Today’s Concepts:

A) LC circuits and Oscillation Frequency
B) Energy
C) RLC circuits and Damping
differential equations killing me. Differential equations have taken over my life. I'm not sure I We've been doing spring stuff in Diff Eq, so this is lining up nicely! I was never good with springs during 2111 so that pre-lecture just was rugged. The pre-lecture said without a battery an LC circuit with a full capacitor would just oscillate back and forth since there is no energy that can be added, but isn't there heat energy being released through the wires that will also dampen the oscillation? For capacitors you told us that huge capacitors were used in car stereo systems. A similar example for how capacitors and inductors are used would be nice.

Can you go over the differential equations again? I find it very convenient that this is pretty much exactly like last semesters spring problems. I'll need practice, but it seems very straight forward!

Can we relate flux to snow?
**LC Circuit**

Circuit Equation: \( \frac{Q}{C} + L \frac{dI}{dt} = 0 \)

\( I = \frac{dQ}{dt} \)

\[ \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \]

\[ \frac{d^2Q}{dt^2} = -\omega^2 Q \]

where \( \omega = \frac{1}{\sqrt{LC}} \)

Solution to this DE: \( Q(t) = Q_{\text{max}} \cos(\omega t + \phi) \)
Just like in 2111

\[ \frac{d^2 Q}{dt^2} = -\omega^2 Q \quad \omega = \frac{1}{\sqrt{LC}} \]

\[ \frac{d^2 x}{dt^2} = -\omega^2 x \quad \omega = \sqrt{\frac{k}{m}} \]

Same thing if we notice that

\[ k \leftrightarrow \frac{1}{C} \quad \text{and} \quad m \leftrightarrow L \]
Also just like in 2111

\[ \frac{1}{2} m v^2 \leftrightarrow \frac{1}{2} k x^2 \]

Total Energy constant (when no friction present)

\[ \frac{1}{2} L I^2 \leftrightarrow \frac{1}{2} \frac{Q^2}{C} \]

Total Energy constant (when no resistor present)

Electricity & Magnetism  Lecture 19, Slide 5
The switch is initially open and the circuit is oscillating with a frequency \( \omega = \frac{1}{\sqrt{LC}} \).

What happens to the frequency \( \omega \) of the oscillations when the switch is closed?

A. Increases (faster oscillations)
B. Remains the same
C. Decreases (slower oscillations)
Charge and Current "90° out of phase"
At time $t = 0$ the capacitor is fully charged with $Q_{\text{max}}$ and the current through the circuit is 0.

What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$

B) $V_L = \frac{Q_{\text{max}}}{C}$ since $V_L = V_C$

C) $V_L = \frac{Q_{\text{max}}}{2C}$

The voltage across the capacitor is $\frac{Q_{\text{max}}}{C}$. Kirchhoff's Voltage Rule implies that must also be equal to the voltage across the inductor.
At time $t = 0$ the capacitor is fully charged with $Q_{max}$ and the current through the circuit is 0.

What is the potential difference across the inductor at when the current is maximum?

A) $V_L = 0$

B) $V_L = Q_{max}/C$

C) $V_L = Q_{max}/2C$

$dI/dt$ is zero when current is maximum
At time $t = 0$ the capacitor is fully charged with $Q_{\text{max}}$ and the current through the circuit is 0.

How much energy is stored in the capacitor when the current is a maximum?

A) $U = \frac{Q_{\text{max}}^2}{2(2C)}$

B) $U = \frac{Q_{\text{max}}^2}{4C}$

C) $U = 0$

Total Energy is constant!

$U_{\text{Lmax}} = \frac{1}{2} LI_{\text{max}}^2$

$U_{\text{Cmax}} = \frac{Q_{\text{max}}^2}{2C}$

$I = \text{max}$ when $Q = 0$
The switch is closed for a long time, resulting in a steady current $V_b/R$ through the inductor.

At time $t = 0$, the switch is opened, leaving a simple LC circuit. Which formula best describes the charge on the capacitor as a function of time?

A. $Q(t) = Q_{\text{max}} \cos(\omega t)$
B. $Q(t) = Q_{\text{max}} \cos(\omega t + \pi/4)$
C. $Q(t) = Q_{\text{max}} \cos(\omega t + \pi/2)$
D. $Q(t) = Q_{\text{max}} \cos(\omega t + \pi)$
After being left in position 1 for a long time, at \( t=0 \), the switch is flipped to position 2.

All circuit elements are “ideal”.

What is the equation for the charge on the upper plate of the capacitor at any given time \( t \)?

How long does it take the lower plate of the capacitor to fully discharge and recharge?
Example 19.1 ( Charged LC circuit )

To figure out the equation for the charge on the upper plate of the capacitor at any given time $t$, I’m going fill in the number in this equation:

$$Q(t) = Q_{\text{max}} \cos(\omega t + \phi)$$

How will I figure out $Q_{\text{max}}$?

A) Find the current through the inductor when the switch is in position 1. Energy will be conserved.

B) Find the rate of energy lost in the resistor when the switch is position 1. Energy will be conserved.

C) Find the voltage drop across the inductor when the switch is in position 1. The inductor and the capacitor will be in parallel.
Example 19.1 (Charged LC circuit)

After being left in position 1 for a long time, at t=0, the switch is flipped to position 2.

All circuit elements are “ideal”.

What is the equation for the charge on the upper plate of the capacitor at any given time $t$?

How long does it take the lower plate of the capacitor to fully discharge and recharge?
The capacitor is charged such that the top plate has a charge \(+Q_0\) and the bottom plate \(-Q_0\). At time \(t = 0\), the switch is closed and the circuit oscillates with frequency \(\omega = 500\) radians/s.

What is the value of the capacitor \(C\)?

- A) \(C = 1 \times 10^{-3} \text{ F}\)
- B) \(C = 2 \times 10^{-3} \text{ F}\)
- C) \(C = 4 \times 10^{-3} \text{ F}\)

\[
\omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(25 \times 10^4)(4 \times 10^{-3})} = 10^{-3}
\]
Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

\[ U_L = \frac{1}{2} LI^2 \]

Energy proportional to \( I^2 \) \( \implies \) \( C \) cannot be negative

Current is changing \( \implies U_L \) is not constant

Initial current is zero
When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

$Q$ is maximum when current goes to zero

$I = \frac{dQ}{dt}$

Current goes to zero twice during one cycle
Switch is flipped to position 2.

Use “characteristic equation”

\[-L \frac{d^2 Q}{dt^2} - R \frac{dQ}{dt} - \frac{1}{C} Q = 0\]

\[at^2 + bt + c = 0\]
Damped Harmonic Motion

\[ Q = Q_{Max} e^{-\beta t} \cos(\omega' t + \varphi) \]

Damping factor
\[ \beta = \frac{R}{2L} \]

Natural oscillation frequency
\[ \omega_o = \frac{1}{\sqrt{LC}} \]

Damped oscillation frequency
\[ \omega'^2 = \omega_o^2 - \beta^2 \]
Remember from 2111?

Overdamped
Critically damped
Under damped

Which one do you want for your shocks on your car?
Example 19.2 (Charged LC circuit)

After being left in position 1 for a long time, at t=0, the switch is flipped to position 2.

The inductor now non-ideal has some internal resistance.

What is the equation for the charge on the capacitor at any given time $t$?

How long does it take the lower plate of the capacitor to fully discharge and recharge?
Example 19.2 (Charged LC circuit)

When we did this problem before without any resistance, we found that it takes 0.57 sec for the lower plate of the capacitor to fully discharge and recharge.

How long will it take now that I am accounting for the resistance of the inductor?

A. more than 0.57sec
B. less than 0.57sec
C. the discharge time is not effected by the resistance
The elements of a circuit are very simple:

\[ V_L = L \frac{dI}{dt} \]

\[ V = V_L + V_C + V_R \]

\[ V_C = \frac{Q}{C} \]

\[ I = \frac{dQ}{dt} \]

\[ V_R = IR \]

This is all we need to know to solve for anything.

But these all depend on each other and vary with time!!
Start with some initial $V, I, Q, V_L$

Now take a tiny time step $dt$ (1 ms)

$dI = \frac{V_L}{L} dt$

$dQ = I dt$

$V_C = \frac{Q}{C}$

$V_R = IR$

$V_L = V - V_R - V_C$

What would this look like?
In the circuit to the right

- \( R_1 = 100 \, \Omega \),
- \( L_1 = 300 \, \text{mH} \),
- \( L_2 = 180 \, \text{mH} \),
- \( C = 120 \, \mu\text{F} \) and
- \( V = 12 \, \text{V} \).

(The positive terminal of the battery is indicated with a + sign. All elements are considered ideal.)

The switch has been closed for a long time. What is the energy stored on the capacitor?

A. 0 J
B. 0.0015 J
C. 0.0034 J
D. 0.0069 J
E. 1.2 J
In the circuit to the right
- \( R_1 = 100 \, \Omega \),
- \( L_1 = 300 \, \text{mH} \),
- \( L_2 = 180 \, \text{mH} \),
- \( C = 120 \, \mu\text{F} \) and
- \( V = 12 \, \text{V} \).
(The positive terminal of the battery is indicated with a + sign. All elements are considered ideal.)

The switch has been closed for a long time. What is the energy stored in the inductors?

A. 0 J  
B. 0.0015J  
C. 0.0034J  
D. 0.0069J  
E. 1.2J
Example 19.3

In the circuit to the right

• \( R_1 = 100 \ \Omega \),
• \( L_1 = 300 \ \text{mH} \),
• \( L_2 = 180 \ \text{mH} \),
• \( C = 120 \ \mu\text{F} \) and
• \( V = 12 \ \text{V} \).

The positive terminal of the battery is indicated with a + sign. All elements are considered ideal. \( Q(t) \) is defined to be positive if \( V(a) - V(b) \) is positive. The switch has been closed for a long time.

What is the charge on the bottom plate of the capacitor 2.82 seconds after switch is open?

- **Conceptual Analysis**
  Find the equation of \( Q \) as a function of time

- **Strategic Analysis**
  Determine initial current
  Determine oscillation frequency \( \omega_0 \)
  Find maximum charge on capacitor
  Determine phase angle
Example 19.4

In the circuit to the right

- $R_1 = 100 \, \Omega$,
- $L_1 = 300 \, \text{mH}$,
- $L_2 = 180 \, \text{mH}$,
- $C = 120 \, \mu\text{F}$ and
- $V = 12 \, \text{V}$.

The positive terminal of the battery is indicated with a $+$ sign. All elements are considered ideal. $Q(t)$ is defined to be positive if $V(a) - V(b)$ is positive.

What is the energy stored in the inductors 2.82 seconds after switch is opened?

- **Conceptual Analysis**
  Use conservation of energy

- **Strategic Analysis**
  Find energy stored on capacitor using information from Ex 19.3
  Subtract of the total energy calculated in the clicker question