Today’s Concepts:

A) The Electric Field
B) Continuous Charge Distributions
Infinite lines of charge and their affect on a point 'p' a distance was difficult to wrap my head around. Particularly the integration explanation of why it was true. 

Good review of AP Physics, although seeing the integral again would be nice.

Infinite lines of charges confused me. 

Its very hard to grasp this concept.. please slow down a little when giving lectures please, gotta write down all the notes

When is the electric field negative (going away or towards another charge)? When is the electric field positive? Why does question 3 have negatives before each of the integrals?

Very confusing, the way the guy explained it. Need to elaborate more on the infinite line charge thing.

How important is it to know and feel comfortable with the derivations of the equations used in this course? (i.e. like the integral getting to the equation Ey = (2kλ/r))

Needs to be more concrete than abstract, and the integrals seem overwhelming

Need more examples of infinite line of charge please!
What if I remove $q_2$? Is there anything at that point in space?

\[ \vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2} \hat{r}_{12} \]

\[ \vec{E}_2 = \frac{\vec{F}_{12}}{q_2} = \frac{kq_1}{r_{12}^2} \hat{r}_{12} \]
If there are more than two charges present, the total force on any given charge is just the \textbf{vector sum} of the forces due to each of the other charges:

\[ \mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} \]

\[ +q_1 \rightarrow -q_1 \Rightarrow \text{direction reversed} \]

**MATH:**

\[ \mathbf{F}_1 = \frac{kq_1q_2}{r_{12}^2} \hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2} \hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2} \hat{r}_{14} \]

\[ \mathbf{E} = \frac{\mathbf{F}_1}{q_1} = \frac{kq_2}{r_{12}^2} \hat{r}_{12} + \frac{kq_3}{r_{13}^2} \hat{r}_{13} + \frac{kq_4}{r_{14}^2} \hat{r}_{14} \]
Electric Field

“What exactly does the electric field that we calculate mean/represent? “
“What is the essence of an electric field? “

The electric field $E$ at a point in space is simply the force per unit charge at that point.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

Superposition

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

Field points toward negative and Away from positive charges.

Field point in the direction of the force on a positive charge.
We never talk about the “gravitational field” in 2111. We just talked about a “gravitational force”.

Let’s say we had. What is the magnitude of the gravitational field close to the surface of the earth?

A) \( g \)
B) \( mg \)
C) \( m/g \)
D) \( g/m \)
Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point A?

A. Up  
B. Down  
C. Left  
D. Right  
E. Zero
What is the direction of the electric field at point A?

A. Up  
B. Down  
C. Left  
D. Right  
E. Zero

B) Since the x value of both Q charges are equal, the force on A is the force in the y direction.  The only force on A is the force in the y direction which points down towards both charges.

C) The vertical components of the forces cancel which leaves only the x component which is to the left.

D) The positive charge will cause a force diagonal to the upper right, and the negative charge will cause a force to the lower right. The resultant would be towards the right.

E) The point is equidistant from the positive and negative particles. So the electric field of the positive and negative charges would be zero.
Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point B?

A. Up
B. Down
C. Left
D. Right
E. Zero
What is the direction of the electric field at point B?

A. Up  
B. Down  
C. Left  
D. Right  
E. Zero

C) The electric charge from (+Q) is to the right and the electric charge from (-Q) is to the left. They are equally strong charges and on the same line but because point B is closer to the negative charge, the direction of the electric field is to the left.

D) The magnitude of the x component due to the negative charge is greater than that of the positive component because it is closer to point B.

E) The positive charge moves out and the negative charge pulls it in so they would cancel.
What is the direction of the electric field at point $P$, the unoccupied corner of the square?

A) $\vec{E} = \hat{i}$

B) $\vec{E} = \hat{j}$

C) $E = 0$

D) Need to know $d$

E) Need to know sign of charge placed at point $P$
Example 2.1 (Field from three charges)

Calculate $E$ at point $P$. 
In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?

A. Case 1
B. Case 2
C. Equal
In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?

- A. Case 1
- B. Case 2
- C. Equal

B) The magnitudes from the like charges add up, while the magnitudes of the electric field from opposite charges will subtract.

C) The magnitudes are equal because all of the charges are equal. However, the sign of the charges affect the direction of the force, but not the magnitude.
Two charges $q_1$ and $q_2$ are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative $y$-axis.

Which of the following statements is true:

A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related
A 6uC charge is placed in an electric field and feels a 0.18N force. If the charge is replaced by a 9uC charge, what force will the new charge feel?

A) 0N  
B) 0.09N  
C) 0.18N  
D) 0.27N  
E) 0.36N
A positive test charge $q$ is released from rest at distance $r$ away from a charge of $+Q$ and a distance $2r$ away from a charge of $+2Q$. How will the test charge move immediately after being released?

A. To the left  
B. To the right  
C. Stay still  
D. Other
A positive test charge \( q \) is released from rest at distance \( r \) away from a charge of \( +Q \) and a distance \( 2r \) away from a charge of \( +2Q \). How will the test charge move immediately after being released?

A. To the left  
B. To the right  
C. Stay still  
D. Other

A) I think it will move to the left. Even though the charge to the right is stronger, it is twice as far away and the strength of the charge falls off significantly with distance.

B) The electric field is inversely proportional to \( r^2 \). So the smaller \( r \) will have the larger electric field. As it will simplify to \( Q/r^2 > Q/2r^2 \)

C) the charge on a particle is inversely contented to the distance. So in this case, double the charge and distance will have the same effect as the closer charge.
A charge of \( q_1 = +4 \mu\text{C} \) is placed at the origin and another charge \( Q_2 = +10 \mu\text{C} \) is placed 0.4m away. At what point on the line connected the two charges is the electric field zero?
Continuous Charge Distributions

“I don't understand the whole dq thing and lambda.”

Summation becomes an integral (be careful with vector nature)

\[ \vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \xRightarrow{\text{WHAT DOES THIS MEAN?}} \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r} \]

Integrate over all charges \((dq)\)

\(r\) is vector from \(dq\) to the point at which \(E\) is defined

Linear Example:

\[ \lambda = \frac{Q}{L} \]

chargess to point for \(E\)

\[ dq = \lambda \, dx \]}
Question: Charge Density

“I would like to know more about the charge density.”

Linear ($\lambda = Q/L$) Coulombs/meter
Surface ($\sigma = Q/A$) Coulombs/meter²
Volume ($\rho = Q/V$) Coulombs/meter³

What has more net charge?.

A) A sphere w/ radius 2 meters and volume charge density $\rho = 2 \text{ C/m}^3$
B) A sphere w/ radius 2 meters and surface charge density $\sigma = 2 \text{ C/m}^2$
C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \rho \frac{4}{3} \pi R^3$$
$$Q_B = \sigma A = \sigma 4\pi R^2$$

Some Geometry

$$A_{sphere} = 4\pi R^2$$
$$A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \frac{4}{3} \pi R^3$$
$$V_{cylinder} = \pi R^2 L$$

$$\frac{Q_A}{Q_B} = \frac{\rho}{\sigma} \frac{4\pi R^3}{4\pi R^2} = \frac{1}{3} \frac{\rho}{\sigma} R$$
Example 2.3 (line of charge)

“Please go over infinite line charge.”

Let’s do one slightly different.

Charge is uniformly distributed along the \( x \)-axis from the origin to \( x = a \). The charge density is \( \lambda \text{ C/m} \). What is the \( x \)-component of the electric field at point \( P: (x,y) = (0,h) \)?
Question: Calculation

Charge is uniformly distributed along the \( x\)-axis from the origin to \( x = a \). The charge density is \( \lambda \ C/m \). What is the \( x\)-component of the electric field at point \( P: (x,y) = (0,h) \)?

We know:

\[
\vec{E} = \int k \frac{dq}{r^2} \hat{r}
\]

What is \( \frac{dq}{r^2} \)?

A) \( \frac{dx}{x^2} \)  
B) \( \frac{dx}{a^2 + h^2} \)  
C) \( \frac{\lambda dx}{a^2 + h^2} \)  
D) \( \frac{\lambda dx}{x^2 + h^2} \)  
E) \( \frac{\lambda dx}{x^2} \)
Question: Calculation

Charge is uniformly distributed along the \( x \)-axis from the origin to \( x = a \). The charge density is \( \lambda \text{ C/m} \). What is the \( x \)-component of the electric field at point \( P: (x,y) = (0,h) \)?

We know:

\[
\vec{E} = \int k \frac{dq}{r^2} \hat{r}
\]

\[
\frac{dq}{r^2} = \frac{\lambda dx}{x^2 + h^2}
\]

\[
E_x = \int dE_x
\]

What is \( dE_x \)?

A) \( dE \cos \theta_2 \)  
B) \(-dE \cos \theta_2 \)  
C) \( dE \sin \theta_1 \)  
D) \( dE \sin \theta_2 \)  
E) \(-dE \sin \theta_2 \)
Charge is uniformly distributed along the \( x \)-axis from the origin to \( x = a \). The charge density is \( \lambda \, \text{C/m} \). What is the \( x \)-component of the electric field at point \( P: (x,y) = (0,h) \)?

\[
\vec{E} = \int k \frac{dq}{r^2} \, \hat{r}
\]

We know:

\[
\frac{dq}{r^2} = \frac{\lambda \, dx}{x^2 + h^2}
\]

\[
E_x = \int dE_x = \int dE \sin \theta_2
\]

What is \( E_x \)?

A) \( -k\lambda a \sin \theta_2 \int_{-\infty}^{\infty} \frac{dx}{x^2 + h^2} \)

B) \( -k\lambda \sin \theta_2 \int_{0}^{a} \frac{dx}{x^2 + h^2} \)

C) neither of the above \( \sin \theta_2 \) DEPENDS ON \( x \)!
We know:

\[ \vec{E} = \int k \frac{dq}{r^2} \hat{r} \]

\[ \frac{dq}{r^2} = \frac{\lambda dx}{x^2 + h^2} \]

\[ E_x = \int dE_x = \int dE \sin \theta_2 \]

**PROBLEM:** We have two variables \( \Theta \) and \( x \)

What is \( x \) in terms of \( \Theta \)?

A) \( x = h \cdot \tan \Theta_2 \)

B) \( x = h \cdot \cos \Theta_2 \)

C) \( x = h \cdot \sin \Theta_2 \)

D) \( x = \frac{h}{\cos \Theta_2} \)
\[ E_x(P) = \frac{k\lambda}{h} \left( \frac{h}{\sqrt{h^2 + a^2}} - 1 \right) \]

Note: \[ \left( \frac{h}{\sqrt{h^2 + a^2}} < 1 \right) \]

\[ E_x < 0 \]

make sense?
For an infinite line of charge, we had:

\[
E_x(P) = -\frac{k\lambda}{h} \int_0^{\theta_f} \sin \theta \, d\theta
\]

How would this integral change if we wanted the y component instead of the x component?

A) The limits would be \( \Theta_1 \) to \(-\Theta_1\)  
B) The limits would be \(+/-\) infinity  
C) The limits would be \(-\pi/2\) to \(\pi/2\)  
D) \(\sin \theta\) would turn to \(\tan \theta\)  
E) \(\sin \theta\) would turn to \(-\cos \theta\)
"Please go over infinite line charge. How does R get outside the integral?"

We had:

\[ E_y(P) = -k\lambda \int_0^{\theta_f} d\theta \frac{h* \sec^2 \theta * (-\cos \theta)}{(h* \tan \theta)^2 + h^2} \]

\[ = \frac{k\lambda}{h} \int_0^{\theta_f} \cos \theta \cdot d\theta \]

How would this integral change if the line of charge were infinite in both directions?

A) The limits would be \( \Theta_1 \) to \(-\Theta_1\)

B) The limits would be \(+\infty\) to \(-\infty\)

C) The limits would be \(-\pi/2\) to \(\pi/2\)

D) \(\sin \theta\) would turn to \(\tan \theta\)

E) \(\sin \theta\) would turn to \(-\cos \theta\)
Note

\[ E_{\text{line}} = \frac{2 \lambda k}{r} \]

\[ k = \frac{1}{4 \pi \varepsilon_o} \]

\[ E_{\text{line}} = \frac{\lambda}{r 2 \pi \varepsilon_o} \]
Two infinite lines of charge are shown below.

Both lines have identical charge densities $+\lambda \text{ C/m}$. Point A is equidistant from both lines and Point B is located a above the top line as shown.

How does $E_A$, the magnitude of the electric field at point A, compare to $E_B$, the magnitude of the electric field at point B?

A. $E_A < E_B$
B. $E_A = E_B$
C. $E_A > E_B$
Two infinite lines of charge are shown below.

How does $E_A$, the magnitude of the electric field at point A, compare to $E_B$, the magnitude of the electric field at point B?

A. $E_A < E_B$
B. $E_A = E_B$
C. $E_A > E_B$
What is the electric field a distance $h$ above the center of ring of uniform charge $Q$ and radius $a$?
In the previous question, what should the electric field be if \( h \gg a \)?

a) \( k \frac{Q}{h^2} \)
b) \( k \frac{Q}{a^2} \)
c) \( k \frac{Q}{(a*h)} \)
d) \( k \frac{Q a*h}{a} \)
What is the electric field a distance $h$ above the center of ring of uniform charge $Q$ and radius $a$?
A total charge $Q$ is uniformly distributed over a half ring with radius $R$. The total charge inside a small element $d\theta$ is given by:

A. $\frac{Q}{2\pi R} d\theta$

B. $\frac{Q}{\pi R} d\theta$

C. $\frac{Q}{\pi R}$

D. $\frac{Q}{\pi} d\theta$

E. $\frac{Q}{2\pi} d\theta$
A total charge $Q$ is uniformly distributed over a half ring with radius $R$. The $Y$ component of electric field created by a short element $d\theta$ is given by:

A. $\frac{kQd\theta}{\pi R^2} \sin \theta$

B. $\frac{kQd\theta}{\pi R^2} \cos \theta$

C. $\frac{kQd\theta}{\pi R^3} \sin \theta$

D. $\frac{kQd\theta}{\pi R^3} \cos \theta$
A total charge $Q$ is uniformly distributed over a half ring with radius $R$. The total electric field at point $C$ is:

A. \( \frac{kQ}{\pi R^2} \)

B. \( \frac{k2Q}{\pi R^2} \)

C. \( \frac{kQ}{R^2} \)

D. 0
Dipole moment = \( \vec{p} = qd \)

Points from negative to positive.

*(opposite the electric field.)*

Water molecule has dipole moment of \( 620 \times 10^{-30} \) Cm

\[ = 2 \times 1.6 \times 10^{-19} \text{C} \times 190 \text{pm} \]
Torque on dipole

\[ \tau = 2 \times (q \vec{E} \times \frac{d}{2}) = \vec{p} \times \vec{E} \]

\[ dU = -dW = -\tau d\Theta = pE^* \sin\Theta \times d\Theta \]

\[ \Delta U = -pE\cos\Theta \]

Define \( U = \) when \( \Theta = \pi/2 \)

\[ U = -\vec{p} \cdot \vec{E} \]
Example 2.6 (Salt Dipole)

The two atoms in a salt (NaCl) molecule are separated by about 500pm. The molecule is placed in an electric field of strength 10N/C at an angle of 20°.

What is the torque on the molecule? What is the potential energy of the molecule?
An electrically neutral dipole is placed in an external field. In which situation(s) does the potential energy of the dipole have the smallest magnitude using the standard definition of U?

A. (a)  
B. (b)  
C. (c)  
D. (d)  
E. (c) and (d)
An electrically neutral dipole is placed in an external field. In which situation(s) does the potential energy of the dipole have the lowest value using the standard definition of $U$?

A. (a)  
B. (b)  
C. (c)  
D. (d)  
E. (c) and (d)