To review Polynomial Long Division, watch the following set of YouTube videos introducing a review of long division, division by a monomial, long division of a polynomial by another polynomial, and synthetic division. The videos are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. [https://www.youtube.com/watch?v=txEyF2QzCqs](https://www.youtube.com/watch?v=txEyF2QzCqs) (review of long division)
2. [https://www.youtube.com/watch?v=nv_djIs0v2U](https://www.youtube.com/watch?v=nv_djIs0v2U) (dividing by a monomial)
3. [https://www.youtube.com/watch?v=lh1wb6AehMI](https://www.youtube.com/watch?v=lh1wb6AehMI) (dividing by a binomial)
4. [https://www.youtube.com/watch?v=FTRDPB1wR5Y](https://www.youtube.com/watch?v=FTRDPB1wR5Y) (dividing polynomials)

Note that this presenter is locating terms in the quotient slightly differently than the presenter in Video 3. Both methods are correct.

5. [https://www.youtube.com/watch?v=lLgRS0mUZLw](https://www.youtube.com/watch?v=lLgRS0mUZLw) (synthetic division)

**Practice problems:** The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact [Math Assistance Area](#).

1. a. Simplify: \( \frac{3x^5}{2x^3} = \) ____________
   b. Simplify: \( \frac{4x^7}{8x^2} = \) ____________

2. Divide: \( \frac{6x^4+9x^3-3x^2+4x+1}{3x^2} \)

3. Divide: \( (x^4 + 3x^3 - 2x^2 - 3x + 1) \div (x + 2) \)

4. \( \frac{x^3-3x^2-4x+12}{x-3} = \) ____________

5. Divide: \( \frac{8x^3+2x^2+1}{2x^2-x} \)

6. Use long division: \( \frac{x^6-64}{x-2} \)
7. Divide: \[ \frac{2x^4+x^3+x-\frac{3}{4}}{x-\frac{1}{2}} \]

8. Divide: \[ \frac{4x^5+13x^4+108x^2-81}{x^2+9} \]

9. Find \((5x^4 + 5x^2 + 5) ÷ (x^2 - x + 1)\)

10. The difference of cubes factoring pattern can be easily forgotten, but can be derived by doing the following long division problem: \( \frac{a^3-b^3}{a-b} = \) \( \) \( \). So \( a^3 - b^3 = (a - b)(\) \() \)

11. Redo the division in Question #3 using synthetic division.

12. Redo Question #6 using synthetic division.

Answers:

1. a. \( \frac{3}{2}x^2 \)  b. \( \frac{x^5}{2} \)  2. \( 2x^2 + 3x - 1 + \frac{4x+1}{3x^2} \)  3. \( x^3 + x^2 - 4x + 5 - \frac{9}{x+2} \)

4. \( x^2 - 4 \) (remainder = 0)

Note: Since \( \frac{x^3-3x^2-4x+12}{x-3} = x^2 - 4 \), then \( x^3 - 3x^2 - 4x + 12 = (x - 3)(x^2 - 4) \).

\((x - 3)\) is a factor of \( x^3 - 3x^2 - 4x + 12 \).

5. \( 4x + 3 + \frac{3x+1}{2x^2-x} \)  6. \( x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32 \)

7. \( 2x^3 + 2x^2 + x + \frac{3}{2} \)  8. \( 4x^3 + 13x^2 - 36x + 9 - \frac{324x}{x^2+9} \)

9. \( 5x^2 + 5x + 5 \)  10. \( (a^2 + ab + b^2) \)

11. (same as Q. 3)  12. (same as Q. 6)
Detailed Solutions

1. \[
\frac{3x^5}{2x^3} = \frac{3x \cdot x \cdot x \cdot x \cdot x}{2 \cdot x \cdot x} = \frac{3x^2}{2}
\]

b. \[
\frac{4x^7}{8x^4} = \frac{4 \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot x \cdot x \cdot x} = \frac{x^3}{2}
\]

2. \[
\frac{2x^2 + 3x - 1}{3x^2 - 6x^4 + 9x^2 - 3x^2 + 4x + 1}
\]

\[
\frac{2x^2}{3x^2} = 2x^2
\]

\[
\frac{x^3}{3x^2} = 3x
\]

\[
\frac{4x + 1}{(x^2 + 3x - 1) \div \frac{4x + 1}{3x^2}} = 2x^2 + 3x - 1 + \frac{4x + 1}{3x^2}
\]

3. \[
\frac{x^3 + x^2 - 4x + 5}{x + 2}
\]

\[
\frac{x^4}{x} = x^3
\]

\[
\frac{x^3}{x} = x^2
\]

\[
-x \cdot 2 \cdot x = -4x
\]

\[
\frac{5x}{x} = 5
\]

\[
\frac{x^4 + 3x^3 - 2x^2 - 3x + 1}{x + 2} = \frac{x^3 + x^2 - 4x + 5 - \frac{9}{x^2}}{x + 2}
\]
4. \[ \frac{x^3 - 3x^2 - 4x + 12}{x-3} \rightarrow \]
\[
\begin{align*}
\frac{x^2 - 4}{x-3} & = \frac{x^3 - 3x^2 - 4x + 12}{x-3} \\
& \quad \frac{x^2 - 4}{x-3} \\
& \quad -(x^3 - 3x^2) \\
& \quad -4x + 12 \\
& \quad \boxed{x^2 - 4} \\
& \quad -(-4x + 12) \\
& \quad 0
\end{align*}
\]

or...

\[ x^3 - 3x^2 - 4x + 12 = (x-3)(x^2 - 4) \]
and \((x-3)\) is a factor of \(x^3 - 3x^2 - 4x + 12\).

5. \[ \frac{4x + 3}{2x^2 - x} \]
\[
\begin{align*}
& \frac{4x + 3}{2x^2 - x} \\
& \quad - \frac{8x^3 + 2x^2 + 0x + 1}{2x^2 - x} \\
& \quad 2x^2 - x \\
& \quad 3x + 1
\end{align*}
\]

Note: the dividend is missing the "x^4" term, so a "0x" has been written in as a placeholder.

\[
\begin{align*}
8x^3 & = 4x \\
2x^2 & = 3 \\
4x(2x^2 - x) & = 8x^3 - 4x^2 \\
3(2x^2x) & = 6x^3 - 3x
\end{align*}
\]

We stop here since the highest power of the divisor is more than the highest power on the remainder.

\[
\frac{8x^3 + 2x^2 + 1}{2x^2 - x} = \frac{4x + 3 + 3x + 1}{2x^2 - x}
\]
\[ \frac{x^4 - 64}{x - 2} \Rightarrow \]

\[ x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32 \]
\[ - 2 \left( x^3 + 2x^2 + x \right) \]
\[ = x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32 \]

or \[ x^4 - 64 = (x - 2)(x^3 + 2x^2 + 4x^3 + 8x^2 + 16x + 32) \]

\[ x - \frac{1}{2} \left[ 2x^2 + x + \frac{3}{2} \right] \]
\[ = 2x^3 + 2x^2 + x + \frac{3}{2} \]

\[ 2x^3 + 2x^2 + x + \frac{3}{2} \]
\[ = \frac{2x^3}{x} = 2x^2 \]
\[ = \frac{2x^2}{x} = 2 \frac{x}{x} = 2 \]
\[ \frac{x}{x} = x \]
\[ \frac{2}{x} \]

\[ \frac{3}{2} \]
\[ \frac{3}{2} \]

\[ \frac{3}{2} \]
8. \[ \frac{4x^3 + 13x^2 - 36x - 9}{x^2 + q} \]

\[ = \frac{4x^5 + 13x^4 + 108x^2 - 81}{x^2 + q} \]

\[ = \frac{4x^3 + 13x^2 - 36x - 9 + \frac{324x}{x^2 + q}}{x^2 + q} \]

9. \[ \frac{5x^4 + 5y^2 + 5}{x^2 - x + 1} \]

\[ = \frac{5(x^2 + y^2) + 5}{x^2 - x + 1} \]

\[ = \frac{5x^2 + 5x + 5}{x^2 - x + 1} \]

\[ = \frac{5(x^2 + x + 1)}{x^2 - x + 1} \]

Note: the "5" could have been factored out of the divisor, making the problem:

\[ 5 \left[ \frac{x^2 + x + 1}{x^2 - x + 1} \right] = 5 \left[ x^2 + x + 1 \right] = 5x^2 + 5x + 5 \]

This would have been the resulting quotient.

Math Assistance Area
Learning Commons: One-stop Academic Support Center
Stop by or call (630) 942-3339
10. \[ \frac{a^3 - b^3}{a - b} \]

(Treat "a" as the variable when including missing terms.)

So, \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

11. \[ \frac{x^4 + 3x^3 - 2x^2 - 3x + 1}{x + 2} \]

Quotient: \[ \frac{x^3 - 4x^2 + 5x - 9}{x + 2} \]
12. \( \frac{x^6 - 64}{x-2} \)

Notes: In synthetic division, the powers of \( x \) are not written, so it is critical that any missing powers of \( x \) are represented with a "0" coefficient.

\[ x^6 - 64 = x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 64 \]

\[
\begin{array}{c|cccccc}
& 1 & 0 & 0 & 0 & 0 & -64 \\
\hline
2 & 2 & 4 & 8 & 16 & 32 & 64 \\
1 & 2 & 4 & 8 & 16 & 32 & 0 \\
\end{array}
\]

\[
\frac{x^6 - 64}{x-2} = x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32
\]

---

**Additional Resources**

Click on the links below to download a free online worksheet for more practice:

1. Go To

2. For more help please contact the [Math Assistance Area](mailto:).