Physical Optics

- Thin Film Interference
- Two Slit Interference
- Single Slit Interference
- Resolution
- Diffraction Gratings
- X-Ray Diffraction
Interference

Constructive

In-phase or shifted by $m \times 2\pi$

just some integer (0, 1, 2, 3, 4……)

Destructive

Difference in phase $(m+1/2) \times 2\pi$

(In terms of distance. In terms of radians is $m \times 2\pi$ or in terms of degrees is $m \times 360^\circ$)
What causes shift in phase?

1) Waves travel difference distances

2) Phase shift at a boundary

\[ \frac{\lambda}{2} \text{ phase shift going from fast to slow (} n_i < n_t \text{)} \]

\[ \text{No phase shift going from slow to fast (} n_i > n_t \text{)} \]
Thin Film Interference

Incoming light beam

Phase shift at air to glass boundary

No phase shift at glass to air boundary

Phase shift due to difference in distance traveled

**Total Phase Shift** = \((2t/\lambda_1 - \frac{1}{2})\cdot2\pi = (2t/(\lambda_0/n) - \frac{1}{2})\cdot2\pi\)
Example 29.1: White Light

White light with wave lengths from 400nm to 690nm shines on a thin film of soapy water (n=1.33) with a thickness of 320nm.

What is the brightness color reflected?

**Total Phase Shift** = \((2t/(\lambda_0/n) - \frac{1}{2})\times 2\pi\)
Example 29.2: White Light

White light with wave lengths from 400nm to 690nm shines on a thin film of soapy water ($n=1.33$) with a thickness of 320nm.

What color would be missing from the reflected light?

Total Phase Shift = $\left(\frac{2t}{\lambda_o/n} - \frac{1}{2}\right) \times 2\pi$
Thin Film Interference around us

Colors in soap bubbles

Colors in gas spills
Physics at work!
Example 29.3: Anti-glare coating

If you have destructive interference, the reflected wave goes away.

But where does the energy go that would have been in that reflected wave?

*It must be transmitted!!*

A glass camera lens has a thin film of magnesium fluoride (MgF$_2$) (n=1.38) for anti-glare. If the purpose is to cut reflection in the middle of the visible spectrum ($\lambda=550$nm), what should the thickness be?
Two slit Interference

A plane wave heading towards to slits

\[ \phi = 2\pi \frac{(R_1 - R_2)}{\lambda} \]

Constructive

\[ m \cdot 2\pi = 2\pi \left( d \sin\theta / \lambda \right) \]

Destructive

\[ (m+1/2) \cdot 2\pi = 2\pi \left( d \sin\theta / \lambda \right) \]

\[(R_1 - R_2) = d \sin\theta\]

Puts a series of bright and dark “fringes” on the screen
Example 29.3: Fringe Spacing

Two narrow slits are 2mm apart. If I shine yellow light ($\lambda = 600\text{nm}$) on the two slits, what will be the spacing between the bright fringes on a screen 3m away?

$$\Delta y = ?$$
Intensity from two slits

Previous formulas tell us positions of min and max intensity, \( I \)

**What is intensity at any general point?**

Just add wave functions

\[
E_1 = E_{\text{max}} \cdot \sin(\omega t) \quad E_2 = E_{\text{max}} \cdot \sin(\omega t + \phi)
\]

\[
\phi = 2\pi \cdot \Delta L / \lambda = 2\pi d \cdot \sin(\theta) / \lambda = \phi
\]

Recall: \( \sin(\alpha) + \sin(\beta) = 2\sin(0.5(\alpha + \beta)) \cdot \cos(0.5(\alpha - \beta)) \)

\[
E_1 + E_2 = \sin(\omega t + \phi/2) \cdot \cos(\phi/2)
\]

\[
I_{\text{tot}} = 4I_0 \cos^2(\phi/2)
\]
Example 29.4: Intensity

Recall in the previous example we found a maximum 0.9mm above the central point and another one at 1.8mm above. What would be the relative intensity 1.2mm above central point?
So far assumed slits were infinitely thin.
Light from slit of finite width can interfere with itself

Think of finite slit as large number of thin slits (Huygen’s Principle)

**Diffractive interference**

Similar to equations for two slits

**Constructive**
\[ m \cdot 2\pi = 2\pi \left( \frac{a}{2} \sin\theta \right)/\lambda \]

**Destructive**
\[ (m+1/2) \cdot 2\pi = 2\pi \left( \frac{a}{2} \sin\theta \right)/\lambda \]

Mostly looking for edges of bright lines (destructive)

\[ m\lambda = a \sin\theta \]
Example 29.6: Single Slit Interference

For what slit width will the first minimum for red light (λ=650nm) appear at 5°?
Example 29.7: Diffractive Interference

Can have both effects at once

405nm light is shone on two slits with a width of 4.05μm that are separated by 19.44μm.

How many bright fringes fit in the central peak of the diffractive envelope?
Can you tell if there is one peak or two peaks here?

Rayleigh’s criterion says you can “resolve” the two different peaks of the max of one falls on the min of the other.

\[ \frac{\lambda}{a} = \sin \theta \]

- Slit width
- Angle between peaks
Question: Resolution with your eye

Criterion for resolution through slits: \( \lambda/a = \sin \theta \)

Through circular holes: \( 1.22\lambda/D = \sin \theta \)

Let’s say you wanted to read some fine print. That is, you wanted to be able to “resolve” one small letter from another.

That means you would want the diameter of the pupil of your eye to:

A. get as small as possible
B. get as large as possible
C. resolution and pupil diameter are unrelated
Example 29.8: Resolving Fine Print

How many dots per inch can you see on a page 40cm from your eye?

Take $D_{\text{eye}} = 2.5\text{mm}$, $\lambda = 400\text{nm}$
A Sunday Afternoon on the Isle of La Grande Jatte
Diffraction Gratings

Like a large number of double slits

For constructive interference, bright lines

\[ d \times \sin \theta = m \lambda \]

In this case

\[ d = \frac{w}{N} \]

- width of entire grating
- Number of “rulings” on entire grating
Example 29.9: Diffraction Grating

Red light ($\lambda=700\text{nm}$) and blue light ($\lambda=450\text{nm}$) shines on a diffraction grating with slit spacing of 2500nm. What are the angles for the first order ($m=1$) lines for these two colors?

What are the angles for the second order ($m=2$) lines for these two colors?

How many orders are possible for the red light?
Resolution on Diffraction Gratings

Can a diffraction grating “resolve” two similar frequencies?
Can it tell that there are two closely spaced lines and not just one?

How wide is each line?

Think of grating as single opening and apply Rayleigh criterion.

\[
\sin \theta = \frac{\lambda}{a}
\]

\[
\Delta \theta = \frac{\lambda}{Nd}
\]

- \(\lambda\): wavelength
- \(a\): total width of grating
- \(N\): number of rulings
- \(d\): spacing between rulings

\( \theta \): angle between lines
Example 29.10: Resolution of Grating

Can a 2.5cm wide diffraction grating with spacing of 25\(\mu\)m tell 470nm light from 480nm light?
Summary on gratings

Wider grating $\rightarrow$ narrower lines $\rightarrow$ better resolution

Narrower spacing $\rightarrow$ bigger bend angle
**X-Ray Diffraction**

Note that for constructive interference:
\[ m\lambda/d = \sin\theta \]

To “study” structure of diffraction grating:
\[ \rightarrow \lambda \text{ and } d \text{ roughly same size} \]

e.g. *What kind of wave would you use to determine the spacing between rulings?*

The spacing between planes of matter \( \sim 10-100 \text{pm} \). What kind of waves would allow you to observe interference?

**X-rays**

Crystalline solid (NaCl)
X-rays scatter off planes of atoms in structure with spacing $d$

Note: not reflection. Rays scatter in all directions.

As before, we show rays we care about.

As always, when $\Delta L = m\lambda$, we get constructive interference.

$$2d \sin \theta = m\lambda$$

Bragg’s Law
Example 29.10: Bragg’s Law

X-rays of wavelength 0.12nm are found to undergo second-order reflection at a Bragg angle of 28° from lithium fluoride crystal.

What is the interplanar spacing of the reflecting planes of the crystal?
1952 - Rosalind Franklin takes X-ray diffraction image of DNA. (Photo 51).

Points to helix nature of DNA

Unknown to her, photo makes its way to James D. Watson and Francis Crick (via Maurice Wilkins and Raymond Gosling).

Create correct model of DNA

1962 - Watson, Francis Crick and Wilkins get Nobel.

Franklin died in 1958.
Same thing, **always:**

Step 1: account for phase shifts at boundary (thin film only)

Step 2: account for difference in distance traveled.

Step 3:  
if sum = $m\lambda$ constructive (light)  
if sum = $(m+1/2)\lambda$ destructive (dark)