Today’s Concept:

Electric Potential Energy
I don't like this return to mechanics and the potential energy concept, but this is conceptually easier.

Can't wait to see it in class. Want to see examples worked out.

Concepts were a lot easier to understand compared to the previous section.

Please do an example of the best way to sort out the values of potential energy in a complex system.

motion of a point charge in an electric field relating to potential energy needs more elaboration

the only thing I didn't get out of this lecture was: the way a point charge would move if released from rest in a region containing an electric field. How do electric fields affect point charges??

very much like the potential energy lecture in mechanics.

its interesting to see how similar the charges can be to what we saw in mechanics

Potential energy of a system of point charges is confusing. Could we go over this please?

done well (:}
Where we’re headed:

Force → Energy

E Field → ???
Recall from Mechanics:

\[ W = \int_{\overrightarrow{r}_1}^{\overrightarrow{r}_2} \mathbf{F} \cdot d\mathbf{r} \quad W_{TOT} = \Delta K \]

- **\( W > 0 \)**: Object speeds up (\( \Delta K > 0 \))
- **\( W < 0 \)**: Object slows down (\( \Delta K < 0 \))
- **\( W = 0 \)**: Constant speed (\( \Delta K = 0 \))
Masses $M_1$ and $M_2$ are initially separated by a distance $R_a$. Mass $M_2$ is now moved further away from mass $M_1$ such that their final separation is $R_b$.

Which of the following statements best describes the work $W_{ab}$ done by the force of gravity on $M_2$ as it moves from $R_a$ to $R_b$?

A. $W_{ab} > 0$
B. $W_{ab} = 0$
C. $W_{ab} < 0$
If gravity does negative work, potential energy increases!

Same idea for Coulomb force... if Coulomb force does negative work, potential energy increases.
A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.
B) In a direction that makes its potential energy decrease.
C) Along a path of constant potential energy.

Nature wants things to move in such a way that PE decreases.
You hold a positively charged ball and walk due east in a region that contains an electric field directed due west.

$W_H$ is the work done by the hand on the ball
$W_E$ is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$
B) $W_H > 0$ and $W_E < 0$
C) $W_H < 0$ and $W_E < 0$
D) $W_H < 0$ and $W_E > 0$
Conservative force: $\Delta U = -W_E$

Not a conservative force. Does not have any $\Delta U$.

Is $\Delta U$ positive or negative?

A) Positive

B) Negative

B) $W_H > 0$ and $W_E < 0$
Two Point Charges

Calculate the change in potential energy for two point charges originally very far apart moved to a separation of “d”

\[ \Delta U = -\int_{\infty}^{d} k \frac{q_1 q_2}{r_{12}^2} \, dr \]

\[ \Delta U = k \frac{q_1 q_2}{d} \]

(Charged particles with the same sign have an increase in potential energy when brought closer together.)

For point charges often choose \( r = \infty \) as “zero” potential energy.

\[ U = k \frac{q_1 q_2}{d} \]
In case A two opposite charges which are equal in magnitude are separated by a distance $d$. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A  
B) Case B
In **case A** two negative charges which are equal in magnitude are separated by a distance $d$. In **case B** the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A
B) Case B
Clicker Question Discussion

\[ U(r) = \frac{q_1 q_2}{4 \pi \varepsilon_0} \frac{1}{r} \]

Case A

\[ U_A = \frac{q^2}{4 \pi \varepsilon_0} \frac{1}{r} \]

Case B

\[ U_A > U_B \]
Example 5.1 (Velocity after a long time)

The two charges shown below are held in place and then released. What is their final velocity after they have moved a great distance apart?

\[ d = 10\text{cm} \]

\[ q_1 = +6\text{nC} \]

\[ q_2 = +8\text{nC} \]

\[ m_1 = 5 \times 10^{-10}\text{kg} \]

\[ m_2 = 7 \times 10^{-10}\text{kg} \]
When we said
   “The ball has \( mgh \) of potential energy.”
we really meant
   “The ball-earth system has \( mgh \) of potential energy.”

Why could we ignore earth?

Why can’t we ignore the other charges now?
A charge of +Q is fixed in space. A second charge of +q was first placed at a distance \( r_1 \) away from +Q. Then it was moved along a straight line to a new position at a distance \( R \) away from its starting position. The final location of +q is at a distance \( r_2 \) from +Q.

What is the change in the potential energy of the two charges during this process?

A. \( \frac{kQq}{R} \)
B. \( \frac{kQqR}{r_1^2} \)
C. \( \frac{kQqR}{r_2^2} \)
D. \( kQq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \)
E. \( kQq\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \)
What is the change in the potential energy of the two charges during this process?

A. \( \frac{kQq}{R} \)
B. \( \frac{kQqR}{r_1^2} \)
C. \( \frac{kQqR}{r_2^2} \)
D. \( kQq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \)
E. \( kQq\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \)

D) The change in potential energy = final potential energy - initial potential energy.

E) The change in potential energy is given as an integral with respect to the radius so there will be two radii in the equation and the change is negative so the before radius and after radius flip.
What is the change in the potential energy of charge \(+q\) during this process?

\[ U_1 = \frac{1}{4\pi \varepsilon_0} \frac{Qq}{r_1} \quad U_2 = \frac{1}{4\pi \varepsilon_0} \frac{Qq}{r_2} \]

\[ \Delta U \equiv U_2 - U_1 = \frac{Qq}{4\pi \varepsilon_0} \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \]

**Note:** \(+q\) moves **AWAY** from \(+Q\).
Its Potential energy **MUST DECREASE**
\[ \Delta U < 0 \]
Two charges are separated by a distance $d$.

What is the change in potential energy when a third charge $q$ is brought from far away to a distance $d$ from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi \varepsilon_0 d} + \frac{qQ_2}{4\pi \varepsilon_0 d}$$

(superposition)

- Don’t “double count”
- No sines and cosines
What is the total work *by the electric field* when three identical charges are brought from infinitely far away to the points on an equilateral triangle.

A) 0

B) \[ W_1 = -\frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \]

C) \[ W_2 = +\frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \]

D) \[ W_3 = -3 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \]

E) \[ W_4 = +3 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \]

\[ \Delta U = -W_{\text{con}} = +3k \frac{Q^2}{d} \]

Work to bring in first charge: \( W_1 = 0 \)

Work to bring in second charge: \( W_2 = -\frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} \)

Work to bring in third charge: \( W_3 = -\frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} - \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} = -\frac{2}{4\pi \varepsilon_0} \frac{Q^2}{d} \)
What is the total work \textit{by the outside force} when it brings three identical charges from infinitely far away to the points on an equilateral triangle?

A) 0

B) $W = -\frac{Q^2}{4\pi\varepsilon_0} \frac{1}{d}$

C) $W = +\frac{Q^2}{4\pi\varepsilon_0} \frac{1}{d}$

D) $W = -3\frac{Q^2}{4\pi\varepsilon_0} \frac{1}{d}$

E) $W = +3\frac{Q^2}{4\pi\varepsilon_0} \frac{1}{d}$
Now let's say the second charge is negative. Now what is the total work done by the field to bring the three charges in?

A) 0

B) \( W = +1 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \)

C) \( W = -1 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \)

D) \( W = +2 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \)

E) \( W = -2 \frac{Q^2}{4\pi \varepsilon_0} \frac{1}{d} \)

\[ W = \sum W_i = + \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} \]

\[ \Delta U = -\frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} \]

\[ \Delta U = -k \frac{Q^2}{d} \]

Work to bring in first charge: \( W_1 = 0 \)

Work to bring in second charge: \( W_2 = + \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} \)

Work to bring in third charge: \( W_3 = + \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} - \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{d} = 0 \)
Two charges which are equal in magnitude, but opposite in sign, are placed at equal distances from point A as shown. If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

A. Potential energy increases  
B. Potential energy decreases  
C. Potential energy does not change  
D. The answer depends on the sign of the third charge
If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

A. Potential energy increases  
B. Potential energy decreases  
C. Potential energy does not change  
D. The answer depends on the sign of the third charge

A) If a positive charge is added, the overall potential energy will become more positive. If a negative charge is added, the overall potential energy will still become more positive, and no matter what charge it is, the overall energy will increase.  
C) Since the original two charges are opposite in direction with equal magnitude, the potential energy between them will not change. No matter what sign the charge goes at point A, the potential energies will cancel out and be left with the initial potential energy between the two original charges.  
D) If a positive charge is placed at A, the potential energy would be negative and vice versa if a negative charge is placed at A.
Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first. Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

A. YES, as long as the third charge is positive  
B. YES, as long as the third charge is negative  
C. YES, no matter what the sign of the third charge  
D. NO
Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

A. YES, as long as the third charge is positive  
B. YES, as long as the third charge is negative  
C. YES, no matter what the sign of the third charge  
D. NO

A) If the third charge approaches the positive charge, the total potential energy can stay at its original state.

C) there will be a point in space where the potential energy of the third charge will be zero from the other two charges so yes, it is possible and it wont matter what's the sign of the actual charge is

D) it is not possible because the only reason it did not change the total potential energy in the last Checkpoint is because the two original charges had the SAME MAGNITUDE and therefore were able to cancel each other's potential energies out. The fact that these have DIFFERENT magnitudes makes that impossible.

LET’S DO THE CALCULATION!
Example 5.2

A positive charge \( q \) is placed at \( x = 0 \) and a negative charge \( -2q \) is placed at \( x = d \).

At how many different places along the \( x \) axis could another positive charge be placed without changing the total potential energy of the system?

- A) 0
- B) 1
- C) 2
- D) 3
Example

At which two places can a positive charge be placed without changing the total potential energy of the system?

Let’s calculate the positions of A and B

A) A & B
B) A & C
C) B & C
D) B & D
E) A & D
Lets work out where $A$ is

$D = 0$

$\Delta U = + \frac{1}{4\pi \varepsilon_0} \frac{Qq}{r} - \frac{1}{4\pi \varepsilon_0} \frac{2Qq}{r + d}$

Set $\Delta U = 0$

$\frac{1}{r} = \frac{2}{r + d}$

Makes Sense!

$Q$ is twice as far from $-2q$ as it is from $+q$

$r = d$
**Let's work out where B is**

Setting $\Delta U = 0$

\[
\frac{1}{r} = \frac{2}{d - r}
\]

\[
2r = d - r
\]

\[
r = \frac{d}{3}
\]

Makes Sense!

$Q$ is twice as far from $-2q$ as it is from $+q$
Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first. Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

A. YES, as long as the third charge is positive
B. YES, as long as the third charge is negative
C. YES, no matter what the sign of the third charge
D. NO
Summary

For a pair of charges:

Just evaluate \( U = k \frac{q_1 q_2}{r} \)

(We usually choose \( U = 0 \) to be where the charges are far apart)

For a collection of charges:

Sum up \( U = k \frac{q_1 q_2}{r} \) for all pairs