Physics 2111
Unit 22

Today's Concept:

Simple Harmonic Motion: *Motion of a Pendulum*

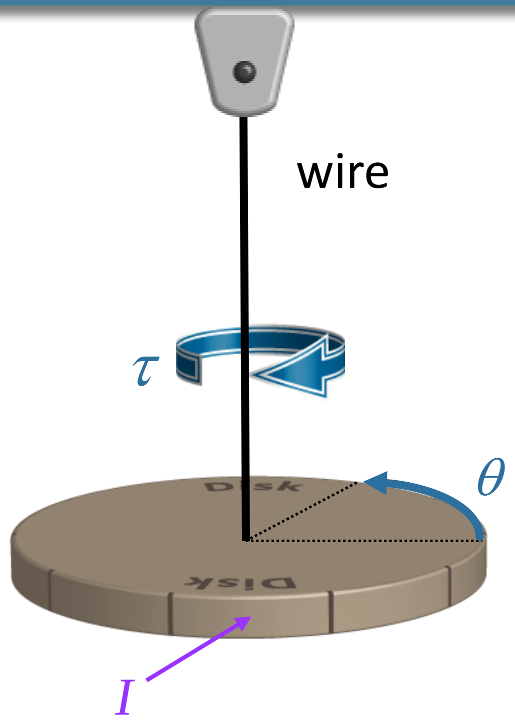
Torsion Pendulum

$$\tau = I\alpha$$

$$-\kappa\theta \qquad I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\omega = \sqrt{\frac{\kappa}{I}}$$



$$\theta(t) = \theta_{\max}\cos(\omega t + \phi)$$

Q: In the prelecture the equation for restoring torque is given as T=-k*theta in clockwise direction..so if the restoring torque is in counter clockwise directions then would T be positive?

CheckPoint

The little wire on a that holds torsion pendulum in this clock has a k value of 0.0012N-m/rad. The torsion pendulum consists of a 500g disk with a 4cm radius and two 50g disks with a 0.5cm radius that are located 2cm from the wire.

What is the period of the pendulum?

Pendulum

$$\tau = I\alpha$$

$$\tau \rightarrow -MgX_{CM}$$

For small θ

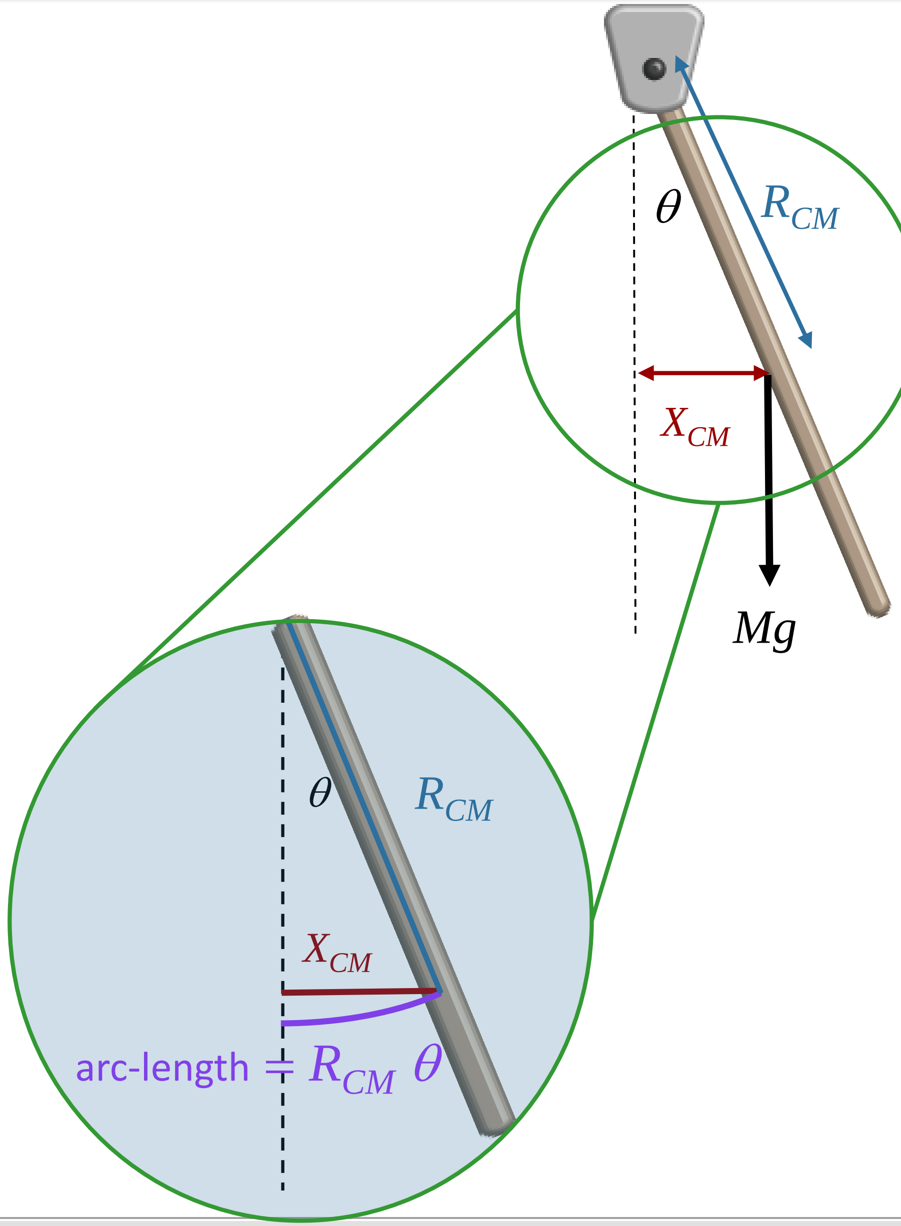
$$-MgX_{CM} \rightarrow -MgR_{CM}\theta$$

$$I\alpha \rightarrow I\frac{d^2\theta}{dt^2}$$

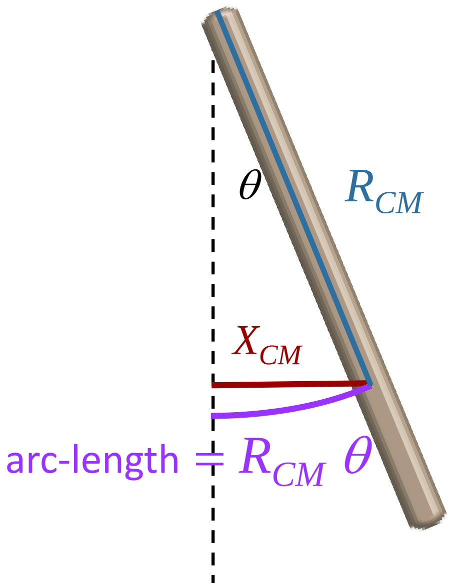
$$\frac{d^2\theta}{dt^2} = -\frac{MgR_{CM}}{I}\theta$$

$$\rightarrow \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

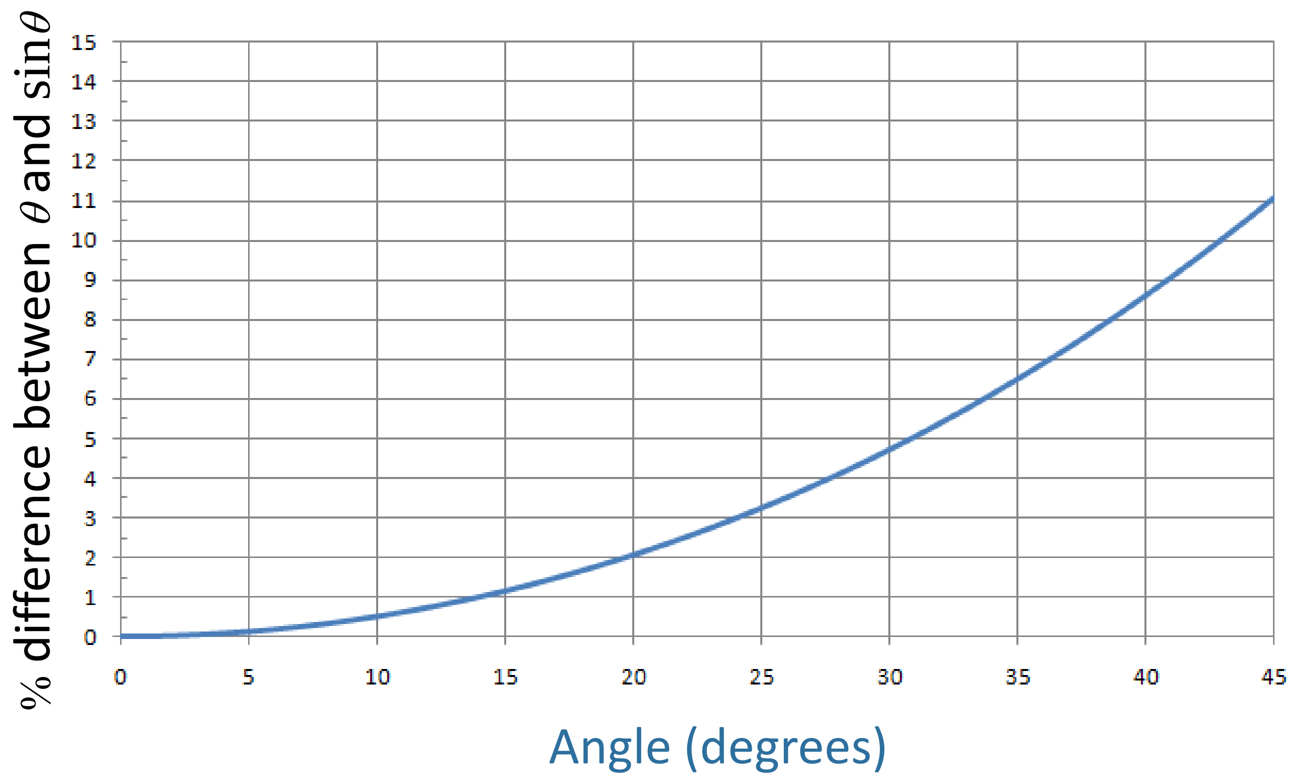
$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$



The Small Angle Approximation

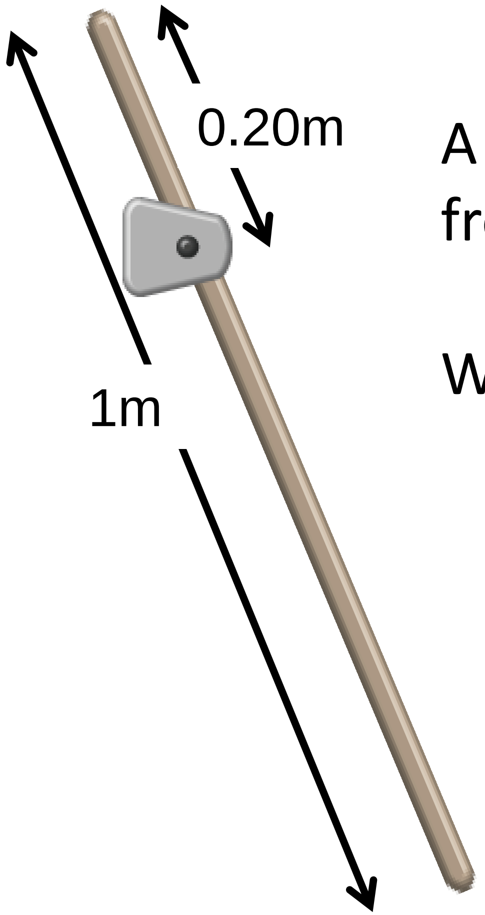


$$\frac{d^2\theta}{dt^2} = -\omega^2 \sin\theta$$



$$\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \ldots = \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \frac{1}{5040}\theta^7 + \ldots$$

Example 22.1 (pinned rod)

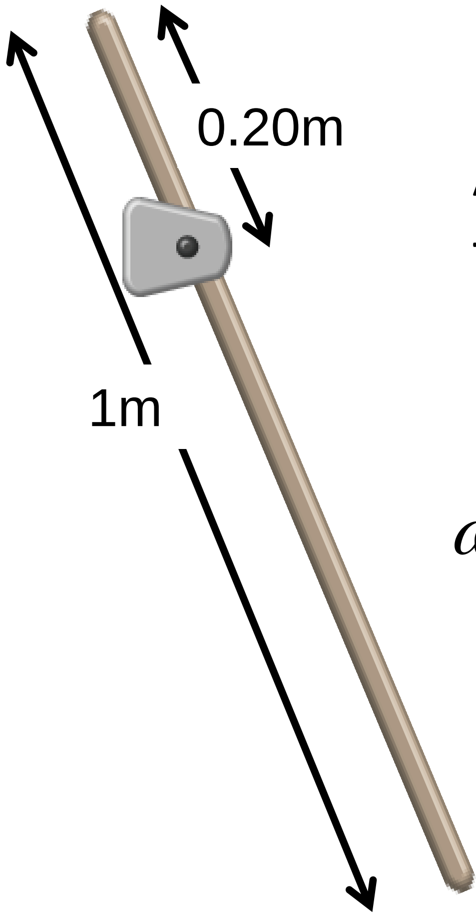


A 1m long rod has a mass of .15kg. It is pinned 20cm from one end and allowed to swing back and forth.

What is its period, T?

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

Question



A 1m long rod has a mass of .15kg. It is pinned 20cm from one end and allowed to swing back and forth.

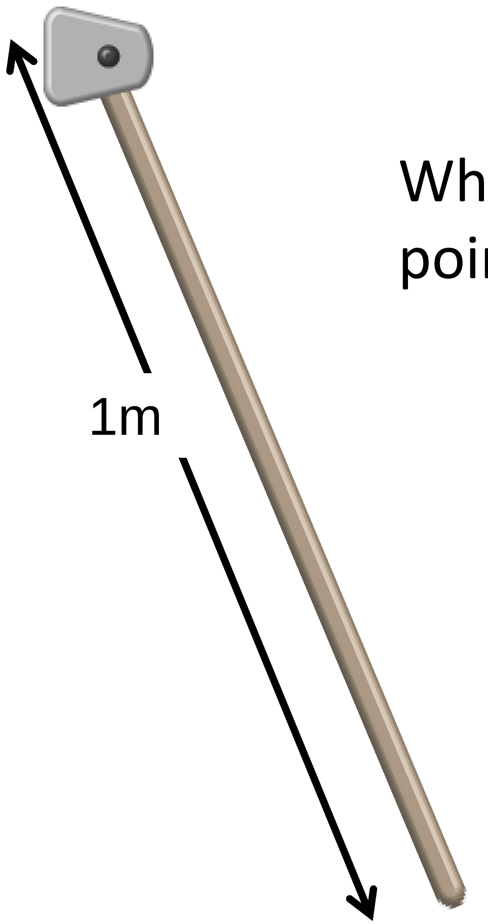
I want to figure out the period, T, using the formula

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

. What is R_{cm} in this case?

A) 0cm
B) 20cm
C) 30cm
D) 50cm
E) 100A

Example 22.2 (pinned rod)

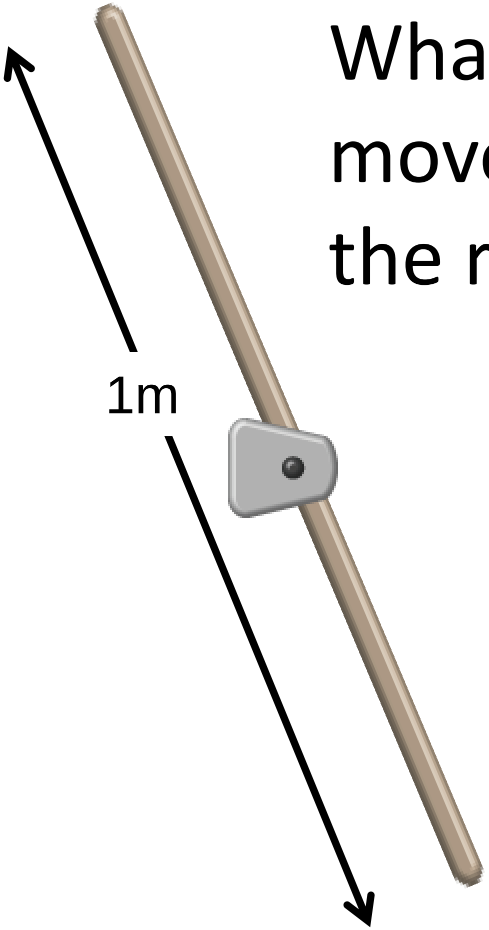


What will happen to the period if I move the pivot point up to one end?

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

Question

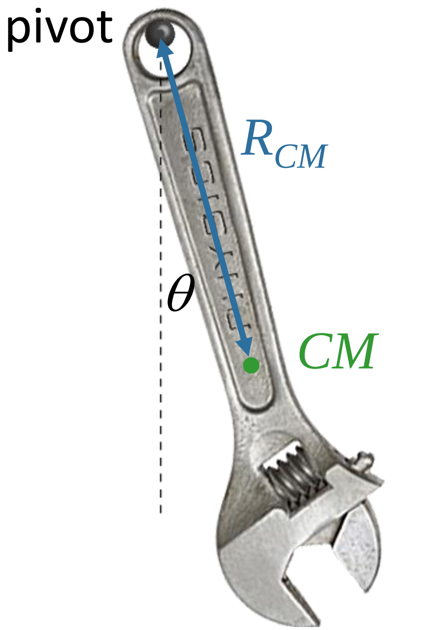
What will happen to the period if I move the pivot point to the middle of the rod?

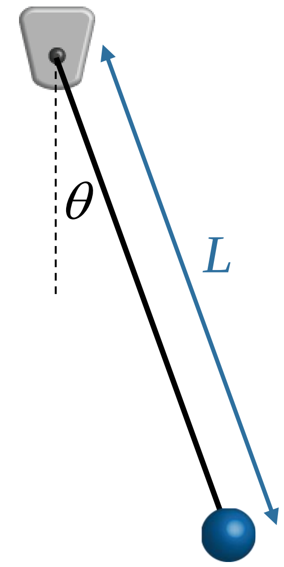


A) It will increase
B) It will decrease
C) It will go to infinity
D) It will go to zero

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

The Simple Pendulum





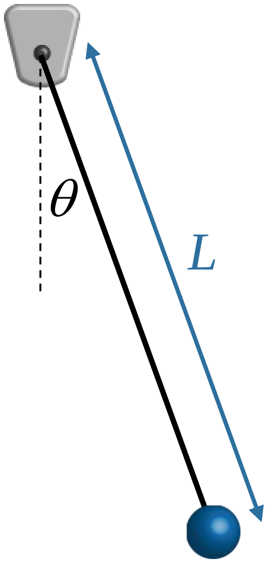
The simple case

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

$$\omega = \sqrt{\frac{MgL}{ML^2}} = \sqrt{\frac{g}{L}}$$

The general case

Example 22.2 (Simple Pendulum)



A 200gram mass is attached to the end of 2.0 meter long string and allowed to swing back and forth.

What is it's period, T?

Question

A pendulum is made by hanging a thin hoola-hoop of diameter D on a small nail.

What is the angular frequency of oscillation of the hoop for small displacements? ($I_{CM} = mR^2$ for a hoop)

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

A) $\omega = \sqrt{\frac{g}{D}}$

B) $\omega = \sqrt{\frac{2g}{D}}$

C) $\omega = \sqrt{\frac{g}{2D}}$