DIY: Graphs of Basic functions; Transformations

To review the basics of the graphs of functions, watch the following set of YouTube videos explaining the basic shapes and variations on these using transformations (translations, reflections, and dilations (stretching/shrinking). They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=i37Zvv-T37Q graphs of basic functions
2. https://www.youtube.com/watch?v=-gwffMEr8i8 Graphing piecewise functions
3. https://www.youtube.com/watch?v=hDNAVh0VtTc piecewise functions, another example
4. https://www.youtube.com/watch?v=3Q5Sy034fok Horizontal and Vertical translations (shifts)
5. https://www.youtube.com/watch?v=UQ3a2QH_-GU Graphing the greatest integer function
   note: notation for the greatest integer function varies. This presenter uses [x], but it can also appear as $\lfloor x \rfloor$ (the floor function) or as $[\![x]\!]$
6. https://www.youtube.com/watch?v=5oQgzup9nx4 Stretch/shrink Part 1 (vertical)
7. https://www.youtube.com/watch?v=kyq0VKPHiJQ Stretch/shrink Part 2 (horizontal)
8. https://www.youtube.com/watch?v=R3fePYOACLE Reflections about coordinate axes

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Graph the following basic functions

a) $f(x)=x$	b) $g(x) = 1/x$	c) $h(x) = x^2$
d) $s(x) = 1/x^2$	e) $t(x)= x^3$	f) $d(x) = \lvert x \rvert$
g) $f(x) = 1/x^3$	h) $a(x) = \ln x$	i) $b(x)= 2^x$
j) $c(x) = [\![x]\!] = \lfloor x \rfloor$	k) $m(x) = x^4$	l) $j(x) = \sqrt{x}$

2. Graph the following piecewise functions:

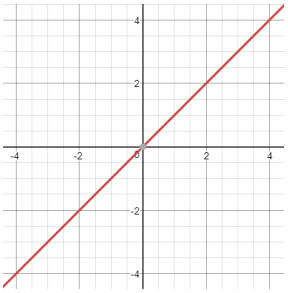
a. $$f(x) = \begin{cases} 2x+1 \ \ if \ x < 1 \\ 0 \ \ if \ \ x = 1 \\ x^2 \ if \ x > 1 \end{cases}$$

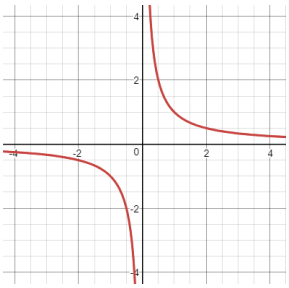
b. $$g(x) = \begin{cases} |x-3| \ if \ -3 < x < 1 \\ x \ \ if \ x = 1 \\ \sqrt{x} \ if \ \ 1 < \ x < \ 3 \end{cases}$$

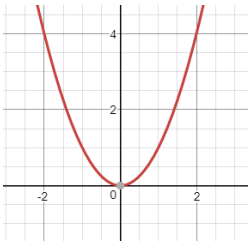
3. Graph the basic functions and the transformed functions on the same graph

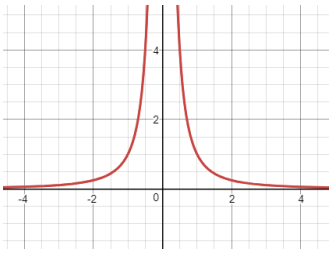
a) $f(x)=2x+6$
b) $g(x)=2/x$
c) $h(x)=(-x)^2+2$
d) $s(x)=(1/x^2)+1$
e) $t(x)=-x^3$
f) $d(x)=|4x|$
g) $f(x)=1/(2x^3)$
h) $a(x)=\ln(x+3)$
i) $b(x)=2^x-5$
j) $c(x)=[\![x-5]\!]$
k) $d(x)=-5x^4+1$
l) $j(x)=-\sqrt{x-2}$

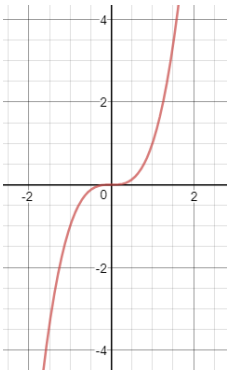
Answers:

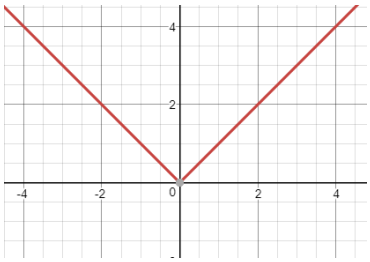
1. a. 

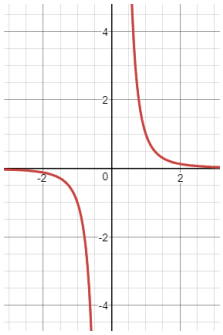
b. 

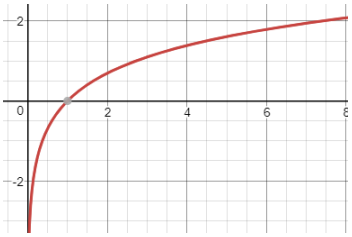
c. 

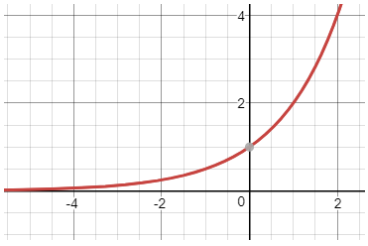
d. 

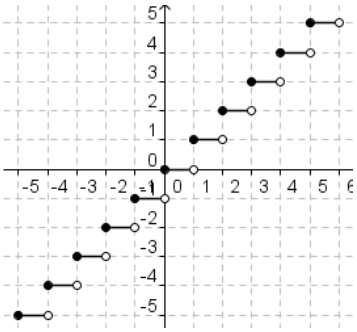
e. 

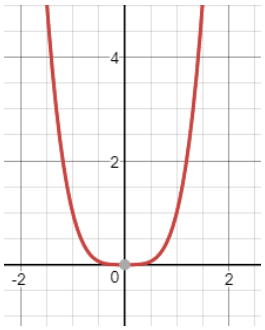
f. 

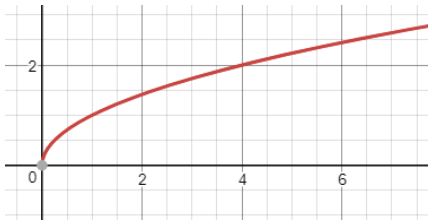
g. 

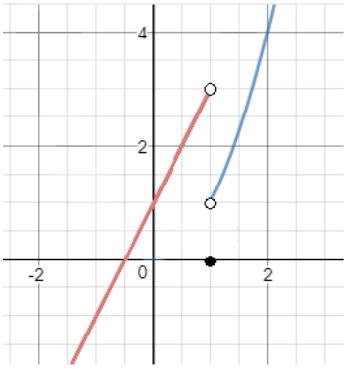
h. 

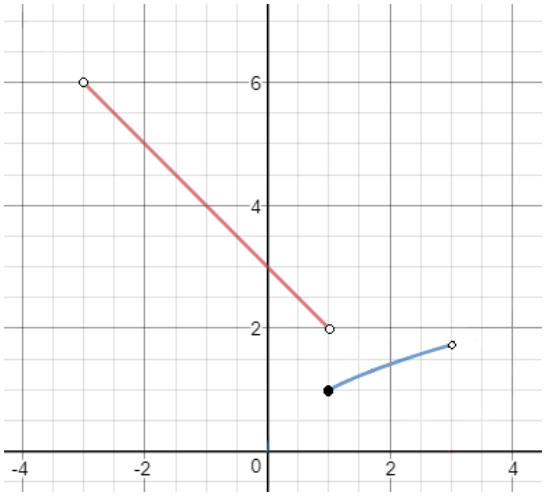
i. 

j. 

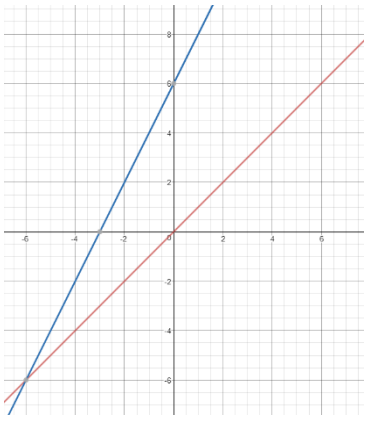
k. 

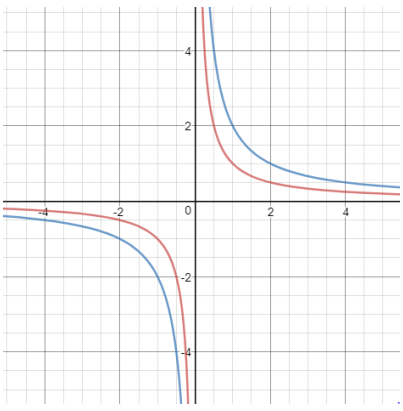
l. 

2. a. 

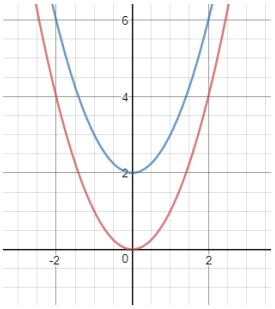
b. 

3. For each function, the function graphed in red is the basic function, while the transformed function is graphed in blue.

a. 

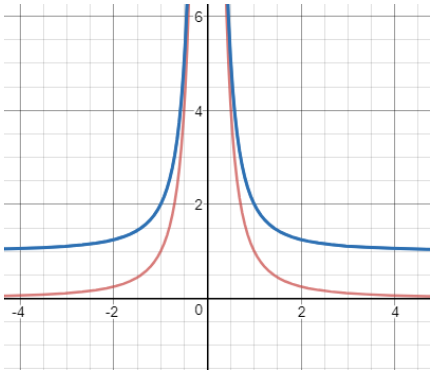
b. 

c.

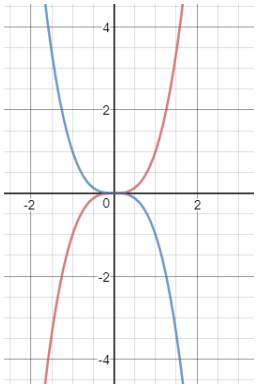


Note: the graphs of $y = x^2$ *and* $y = (-x)^2$ would be identical since $(-x)^2 = x^2$

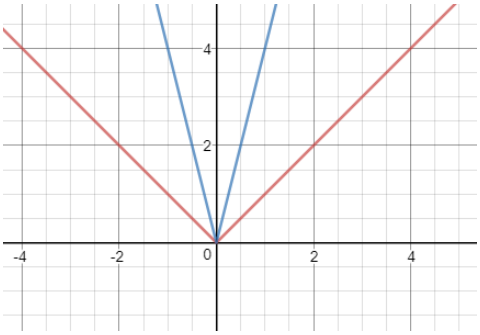
d.



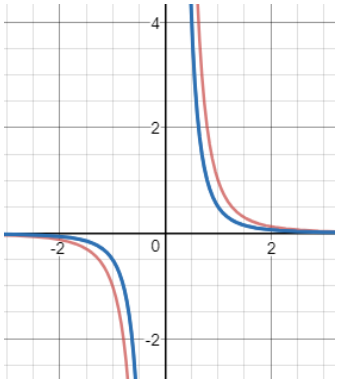
e.



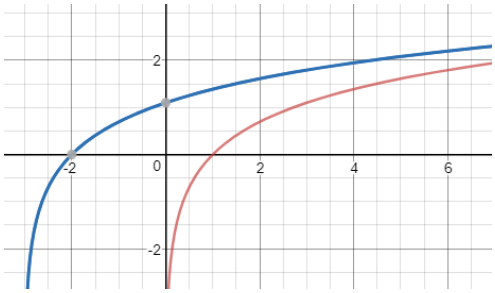
f.



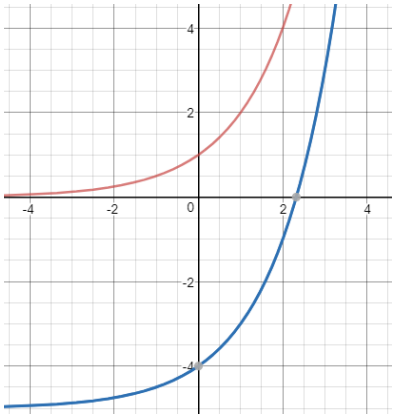
g.

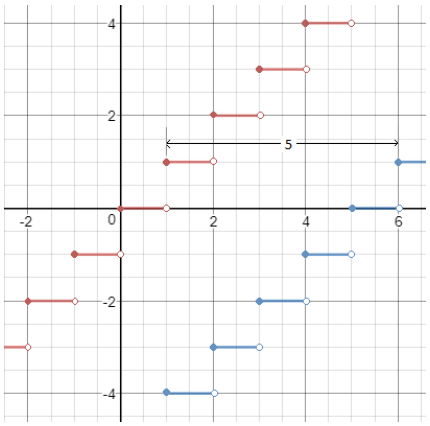


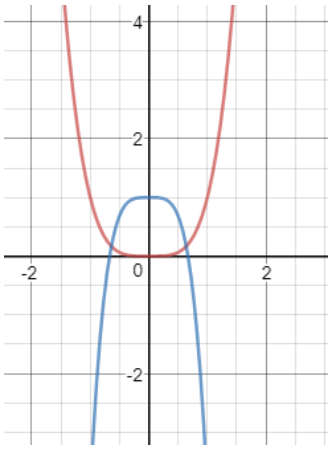
h.

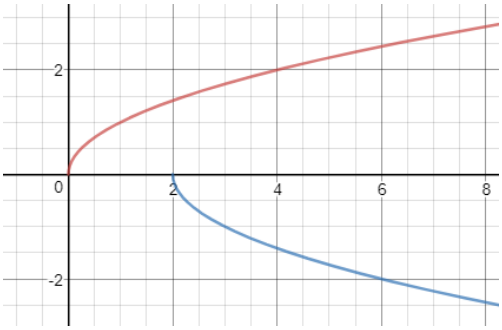


i.



j. 

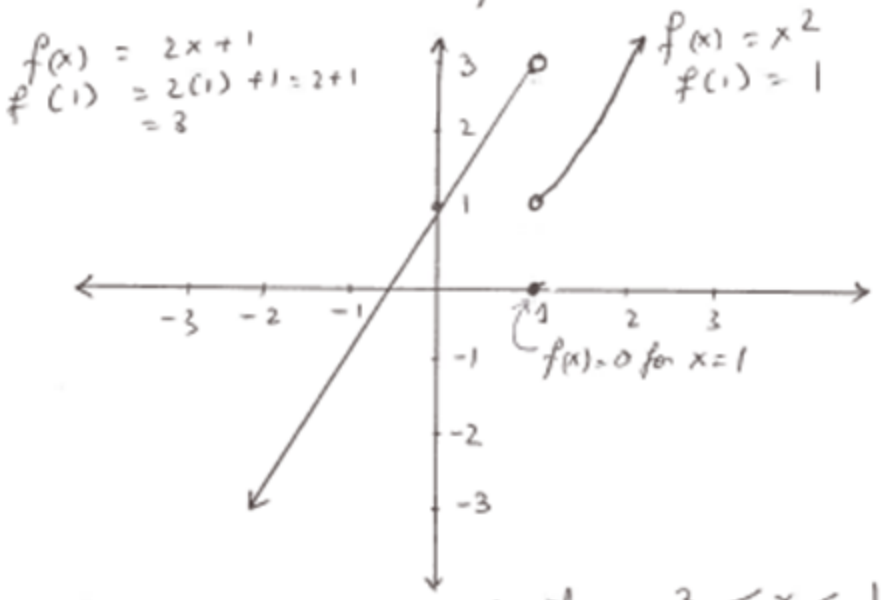
k. 

l. 

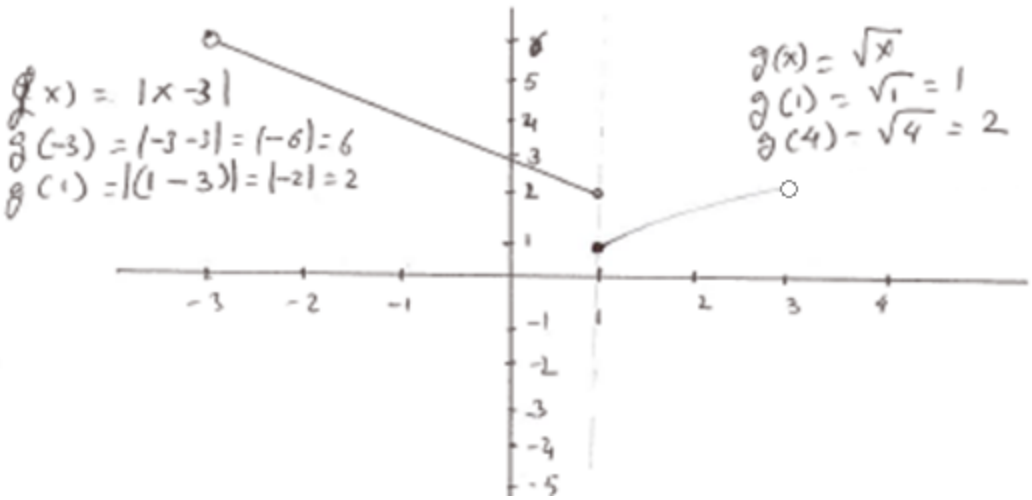
Detailed Solutions

1. (see graphs in Answers section)

2 a) $f(x) = \begin{cases} 2x+1 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$



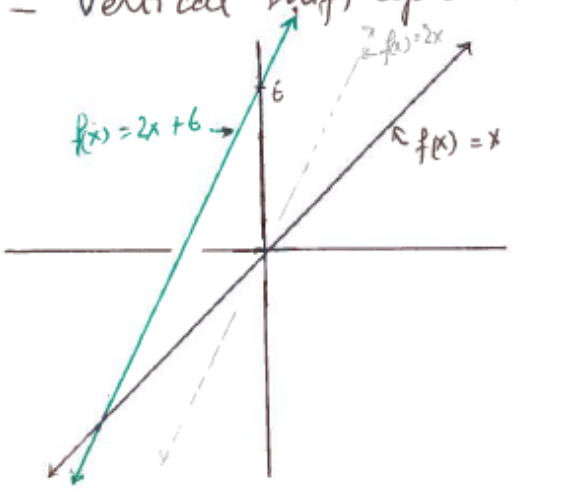
2 b) $g(x) = \begin{cases} |x-3| & \text{if } -3 < x < 1 \\ x & \text{if } x = 1 \\ \sqrt{x} & \text{if } 1 < x < 3 \end{cases}$



3a) $f(x) = 2x + 6$

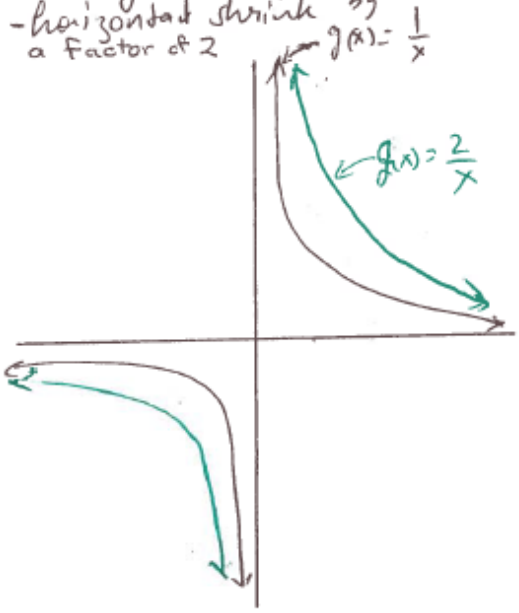
The basic function is $f(x) = x$

$f(x) = 2x + 6$ involves
- horizontal shrink by a factor of 2 (or vertical stretch)
- Vertical shift up 6 units



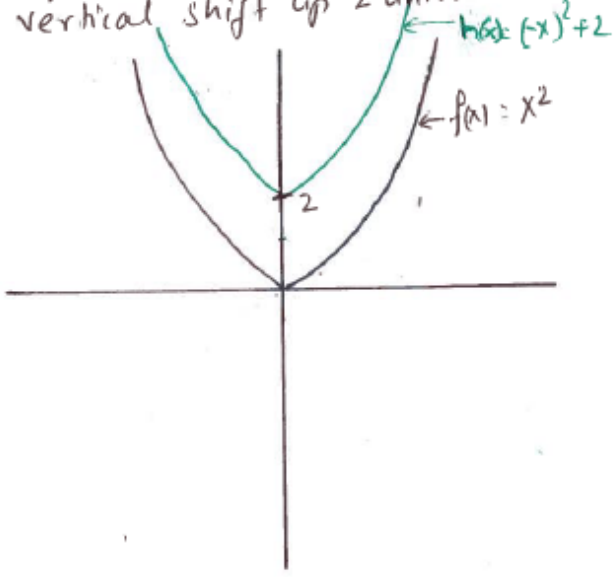
3b) $g(x) = \frac{2}{x}$

The basic function is $g(x) = \frac{1}{x}$
- horizontal shrink by a factor of 2



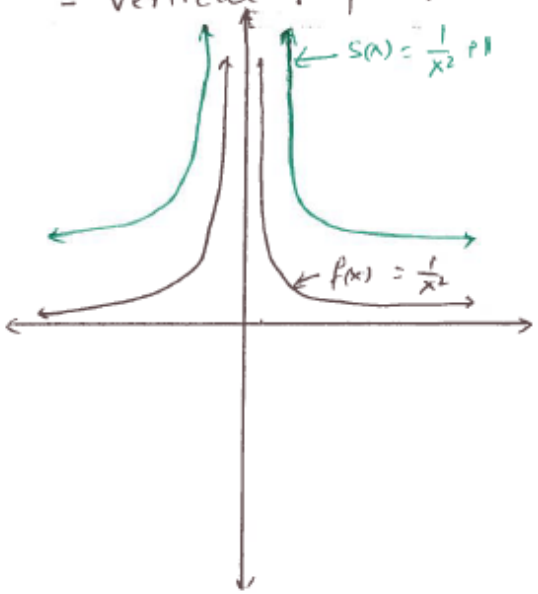
3c) $h(x) = (-x) + 2$

The basic function is $f(x) = x$
- Reflection about y-axis
- vertical shift up 2 units



3d) $s(x) = \frac{1}{x^2} + 1$

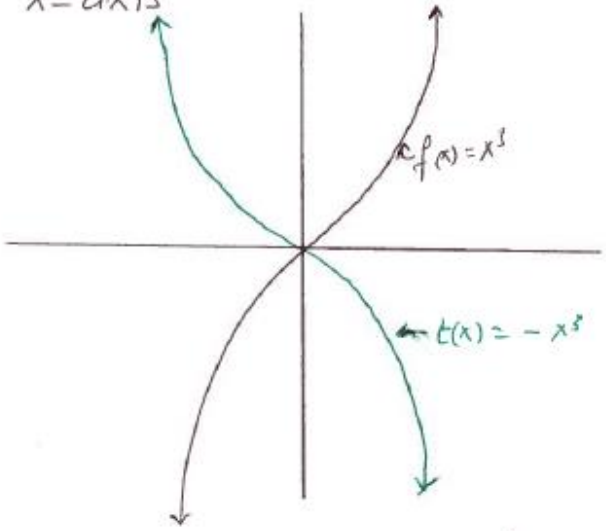
The basic function is $f(x) = \frac{1}{x^2}$
- vertical shift up 1 unit



3e) $t(x) = -x^3$

The basic function is $f(x) = x^3$

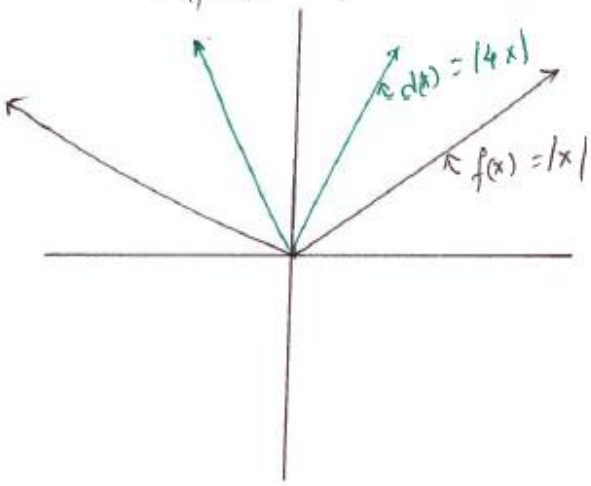
- Reflection about the x-axis



3f) $d(x) = |4x|$

The basic function is $f(x) = |x|$

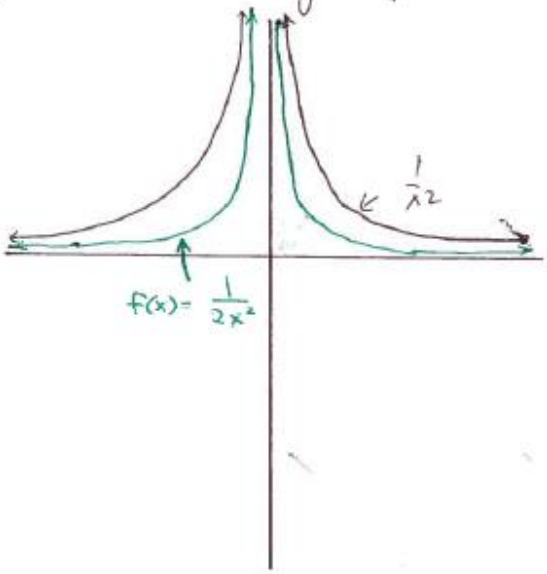
- Horizontal shrink by a factor of 4



3g) $f(x) = \frac{1}{2x^2} = \frac{1}{2}\left(\frac{1}{x^2}\right)$

The basic function is $f(x) = \frac{1}{x^2}$

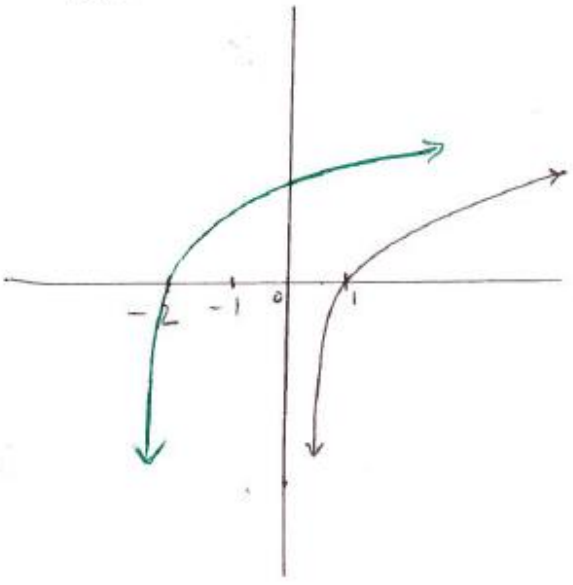
- Vertical "stretch" by a factor of ½



3h) $a(x) = \ln(x+3)$

The basic function is $f(x) = \ln(x)$

- horizontal shift of 3 units to the left

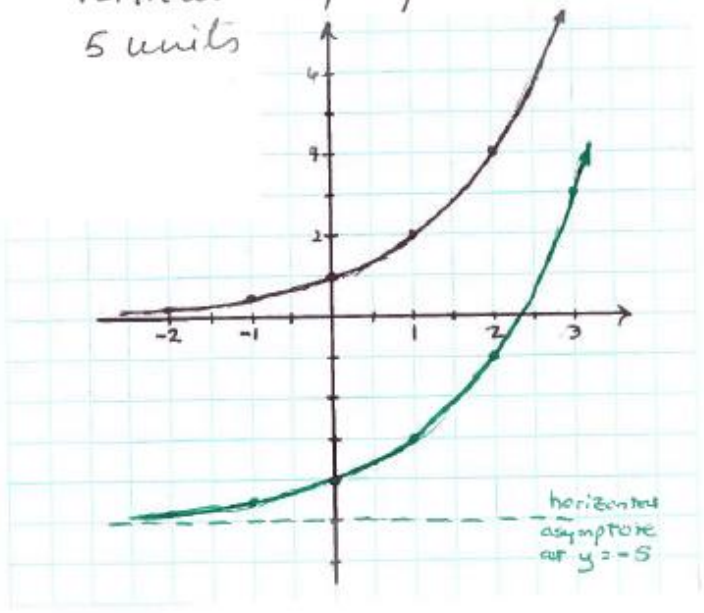


3i) $b(x) = 2^x - 5$

The basic function is

$f(x) = 2^x$

- Vertical shift of down 5 units

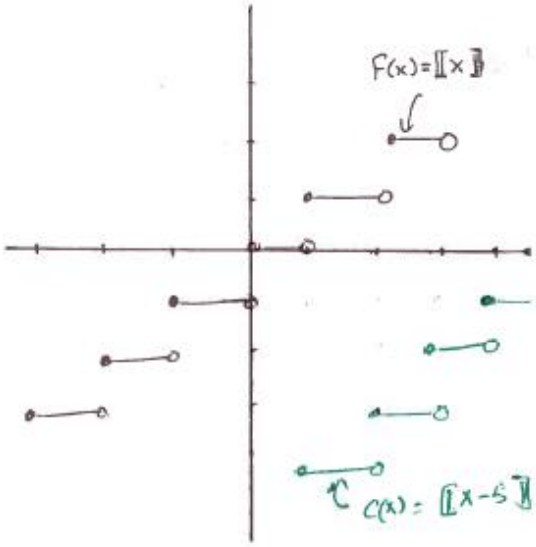


3j) $c(x) = [\![x - 5]\!]$

The basic function is

$f(x) = [\![x]\!]$

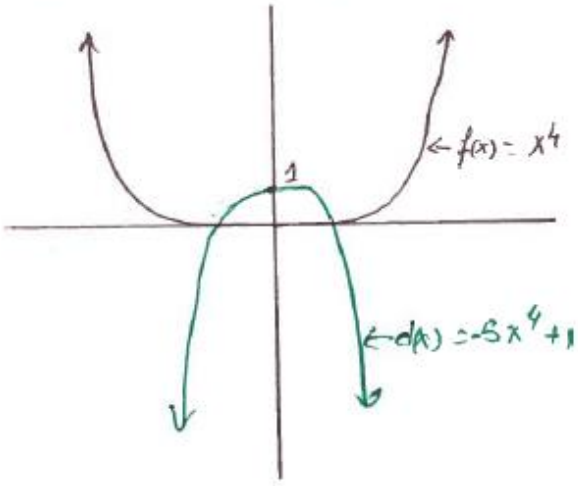
- horizontal shift of 5 units to the right.



3k) $d(x) = -5x^4 + 1$

The basic function is $f(x) = x^4$

- by a factor of 5
- Reflection about the x-axis
- vertical shift up 1 unit.

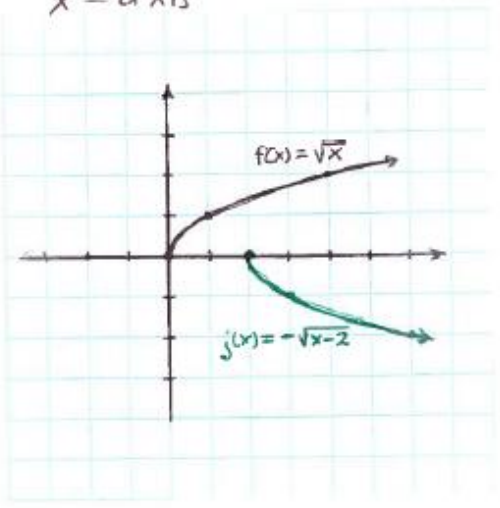


3l) $j(x) = -\sqrt{(x-2)}$

The basic function is

$f(x) = \sqrt{x}$

- Horizontal shift to the right for 2 units
- Reflection about the x-axis



Additional Resources

Click on the links below to download worksheets under "Linear Equations and Inequalities":

- Transformations of graphs

Alternatively;

1. Go to http://www.kutasoftware.com/freeipc.html

2. Under **"Functions"** find**:**
   - Transformations of graphs
3. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets
4. For help please contact the ***Math Assistance Area*****.**