

Physics 2111

Unit 14

Today's Concepts:

- a) Rotational Motion
- b) Moment of Inertia

Which Method to Use (some hints)

- Force is constant in direction and magnitude
 - **Newton's Second Law**
- Force changes in direction or magnitude
 - **$W_{\text{tot}} = \Delta KE$**
- No non-conservative forces (e.g. no friction)
 - **$W_{\text{NC}} = \Delta ME$ (or $ME_o = ME_f$)**
- There is a collision or an explosion
 - **$\vec{J}_{\text{NET}} = \Delta \vec{p}$ ($\vec{p}_o = \vec{p}_f$ if $\vec{J}_{\text{NET}} = 0$)**
- Two objects are isolated (e.g Karen in boat)
 - **COM**
- Involves **time** in the question or given information
 - **Newton's Second Law**

Where we are.....

Thursday, October 18 | 7:32 AM cartert@cod.edu | account | log off

smartPhysics Physics 2111
College of DuPage

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- + Linear Dynamics
- + Conservation Laws
- Rotational Dynamics [Edit Title](#)
 - 14. Rotational Kinematics & Moment of Inertia ← **Some new definitions**
 - 15. Parallel-Axis Theorem and Torque
 - 16. Rotational Dynamics
 - 17. Rotational Statics
 - 18. Rotational Statics: Part II
 - 19. Angular Momentum
 - 20. Angular Momentum Vector and Precession
- + Applications

Daily Planner

Thursday, October 18

- 8:00 am [Homework - Center Of Mass](#)
- 8:00 am [Homework - Conservation Of Momentum](#)
- 8:00 am [PreLecture - Kinematics And Moment Of Inertia](#)
- 8:00 am [Checkpoint - Rotational Kinematics & Moment Of Inertia](#)


Monday, October 22

- 8:00 am [PreLecture - Parallel Axis Theorem And Torque](#)
- 8:00 am [Checkpoint - Parallel-Axis Theorem And Torque](#)

Wednesday, October 24

Announcements

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Your Comments

Why, when calculating moment of inertia for the solid cylinder, was the $d(\phi)$ component of the dV multiplied by r ?

Can we go over the triangle of spheres problem?

not too bad

The rotating hoop question was a bit confusing.

derivation for the moment of inertia of solid masses using densities

Calculating moment of inertia and moment of inertia around different axes.

I cranked this guy out, We got this guy.

Going over all the different types of acceleration, what they mean, and in what problems you find them in would be a nice refresher. Going over that part i realize how much I started to forgot about the other 2 when trying to mix them in with the new acceleration(s).

Please explain how to find moment of inertia for continuous bodies

Can we use the Parallel Axis Theorem? ;)

would like to see moment of inertias in rotation of just three points like the triangle of spheres checkpoint question, do we have to solve for the actual weight from both the 2L sides connecting the one dot on the triangle?

Summary of Rotations

$$x \longrightarrow \theta$$

$$dx/dt = v \longrightarrow \omega = d\theta/dt$$

$$dv/dt = a \longrightarrow \alpha = d\omega/dt$$

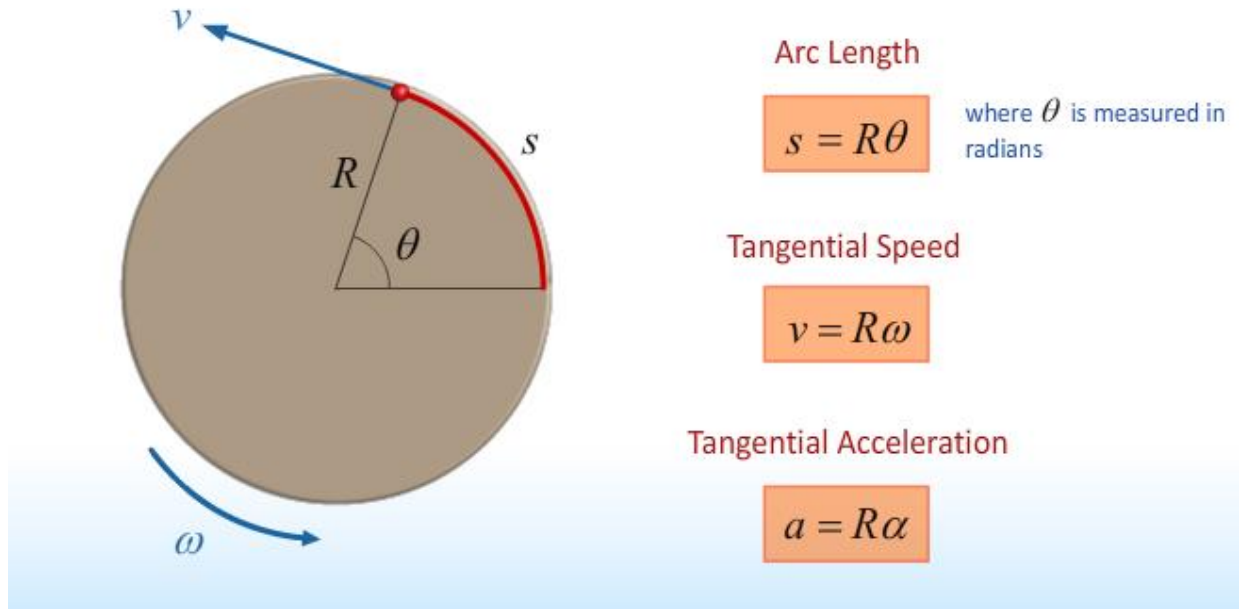
For constant angular acceleration

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$$

Rotational → Translational



Angular velocity ω is measured in radians/sec

Frequency f is measured in revolutions/sec

1 revolution = 2π radians

$$\omega = 2\pi f \xrightarrow{\text{Period } T = 1/f} \omega = \frac{2\pi}{T}$$

Question



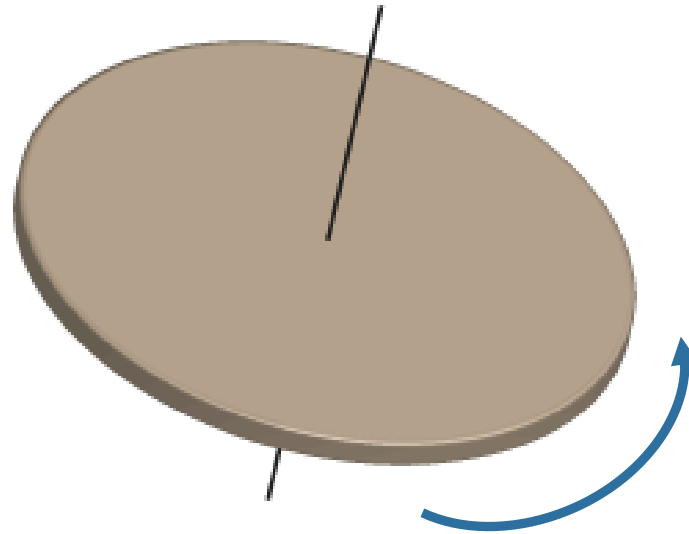
A disk spins at 2 revolutions/sec.

What is its angular velocity?

A) $\omega = 2\pi$ rad/sec

B) $\omega = \frac{\pi}{2}$ rad/sec

C) $\omega = 4\pi$ rad/sec



Example 14.1 (LP Record)

If you have an old LP record that's rotating at $33 \frac{1}{3}$ rpm, what is ω in rad/sec?

If you turn off the turntable and it completes two revolutions before coming to a stop, what is α ?

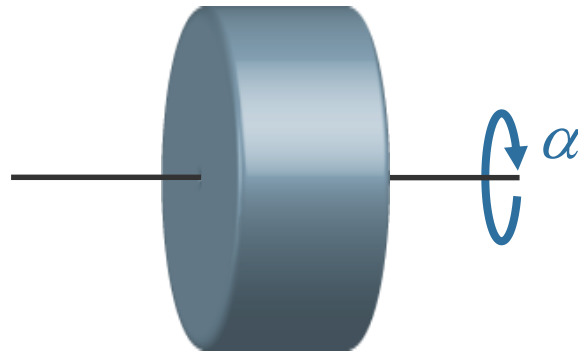
CheckPoint



A wheel which is initially at rest starts to turn with a constant angular acceleration. After 4 seconds it has made 4 complete revolutions.

How many revolutions has it made after 8 seconds?

- A) 8 B) 12 C) 16



Checkpoint 2

Remember this one?

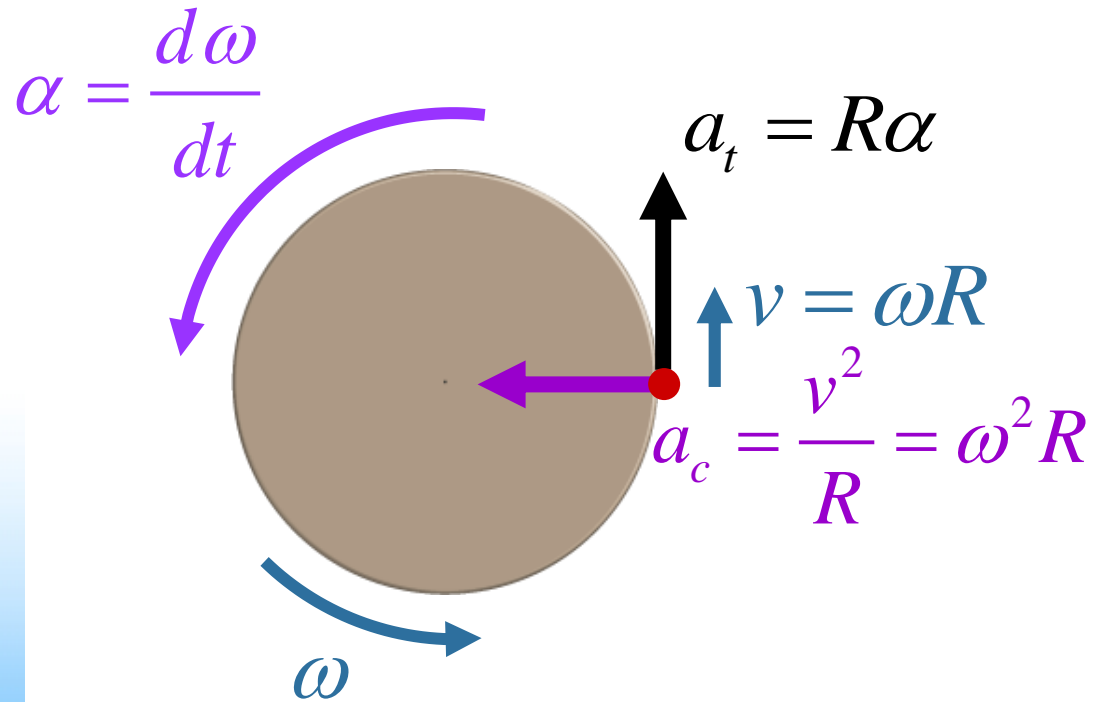


At $t = 0$ a ball, initially at rest, starts to roll down a ramp with constant acceleration. Suppose it moves 1 foot between $t = 0$ sec and $t = 1$ sec.

How far does it move between $t = 1$ sec and $t = 2$ sec?

- A) 1 foot B) 2 feet C) 3 feet D) 4 feet E) 6 feet

Lots of “accelerations”



For constant angular acceleration

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$$

Constant α does not mean constant ω

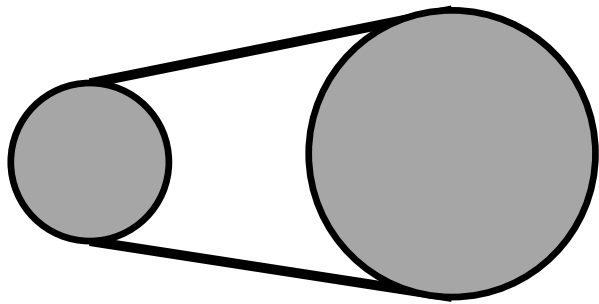
Toy Gyroscope



A small toy gyroscope has a radius of 2.8 cm and is accelerated from rest at 14.2 rad/sec^2 until its angular speed is 2760 rev/min.

- What is the tangential acceleration of a point on the rim of the gyroscope during this spin up?
- What is the centripetal acceleration of this point when it is spinning at full speed?
- What distance does this point travel during the spin-up?

Belt and Wheels



A small wheel with a radius of 10cm is coupled to a larger wheel with a radius of 25cm via belt.

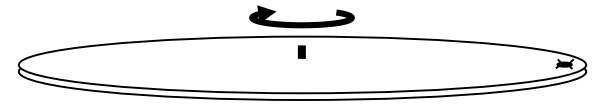
The smaller wheel starts from rest and increases its angular speed at a rate of 1.6rad/sec^2 .

How long does it take the large wheel to reach an angular speed of 100rev/min assuming the belt does not slip?

How many revolutions does the smaller wheel turn during this time?

Example 14.2 (Lady bug's acceleration)

A ladybug is sitting on the edge of Dr. Carter's Bruce Springsteen record album ($r=25\text{cm}$). The album starts from rest and uniformly accelerates up to its final ω_f of 4rad/sec over a period of 5 seconds.



What is a_{tot} of the ladybug 2 seconds after the record started to move?

What is a_{tot} of the ladybug 4 seconds after the record started to move?

summary so far

$$x \longrightarrow \theta$$

$$dx/dt = v \longrightarrow d\theta/dt = \omega$$

$$dv/dt = a \longrightarrow d\omega/dt = \alpha$$

$$m \longrightarrow ??$$

Rotational Kinetic Energy

$$K_{system} = \frac{1}{2} I \omega^2$$

Example 14.3 (MOI of mass on rod)

Moment of Inertia

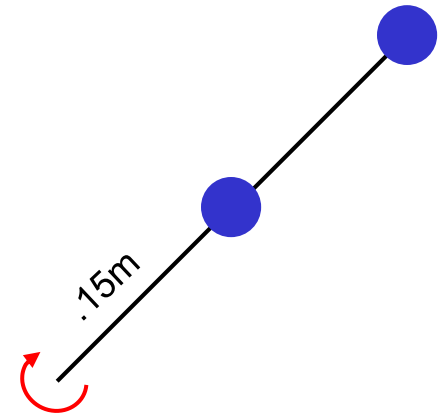
For Discrete Distributions

$$I \equiv \sum m_i r_i^2$$

For Continuous Distributions

$$I = \int r^2 dm$$

What is the moment of inertia of a 2kg mass attached to a 15cm rigid massless rod if it is rotated about the far end of the rod?



What if a second rod/mass combination is added?

Calculation Moment of Inertia

Moment of Inertia

For Discrete Distributions

$$I \equiv \sum m_i r_i^2$$

For Continuous Distributions

$$I = \int r^2 dm$$

Solid Cylinder

$$I = \frac{1}{2} MR^2$$



Cylindrical Shell

$$I = MR^2$$



Solid Sphere

$$I = \frac{2}{5} MR^2$$



Spherical Shell

$$I = \frac{2}{3} MR^2$$



Bigger when
the mass is
further out



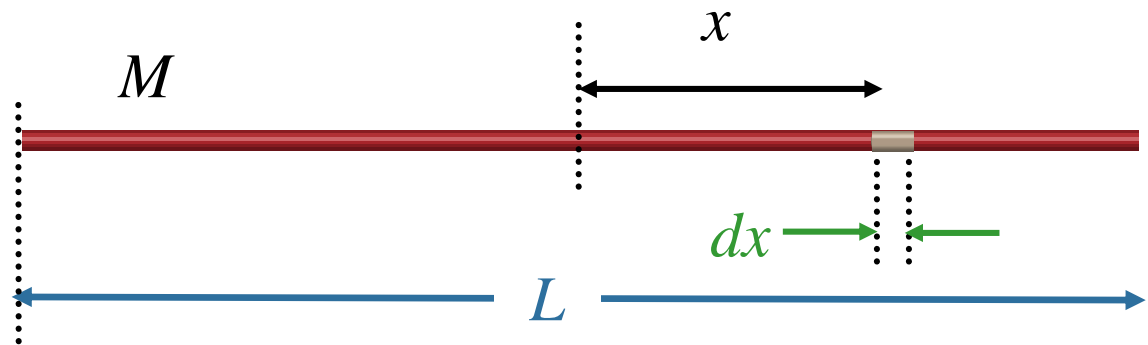
A mass M is uniformly distributed over the length L of a thin rod. The mass inside a short element dx is given by:

A) $M dx$

B) $\frac{dx}{M}$

C) $\frac{M}{L} dx$

D) $\frac{L}{M} dx$



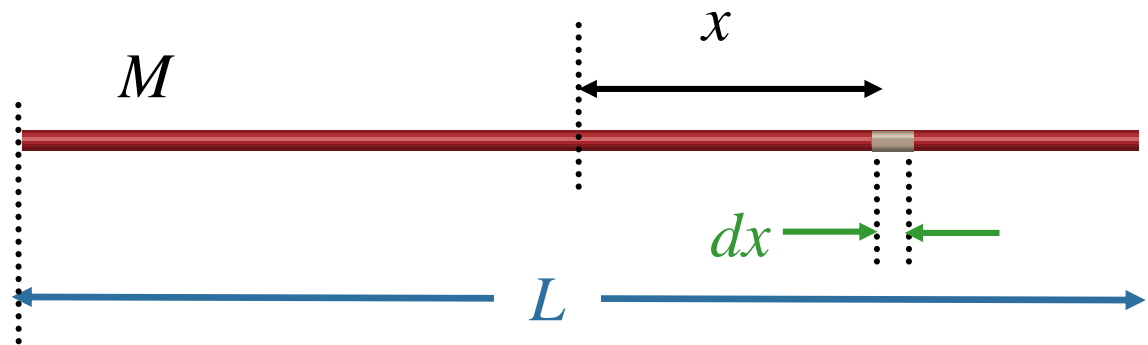


A mass M is uniformly distributed over the length L of a thin rod. The contribution to the rod's moment of inertia provided by element dx is given by:

A) $x^2 \frac{M}{L} dx$

B) $\frac{1}{x^2} \frac{M}{L} dx$

C) $\frac{M}{L} dx^2$

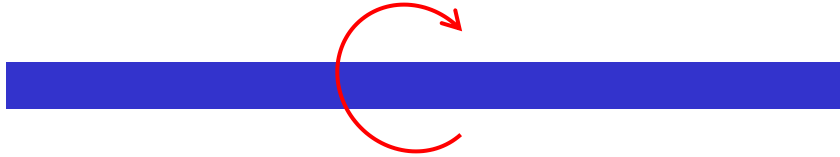


Example 14.4 (MOI of rod)



What is the moment of inertia of a 10kg rod, 0.5m in length, rotated about one end?

Example 14.4 (MOI of rod)



What is the moment of inertia of a 10kg rod, 0.5m in length, rotated about its center?

Example 14.5 (MOI of rod and mass)



What is the moment of inertia of our 10kg rod if a 20kg mass is attached and it is rotated about the far end?

Example 14.5 (MOI of rod and mass)



What is the moment of inertia of our 10kg rod if a 20kg mass is attached and it is rotated about the near end?

Equation Sheet

You are expected to understand the concept of moment of inertia and how to derive it for simple one dimensional cases.

Basic formulas on the equation sheet.

Rotational Dynamics

$$I = \sum m_i r_i^2$$

$$I_{\text{parallel}} = I_{\text{CM}} + MD^2$$

$$I_{\text{disk}} = I_{\text{cylinder}} = \frac{1}{2}MR^2$$

$$I_{\text{hoop}} = MR^2$$

$$I_{\text{solid-sphere}} = \frac{2}{5}MR^2$$

$$I_{\text{spherical shell}} = \frac{2}{3}MR^2$$

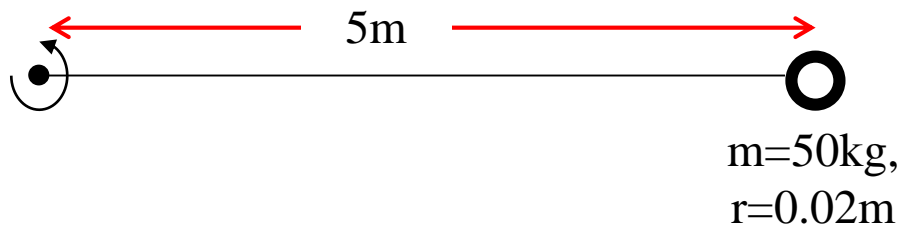
$$I_{\text{rod-cm}} = \frac{1}{12}ML^2$$

$$I_{\text{rod-end}} = \frac{1}{3}ML^2$$

$$\tau = I\alpha \text{ (rotation about a fixed axis)}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, |\tau| = rF\sin\phi$$

Question



A thin walled hollow sphere of mass 50kg and radius 2cm is attached to one end of a massless, rigid rod of length 5m . If the rod and sphere are rotated about the opposite end of the rod, which answer below is roughly the moment of inertia of the system.

$$I = \sum m_i r_i^2$$

$$I_{\text{parallel}} = I_{\text{CM}} + MD^2$$

$$I_{\text{disk}} = I_{\text{cylinder}} = \frac{1}{2}MR^2$$

$$I_{\text{hoop}} = MR^2$$

$$I_{\text{solid-sphere}} = \frac{2}{5}MR^2$$

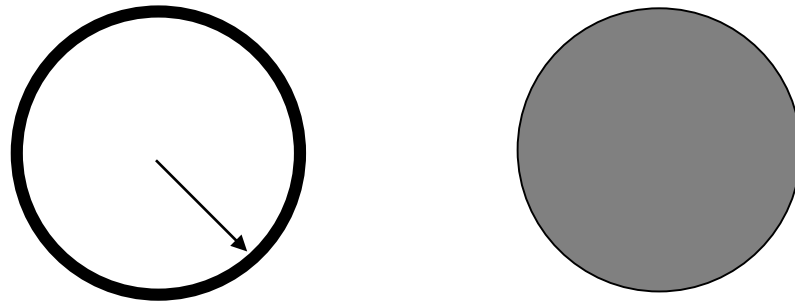
$$I_{\text{spherical shell}} = \frac{2}{3}MR^2$$

$$I_{\text{rod-cm}} = \frac{1}{12}ML^2$$

$$I_{\text{rod-end}} = \frac{1}{3}ML^2$$

- A. $\frac{2}{5} (50\text{kg})(0.02\text{m})^2$
- B. $\frac{2}{3} (50\text{kg})(0.02\text{m})^2$
- C. $\frac{2}{3} (50\text{kg})(5\text{m})^2$
- D. $(50\text{kg})(5\text{m})^2$
- E. $\frac{7}{5}(50\text{kg})(5\text{m})^2$

Question



A hoop and a solid disk both have the same mass, M , and the same radius, R . Which one of them has the greater moment of inertia when they are rotated about their centers?

- A. the hoop
- B. the disk
- C. both have the same moment of inertia

Checkpoint

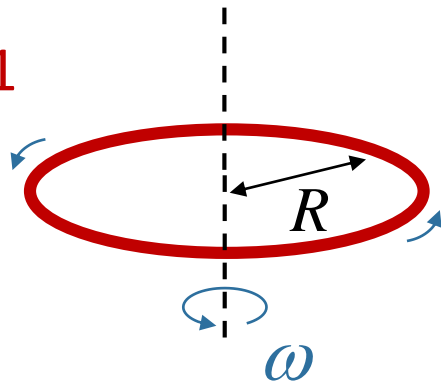


In both cases shown below a hula hoop with mass M and radius R is spun with the same angular velocity about a vertical axis through its center. In **Case 1** the plane of the hoop is parallel to the floor and in **Case 2** it is perpendicular.

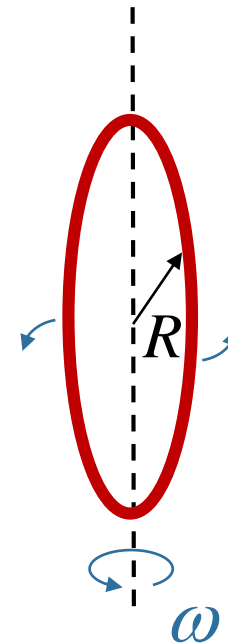
In which case does the spinning hoop have the most kinetic energy?

- A) Case 1 B) Case 2 C) Same

Case 1

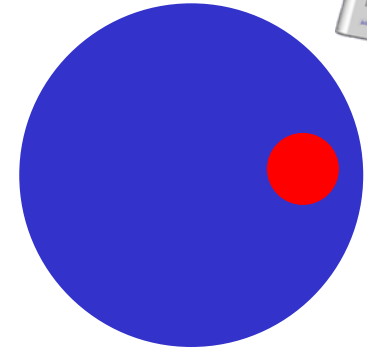
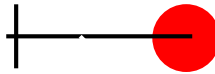
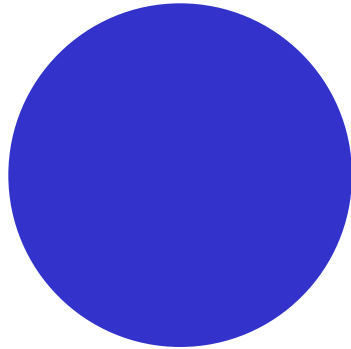


Case 2



Only about half got this right so lets try again...

Adding MOI



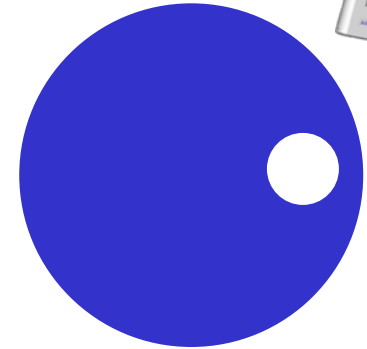
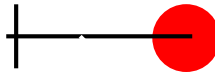
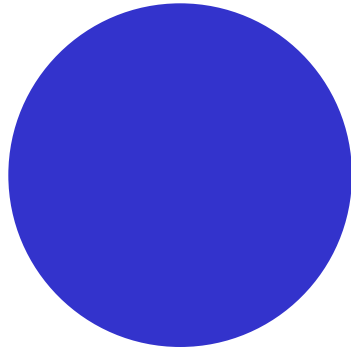
The moment of inertia of a disk with a radius of 7cm rotated about its center is 0.9 kg m^2 . The moment of inertia of a disk made of the same material with a radius of 1cm rotated about a point 5cm away is 0.1 kg m^2 .

What is the moment of inertia of a 7cm disk rotated about its center with a 1cm disk added to it at a distance 5cm from its center?

- A) 1.0 kg m^2
- B) 0.9 kg m^2
- C) 0.8 kg m^2

- D) 0.7 kg m^2
- E) 0.0 kg m^2

Adding MOI



The moment of inertia of a disk with a radius of 7cm rotated about its center is 0.9 kg m^2 . The moment of inertia of a disk made of the same material with a radius of 1cm rotated about a point 5cm away is 0.1 kg m^2 .

What is the moment of inertia of a 7cm disk rotated about its center with a 1cm hole cut in it at a distance 5cm from its center?

- A) 1.0 kg m^2
- B) 0.9 kg m^2
- C) 0.8 kg m^2

- D) 0.7 kg m^2
- E) 0.0 kg m^2