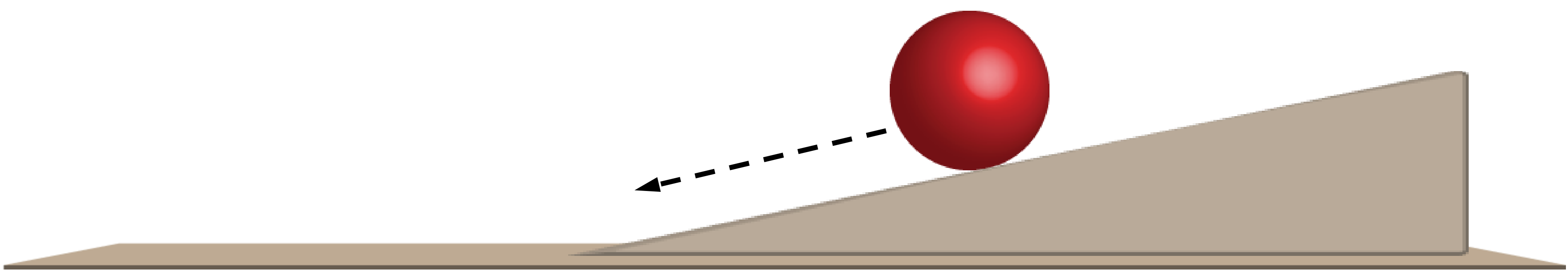


Question

A ball rolls across the floor, and then starts up a ramp as shown below. In what direction does the angular acceleration vector point when the ball is rolling back down the ramp?

A) into the page

B) out of the page



summary so far

$x \longrightarrow \theta$

$dx/dt = v \longrightarrow d\theta/dt = \omega$

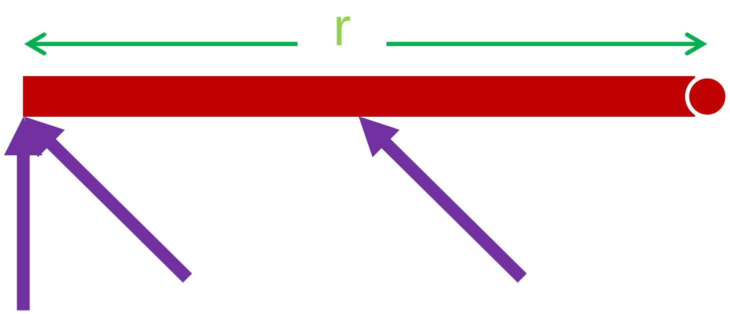
$dv/dt = a \longrightarrow d\omega/dt = \alpha$

$m \longrightarrow I$

$F \longrightarrow ??$

Torque

Image after physics class, you wanted open one of the exterior doors to the BIC (one of the ones with crash bars) and it sticks. What could you do?



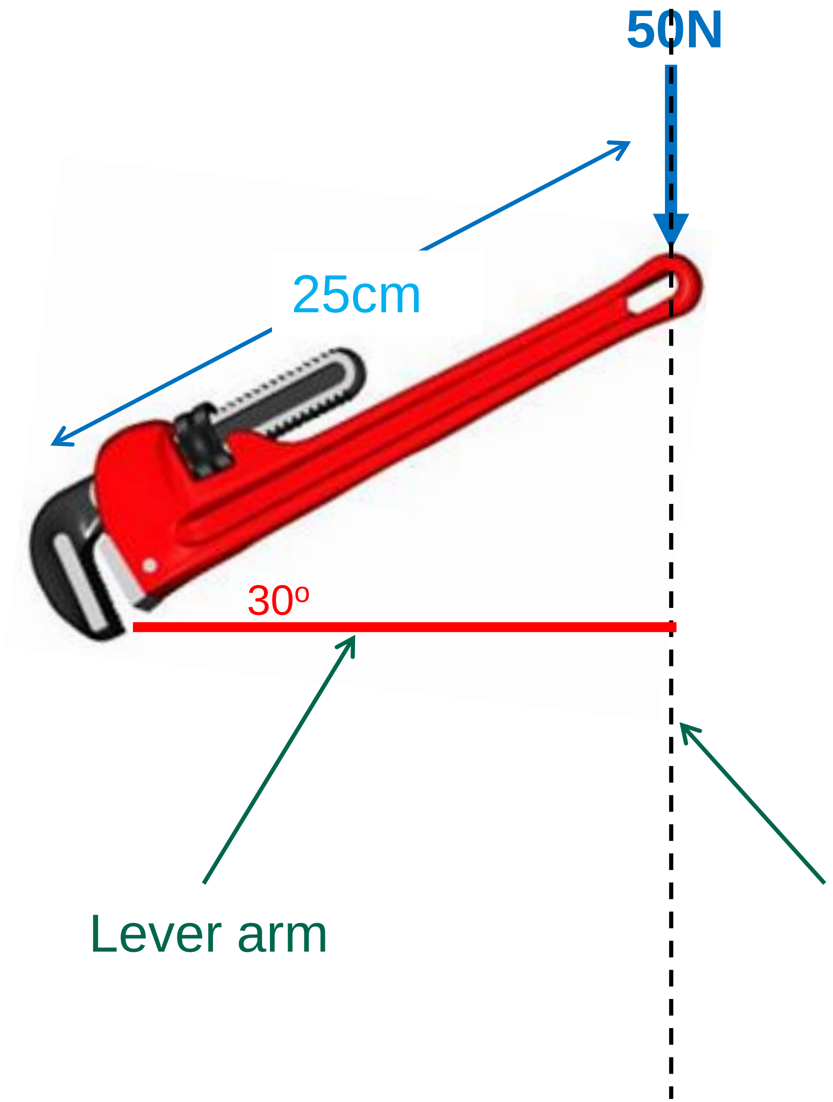
1) You could hit it harder (make F bigger)

2) You could hit it farther from the hinge (make r bigger)

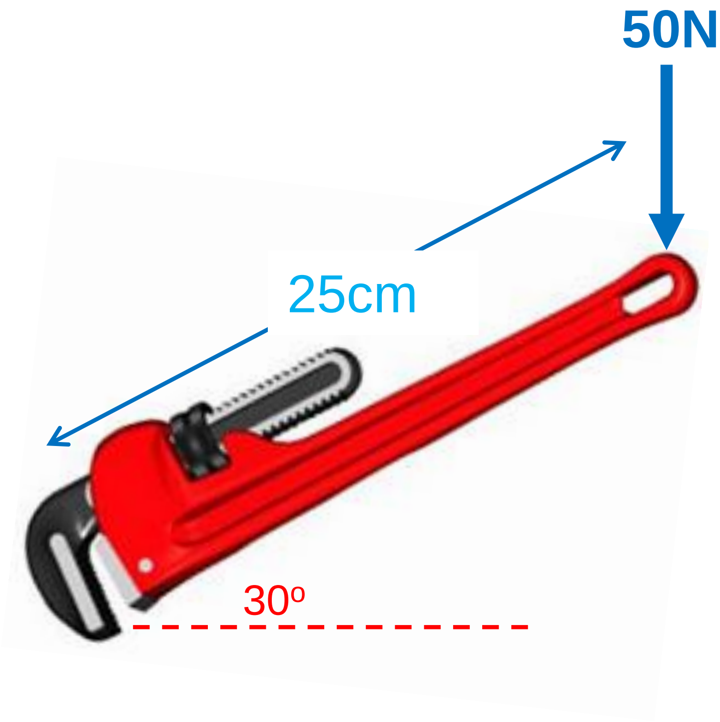
3) You could hit it more perpendicular to the door (make sin(θ) bigger)

$$|\tau| = |r||F|\sin(\theta) \qquad \vec{\tau} = \vec{r} X \vec{F}$$

Example 15.2 (torque on a wrench)



A 50N force is placed on one end of a 25cm long wrench as shown to the left. What is the torque applied by this force if it rotates about the other end?



Keeping in mind that $\vec{\tau} = \vec{r} X \vec{F}$, what is the direction for the torque in this case?

A) down
B) up
C) into the screen
D) out of the screen
E) right

Question

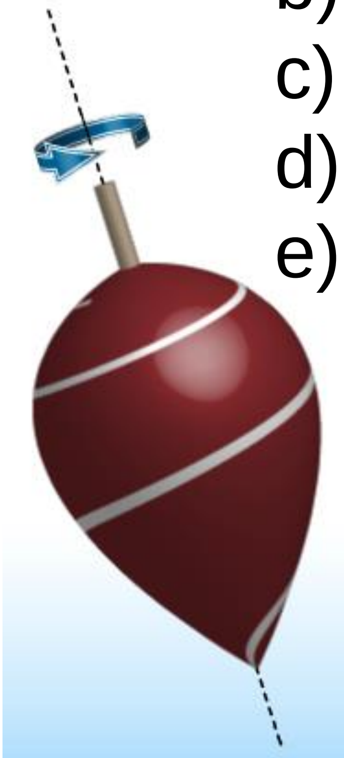
The cart below is moving to the right with a velocity v. In order to increase the linear velocity, of the cart, I should apply a force

a) in the same direction as v
b) in the opposite direction to v
c) perpendicular to v.

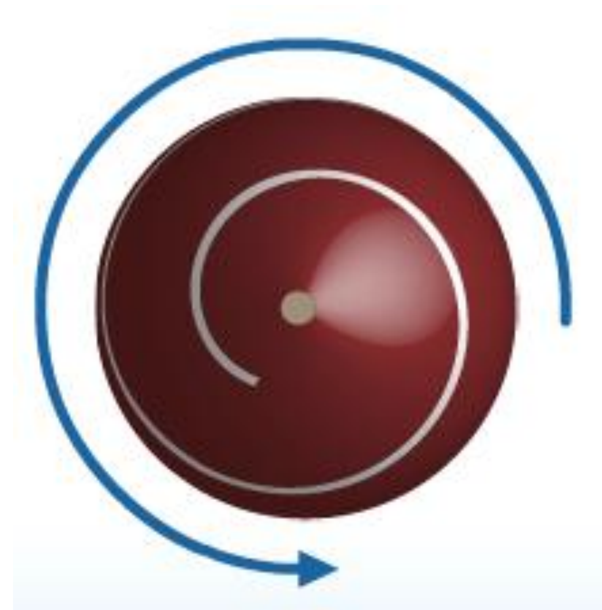
Question

The direction for ω in the top view to the right is:

a) out of the screen
b) into the screen
c) to the right
d) to the left
e) up



Side view



top view

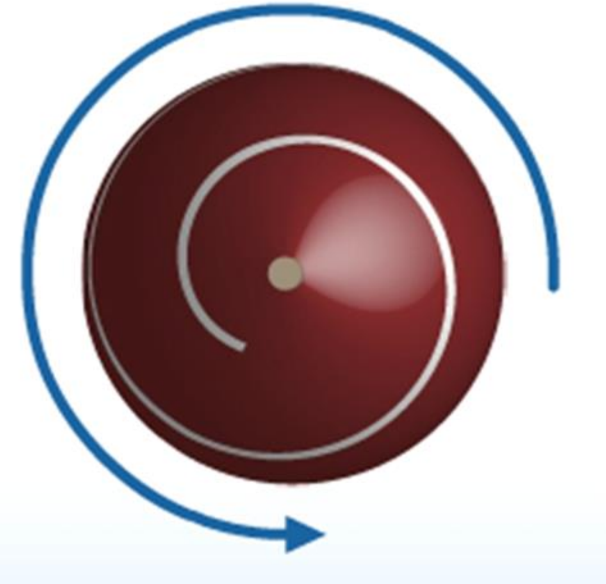
Question

The top below is rotating with a rotational velocity, ω, out of screen. In order to increase the rotational velocity, of top, I should apply a torque

a) in the same direction as ω

b) in the opposite direction to ω

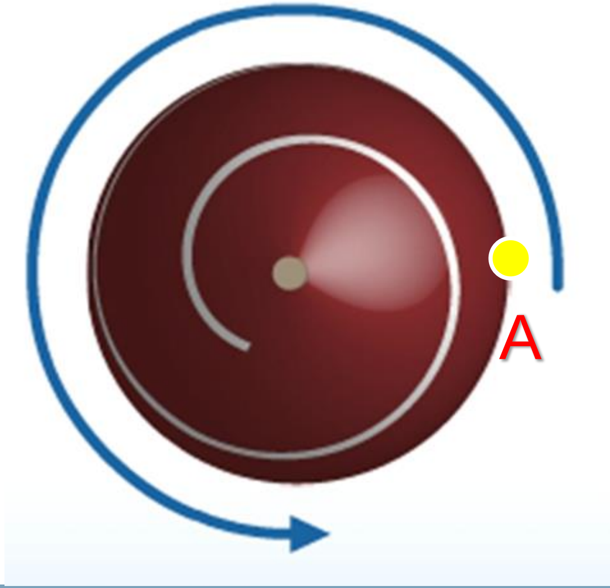
c) perpendicular to ω.



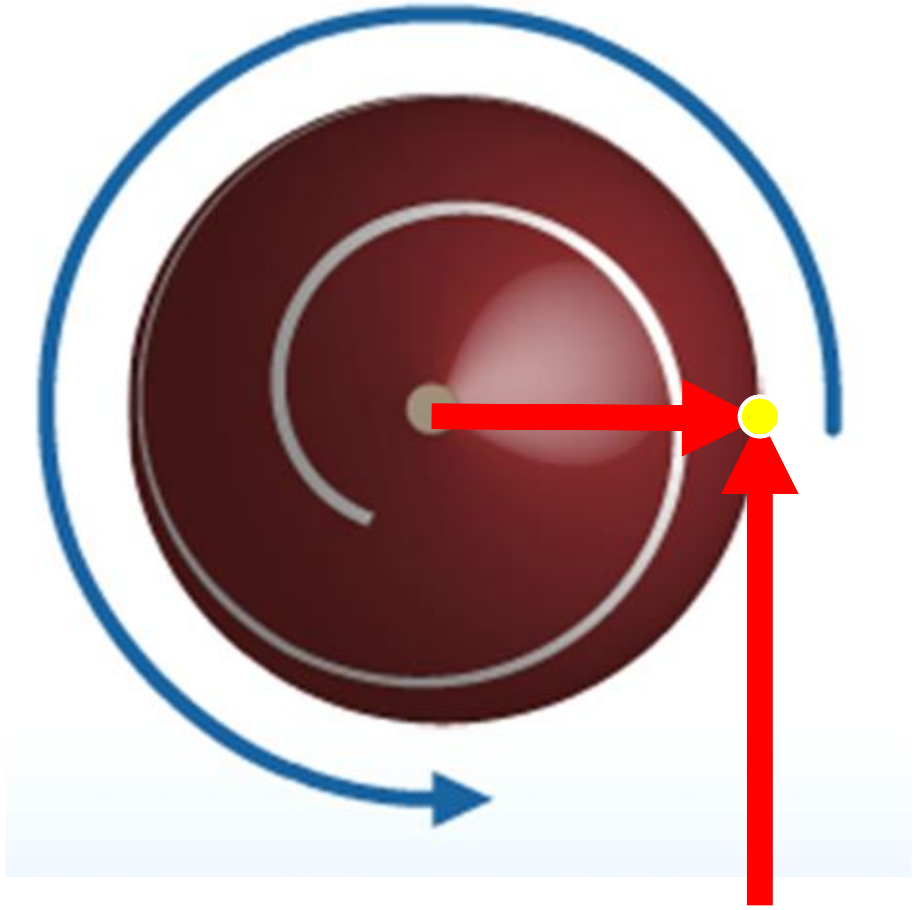
Question

In order to increase the rotational velocity, w of top by applying a force at point A, I should apply the force:

a) up
b) down
c) left
d) right
e) out of the screen



Direction of torque



Example 15.3 (torque on pulley)

A torque of 20Nm is applied to a pulley of mass 10kg with a radius of 25cm. The pulley is initially at rest. What is the angular velocity of the pulley 4 seconds after the torque is applied?

$I = \Sigma m_i r_i^2$

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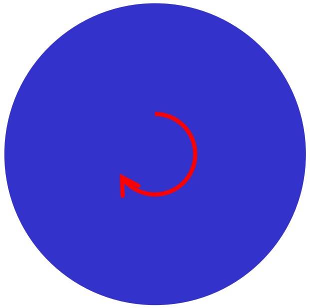
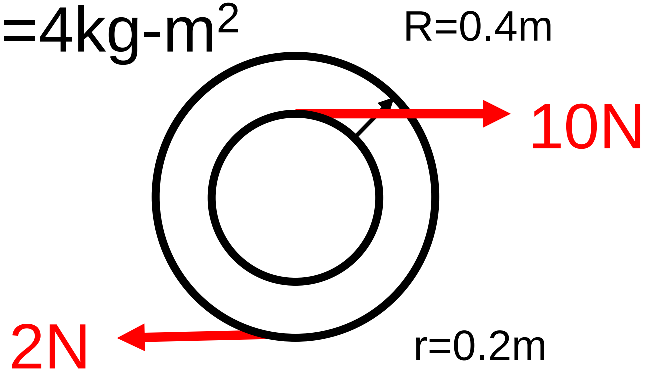


Question

m=12kg

I=4kg-m^2



Top view

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Two disks of different radii are glued together and are laying flat on a frictionless table.

- The inner disk with a radius of 0.2m is subjected to a force of 10N to the right.
- The outer disk with a radius of 0.4m is subjected to a force of 2N to the left.
- The disks have a combined mass of 12kg and combined moment of inertia of 4kg-m^2.

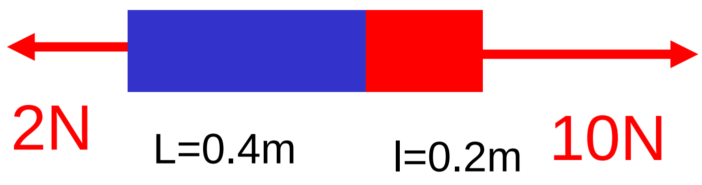
At this moment, what is linear acceleration of the center of mass of the combined disks?

A. 4m/sec^2
B. 2m/sec^2
C. 1m/sec^2
D. 0.66 m/sec^2
E. 0 m/sec^2

Question

m=12kg

I=4kg-m^2



Top view

A. 4m/sec^2
B. 2m/sec^2
C. 1m/sec^2
D. 0.66 m/sec^2
E. 0 m/sec^2

Two blocks of different lengths are glued together and are laying flat on a frictionless table.

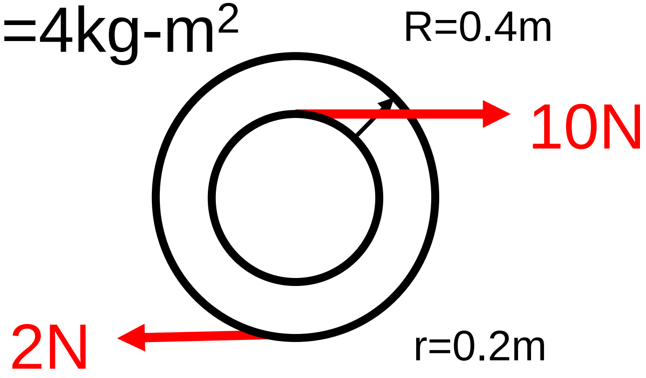
- The right block with a length of 0.2m is subjected to a force of 10N to the right.
- The left block with a length of 0.4m is subjected to a force of 2N to the left.
- The blocks have a combined mass of 12kg and combined moment of inertia of 4kg-m^2.

What is linear acceleration of the center of mass of the combined blocks?

Question

m=12kg

$I=4kg\text{-}m^2$



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Two disks of different radii are glued together and are laying flat on a frictionless table.

- The inner disk with a radius of 0.2m is subjected to a force of 10N to the right.
- The outer disk with a radius of 0.4m is subjected to a force of 2N to the left.
- The disks have a combined mass of 12kg and combined moment of inertia of $4kg\text{-}m^2$.

At this moment, what is linear acceleration of the center of mass of the combined disks?

A. $4m/sec^2$
B. $2m/sec^2$
C. $1m/sec^2$
D. $0.66\ m/sec^2$
E. $0\ m/sec^2$

IF YOU DID NOT UNDERSTAND THE ANSWER TO THE PREVIOUS CLICKER QUESTION, PLEASE ASK ABOUT IT.

IT HAS BEEN THE POINT OF *CONSIDERABLE* CONFUSION IN THE PAST.

Example 15.4 (pulley and mass)

A long length of string is wrapped around 5kg drum with a radius of 30cm. The drum is free to spin around a frictionless axle. The other end of the string is attached to a 10kg mass. If the mass is allowed to drop, what is its acceleration?

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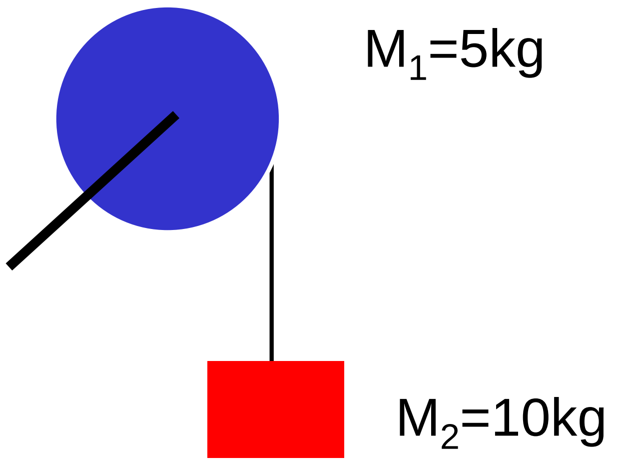
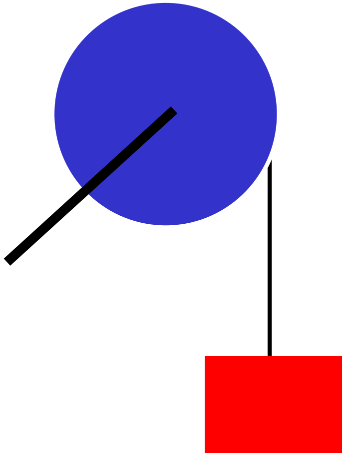
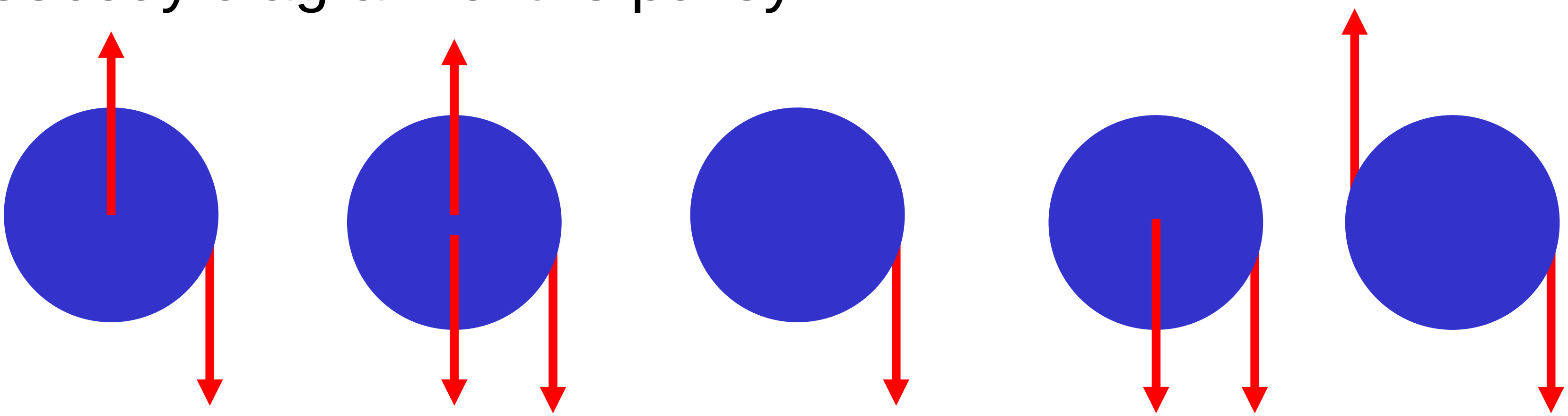


Question

A long length of string is wrapped around 5kg drum with a radius of 30cm. The drum is free to spin around a frictionless axle. The other end of the string is attached to a 10kg mass.



Which of the following best represents the freebody diagram of the pulley?

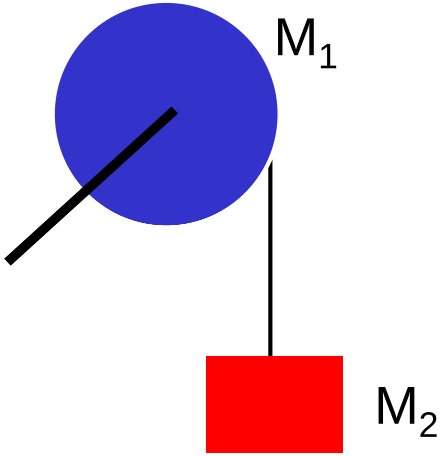


(A) **(B)** **(C)** **(D)** **(E)**

Limiting Case Check

A long length of string is wrapped around drum mass M_1 and radius of R. The drum is free to spin around a frictionless axle. The other end of the string is attached to a M_2 mass. If the mass is allowed to drop, what is its acceleration if **M_2>>>M_1**?

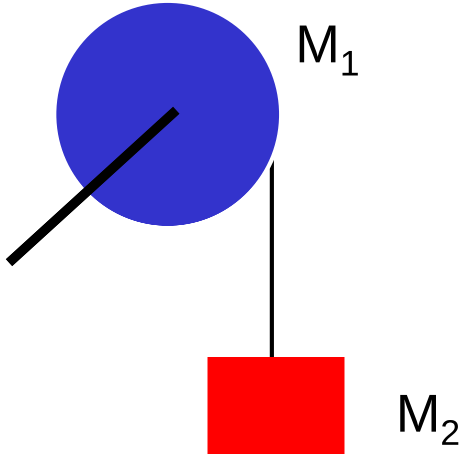
A. 0
B. infinity
C. g
D. g/2
E. 2g



Limiting Case Check

A long length of string is wrapped around drum mass M_1 and radius of R. The drum is free to spin around a frictionless axle. The other end of the string is attached to a M_2 mass. If the mass is allowed to drop, what is its acceleration if **$M_1 >>> M_2$**?

A. 0
B. infinity
C. g
D. g/2
E. 2g



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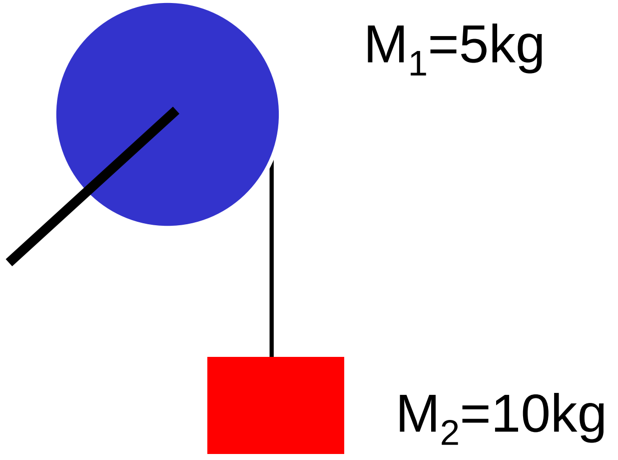
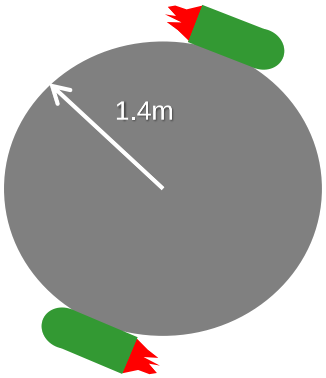


Example 15.5: Stopping a satellite



A cylindrical satellite is 1.4 meters in diameter with its 940kg mass uniformly distributed. The satellite is spinning at 10rpm but must be stopped so that astronauts can make repairs. Two small gas jets each with a 20N thrust are mounted on opposite sides of the satellite and fire tangent to the satellite's rim.

How long must the jets be fired in order to stop the satellite's rotation?

Example 15.6: Robot

A 70 pound robot is powered by four electric motors that can each put out 5 ft-lbs of torque to 6” wheels.

What is the maximum acceleration of the robot?