DIY: Trigonometry-Graphs of Trig Functions

To review basic Trigonometric concepts, watch the following set of YouTube videos introducing concept of basic unit circle, values of special angles, graphs of various trigonometric ratios and the inverse trigonometric functions. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. Unit Circle :- https://www.youtube.com/watch?v=1m9p9iubMLU
2. Values of sine and cosine of special angles on the unit circle https://www.youtube.com/watch?v=weFditQFWpY
3. Graph of sine and cosine functions https://www.youtube.com/watch?v=uKyBl1FOaks
4. Graphing sine and cosine functions with different amplitudes, periods and phase shifts https://www.youtube.com/watch?v=80c_F0-7ZxE
5. Graph of tan function https://www.youtube.com/watch?v=FK6-tZ5D7xM
6. Graph of tan with transformations: https://www.youtube.com/watch?v=n0cVo54OfXI
7. Graph of sec and csc function https://www.youtube.com/watch?v=2o_dxyKAXLQ
8. Writing Equations for Trig Graphs https://www.youtube.com/watch?v=MqzCD_zE0Aw
9. Finding a Formula for a Trigonometric Graph, Ex 2: https://www.youtube.com/watch?v=Nu6QUnlH79U
10. Graphing a Secant Function, EX 1: https://www.youtube.com/watch?v=c6bDt07i5n0
11. Graph of inverse trig functions https://www.youtube.com/watch?v=bBBUMHe900U

Practice problems: The following problems use the techniques demonstrated in the above videos.

1) Evaluate of the following without using calculator:

a) $\sin 120° =$ b) $\cos 45° =$ c) $\tan 2\pi =$

d) $\sec(-60°) =$ e) $\cot\left(\frac{\pi}{4}\right) =$ f) $\csc\left(-\frac{\pi}{2}\right) =$

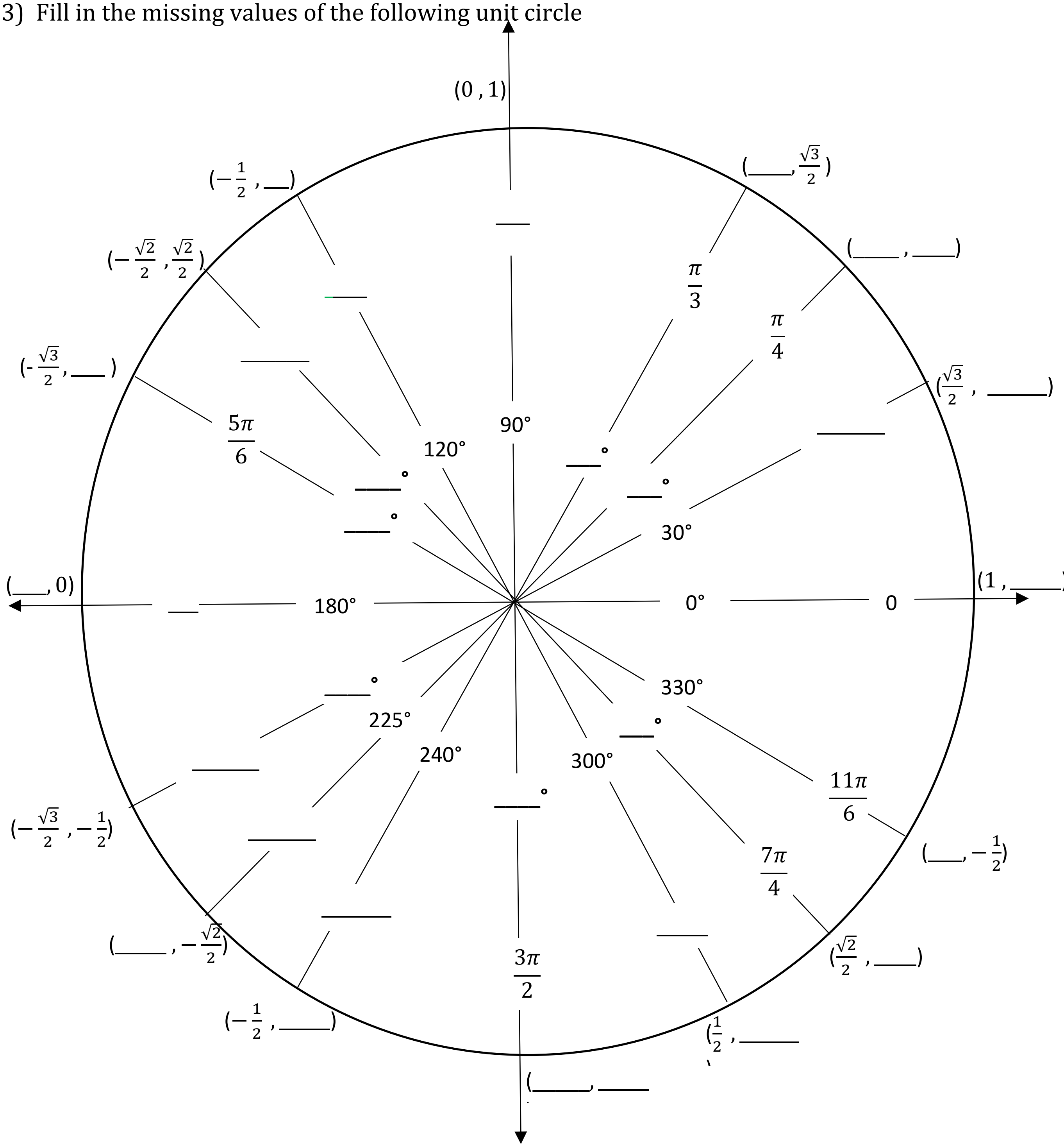
g) $\sin(135°) =$ h) $\cos\left(\frac{3\pi}{4}\right) =$ i) $\sec(-330°)$

2) Draw the basic curves of the following trig functions over two periods:

a) $y = \sin\theta$ b) $y = \cos\theta$ c) $y = \tan\theta$

d) $y = \sec\theta$ e) $y = \csc\theta$ f) $y = \cot\theta$

3) Fill in the missing values of the following unit circle



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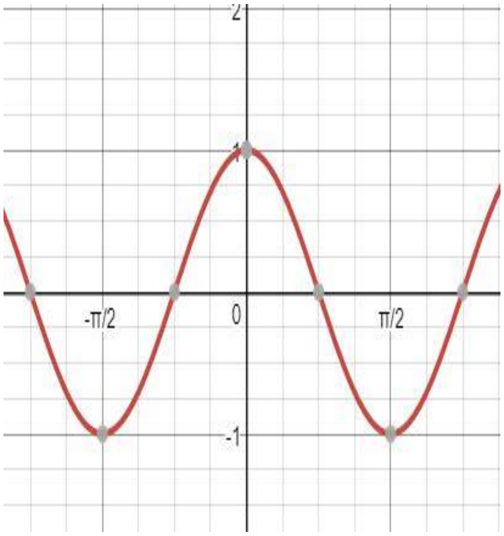
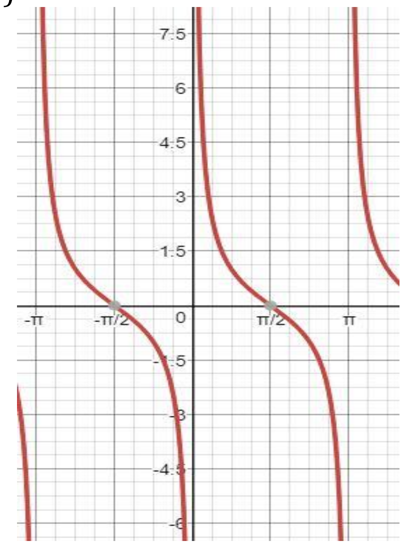
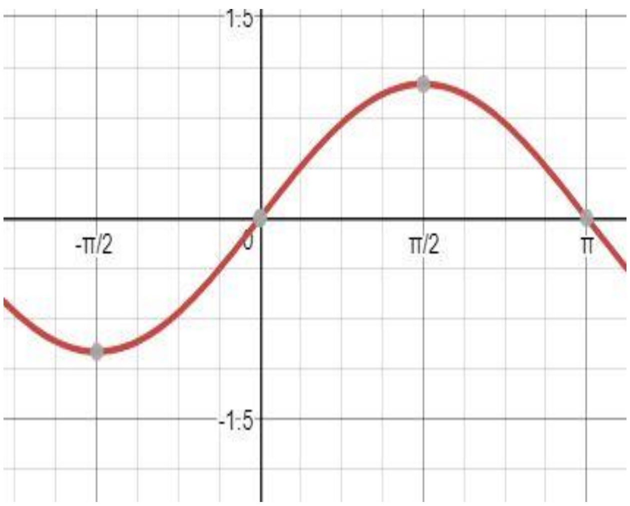
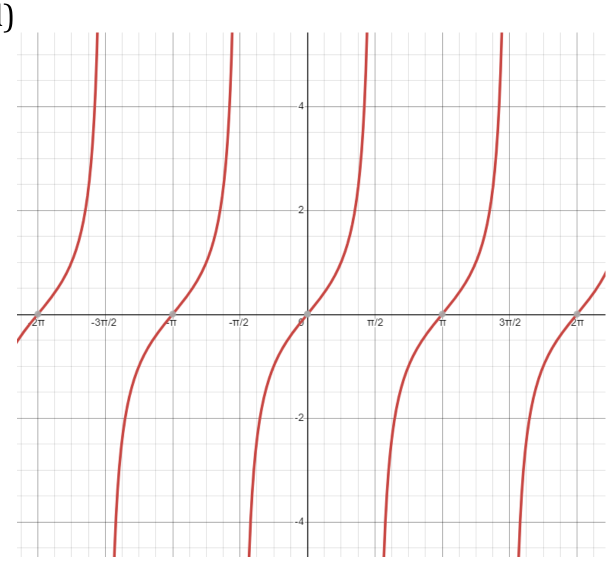
4) Identify the following graphs as sine, cosine, tangent or cotangent graphs

a) 	b) 
c) 	d) 

5) Draw the graphs of the following trigonometric functions over one period.

a) $y = 1+\sin x$ b) $y = \cos(\theta+\pi)$ c) $z = \tan 2\beta$

d) $y = 2\sec\alpha$ e) $s = \csc(2t) + 3$ f) $f(x) = \cot(3x)$

g) $y = 2\sin(2x)$ h) $y = 5\cos(\frac{1}{2}\theta - \frac{\pi}{4})$ i) $y = \frac{1}{2} + \tan\frac{\theta}{2}$

j) $y = \cos(2x+\pi) - \frac{1}{4}$ k) $y = 1 + \frac{1}{\tan x}$ l) $y = 2 - \sin(4\theta+\pi)$

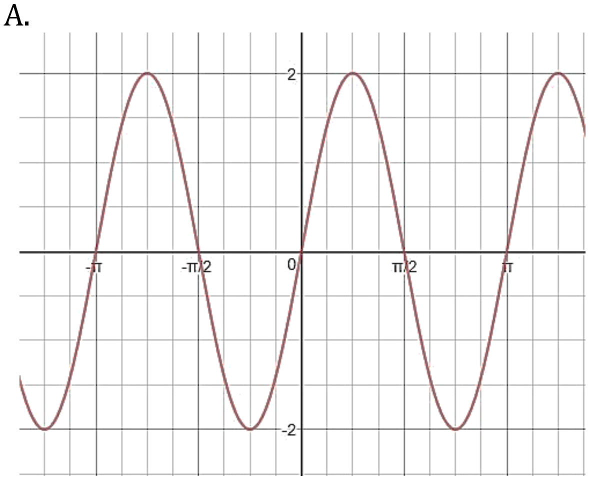
6) Find the amplitude (if applicable), period, phase shift and vertical translation of the following functions:

a) $6y = 3 + \cos(24x - 72\pi)$ b) $23 - y = 24 + \sin(2\theta)$ c) $10 - 3\tan(x-2\pi) = 1-3y$

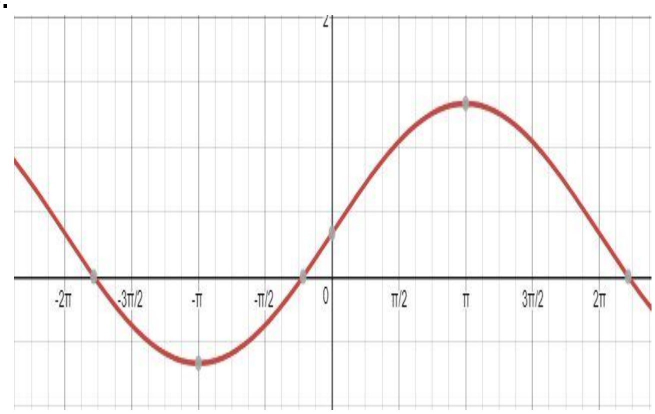
d) $y = \sin(\pi\theta) + 23$ e) $5y = \tan(\frac{x}{2} + 4\pi)$ f) $y = -\frac{1}{4}\cos(\frac{3}{4}x + \frac{\pi}{8})$

7) Match the following graphs to the corresponding function:

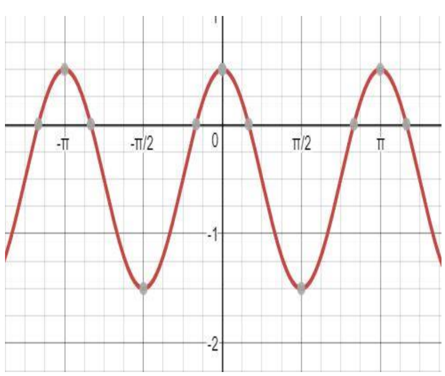
A.



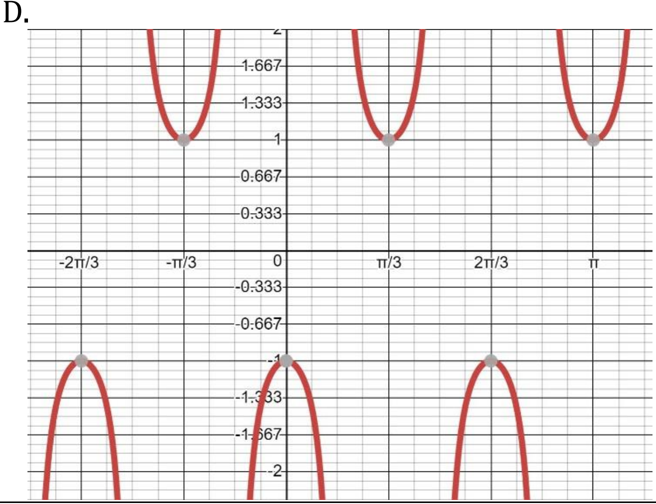
B.



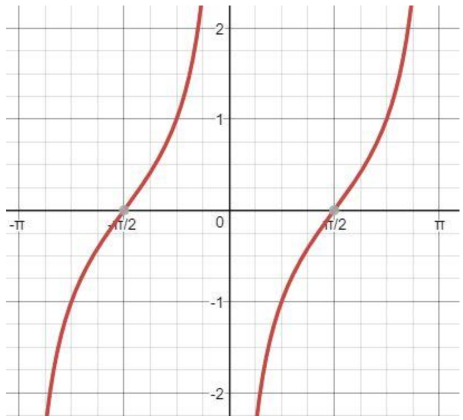
C.



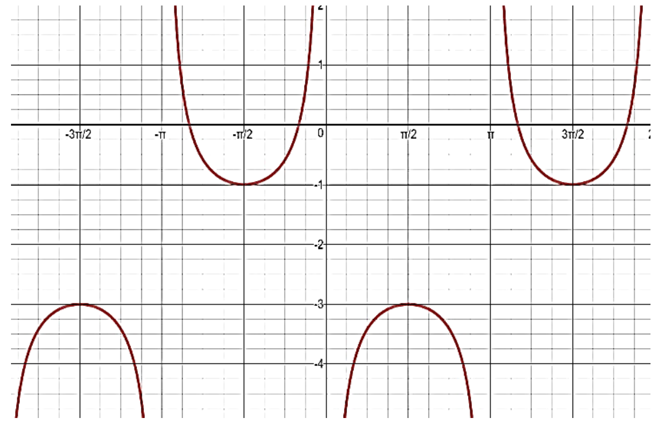
D.



E.



F.

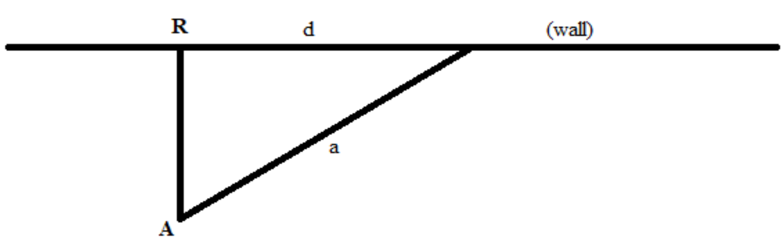


a) y =-2+sec(x + $\frac{\pi}{2}$)	b) y = $-\frac{1}{2} + \cos(2\theta + 2\pi)$
c) y = $\frac{1}{3} + \sin(\frac{\theta}{2} + 2\pi)$	d) y = 2sin(2θ)
e) y = - csc(3θ + $\frac{\pi}{2}$)	f) y = tan(θ − $\frac{\pi}{2}$)

8) Write the equation and then draw a sine function with the following characteristics:

a) Amplitude of 2 units
b) A period of 4π.
c) Has a phase shift of $\frac{\pi}{2}$ to the right
d) A Vertical Translation of $\frac{1}{2}$ down

9) A rotating beacon is located at point ***A*** next to a wall as shown in the figure below. The beacon 4m from the wall.



a) The distance d is given by $d = 4\tan(2\pi t)$, where *t* is time measured in seconds since the beacon started rotating. When t = 0, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value d is positive; *d* is negative if the beacon is aimed to the left of *R*. Find d for each of the following time: (*Round all answers to one decimal place*)

i. *t*=0
ii. *t* =0.1
iii. *t*= 0.2
iv. *t*= 0.8
v. Explain why t-values between 0.25 and 0.75 are meaningless.

b) The distance a is given by

$$a = 4\,|\,\sec 2\pi t\,|$$

Find a for each of the following times: (*Round all answers to one decimal place*)

i. $t = 0$
ii. $t = 0.86$
iii. $t = 1.24$

10) The distance of a weight attached to a spring above its equilibrium position is

$$s(t) = -\,2\cos(20t)$$

inches after t seconds. ($s<0$ means the weight is below its equilibrium position.)

a) What is the maximum height that the weight rises above the equilibrium position?
b) What are the frequency and period?(*Hint: frequency is reciprocal of period*)
c) When does the weight first reach its maximum height?

11) Draw the graph of the following inverse functions:

a) $y = \sin^{-1}(x)$ b) $y = \cos^{-1}(x)$ c) $y = \tan^{-1}(x)$

12) Evaluate the following: (Use calculator wherever necessary)

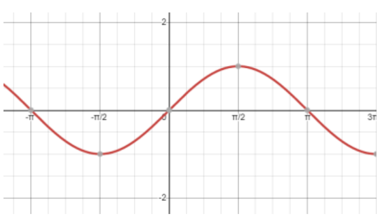
a) $\sin^{-1}(\sin x)$, $-\pi/2 \le x \le \pi/2$ b) $\arccos(\cos 2\pi)$
c) $\cos^{-1}(0.289)$ d) $\tan(\arctan(0))$ e) $\operatorname{arccot}(75)$

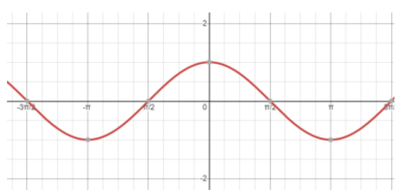
Answers

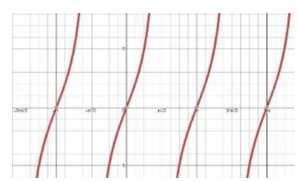
1)

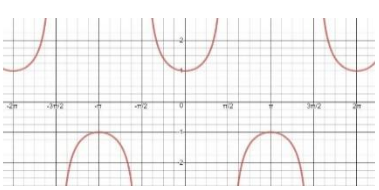
a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) 0 d) 2 e) 1 f) -1 g) $\frac{\sqrt{2}}{2}$ h) $-\frac{\sqrt{2}}{2}$ i) $\frac{2\sqrt{3}}{3}$

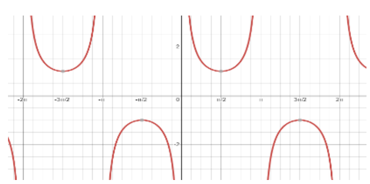
2)

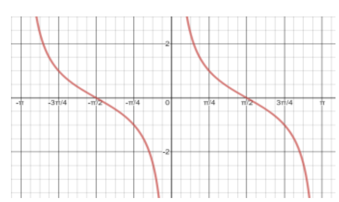

a)


b)

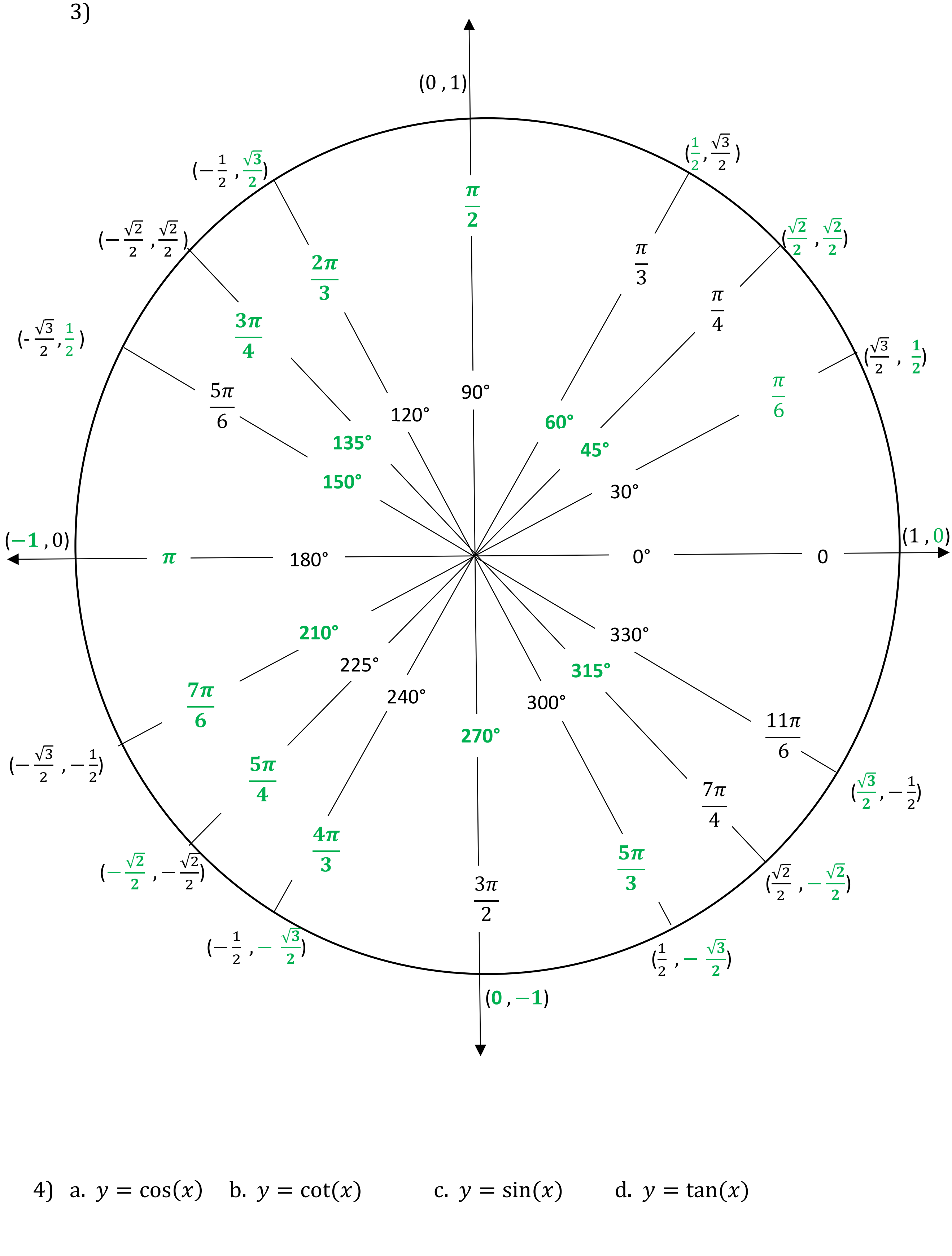
c)


d)


e)


f)


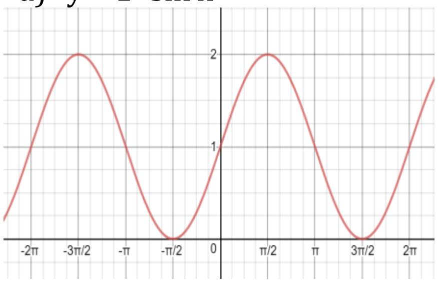
3)



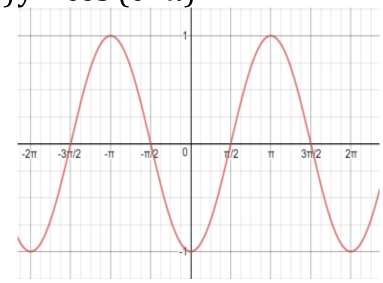
4) a. $y = \cos(x)$ b. $y = \cot(x)$ c. $y = \sin(x)$ d. $y = \tan(x)$

5)

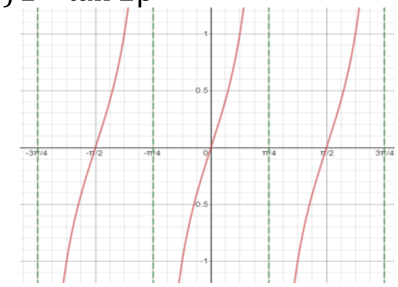
a) y = 1+sin x



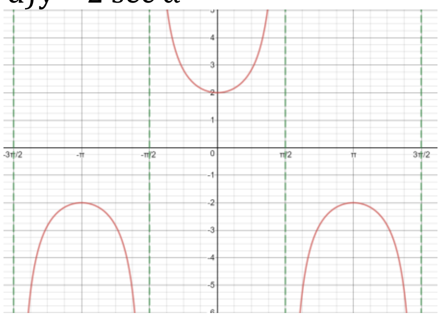
b) y = cos (θ+π)



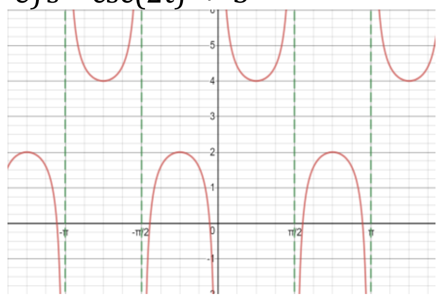
c) z = tan 2β



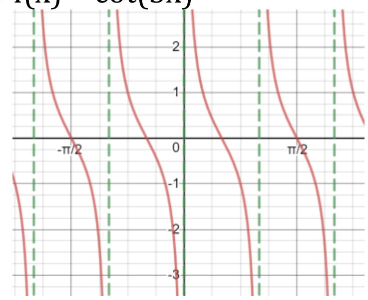
d) y = 2 sec α



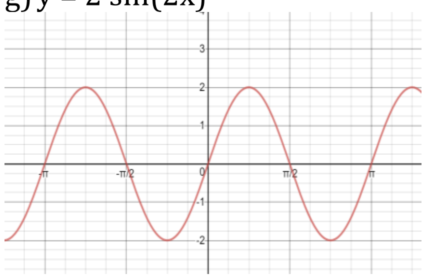
e) s = csc(2t) + 3



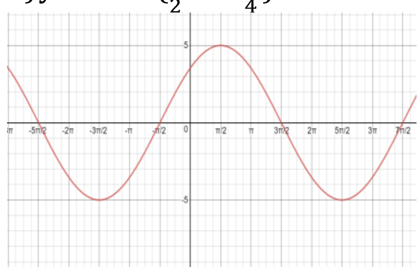
f) f(x) = cot(3x)



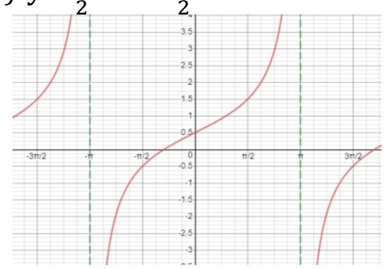
g) y = 2 sin(2x)



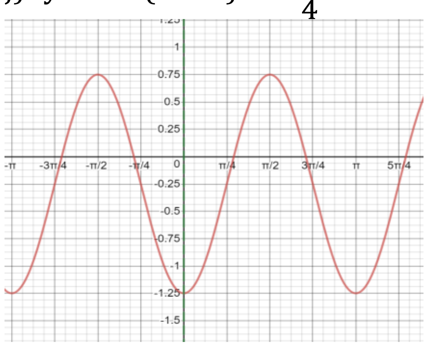
h) $y = 5\cos(\frac{1}{2}\theta - \frac{\pi}{4})$



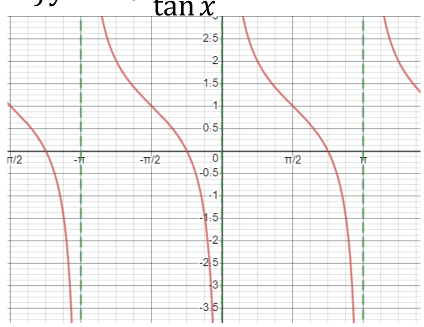
i) $y = \frac{1}{2} + \tan\frac{\theta}{2}$



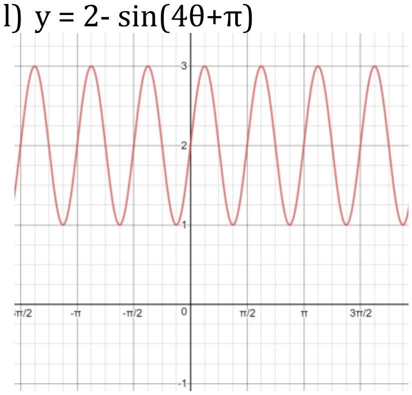
j) $y = \cos(2x+\pi) - \frac{1}{4}$



k) $y = 1 + \frac{1}{\tan x}$



l) y = 2- sin(4θ+π)



6)

a) Period = $\frac{\pi}{12}$
Phase Shift = 3π to the right
Amplitude = $\frac{1}{6}$
Vertical Translation = $\frac{1}{2}$ unit up

b) Period = π
Phase Shift = 0
Amplitude =1
Vertical Translation = 1 unit down

c) Period =π
Phase Shift = 2π right
Amplitude = not applicable
Vertical Translation =3 units down

d) Period = 2
Phase Shift = none
Amplitude = 1
Vertical Translation = 23 units up

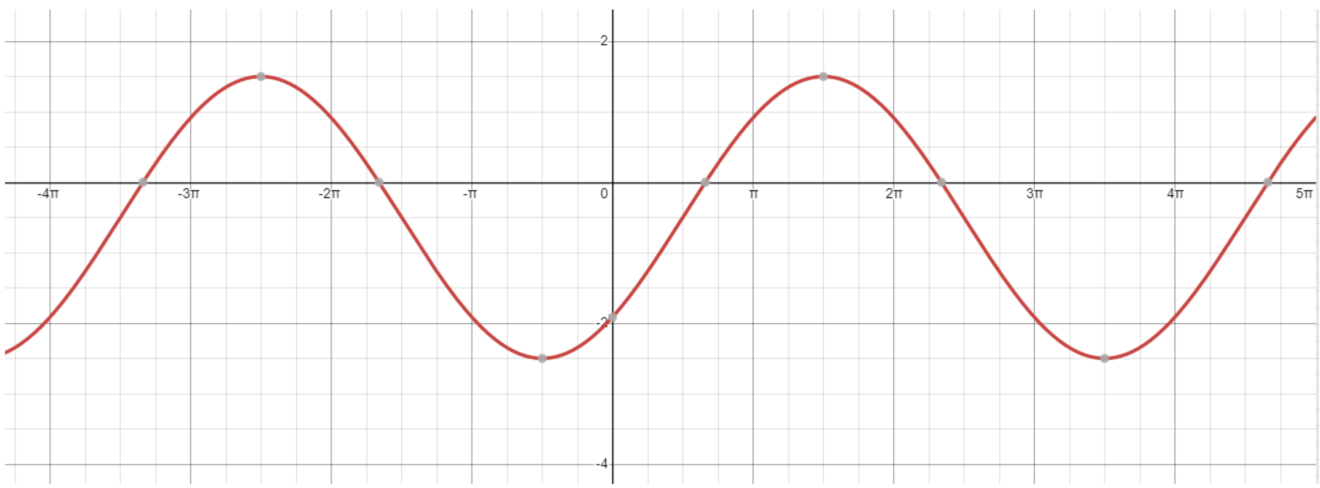
e) Period = 2π
Phase Shift = 8π left
Amplitude = not applicable
Vertical Translation = none

f) Period =$\frac{8\pi}{3}$
Phase Shift =$\frac{\pi}{6}$ left
Amplitude = $\frac{1}{4}$ with reflection about x-axis
Vertical Translation = none

7)

A. d
B. c
C. b
D. e
E. f
F. a

8) $y = -\frac{1}{2} + 2\sin\frac{1}{2}(x - \frac{\pi}{2})$ or $y = -\frac{1}{2} + 2\sin(\frac{x}{2} - \frac{\pi}{4})$



9) a)

i. 0 ii. 2.9m ii. 12.3 m iv. -12.3 m v. During that time, the beacon is pointing away from the wall.

b)

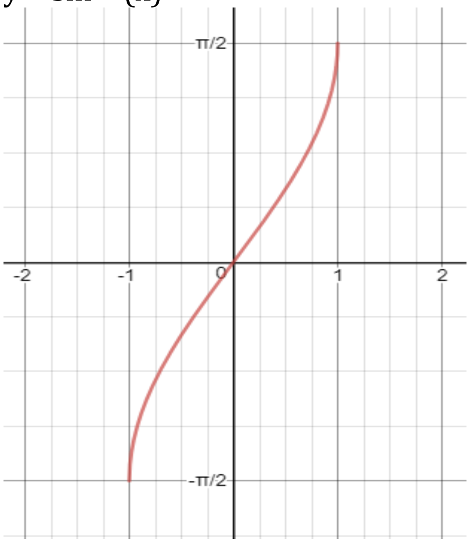
i. 4m ii. 6.3 m ii. 63.70m

10)

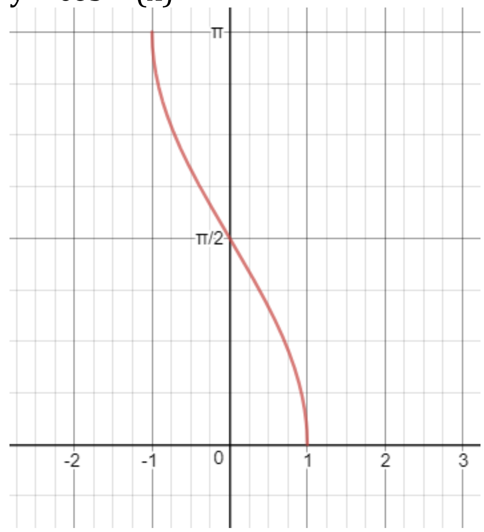
a) 2 inches b) Frequency = $\frac{10}{\pi}$ cycles per second, Period $= \frac{\pi}{10}$ seconds c) $t = \frac{\pi}{20}$ seconds

11)

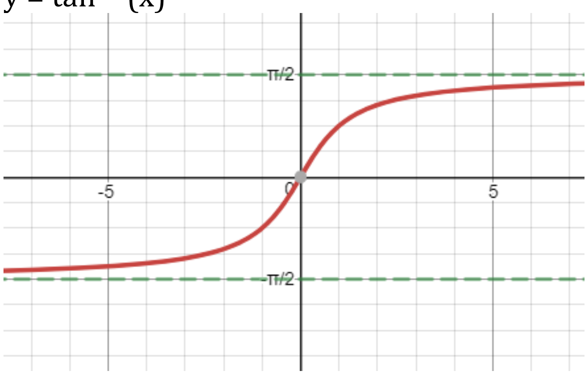
a) $y = \sin^{-1}(x)$



b) $y = \cos^{-1}(x)$



c) $y = \tan^{-1}(x)$

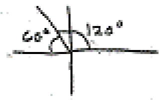


12)

a) x
b) 0
c) 73.2° or 1.28 radians
d) 0
e) 0.764° or 0.013 radians

Detailed Solutions

1. a) $\sin 120° = \sin 60° = \boxed{\sqrt{3}/2}$



b) $\cos 45° = \boxed{\sqrt{2}/2}$

c) $\tan 2\pi = \tan 0 = \boxed{0}$

d) $\sec(-60°) = \sec 60° = \dfrac{1}{\cos 60°} = \dfrac{1}{1/2} = \boxed{2}$

e) $\cot(\pi/4) = 1/\tan \pi/4 = 1/1 = 1$

f) $\csc\left(-\frac{\pi}{2}\right) = \dfrac{1}{\sin(-\pi/2)} = -\dfrac{1}{\sin \pi/2} = -\dfrac{1}{1} = -1$

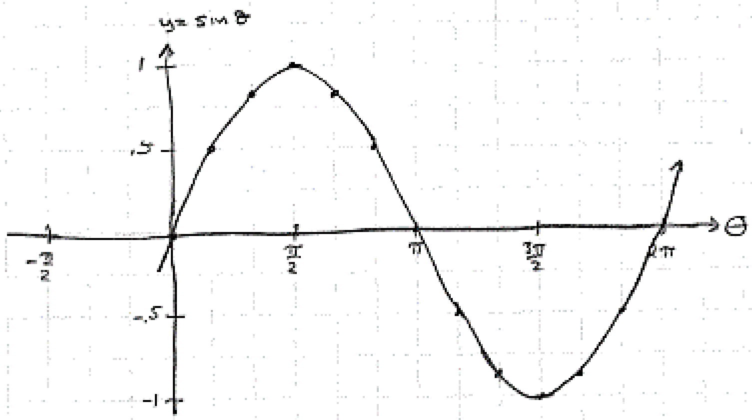
g) $\sin 135° = \sin(180° - 135°) = \sin 45° = \boxed{\sqrt{2}/2}$

h) $\cos 3\pi/4 = -\cos\left(\pi - \frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = -\sqrt{2}/2$

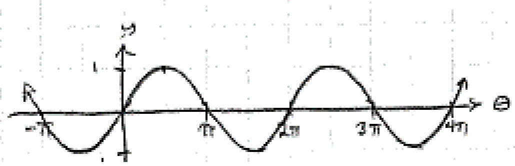
i) $\sec(-330°) = \sec(360° - 330°) = \sec 30° = \dfrac{1}{\cos 30°} = \dfrac{1}{\sqrt{3}/2} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{2\sqrt{3}}{3}}$

2\. a. $y = \sin \theta$

θ	y
0	0
$\pi/6$	$1/2$
$\pi/3$	$\sqrt{3}/2 \approx .87$
$\pi/2$	1
$2\pi/3$	$.87$
$5\pi/6$	$1/2$
π	0
$7\pi/6$	$-1/2$
$3\pi/2$	-1
$11\pi/6$	$-1/2$
2π	0

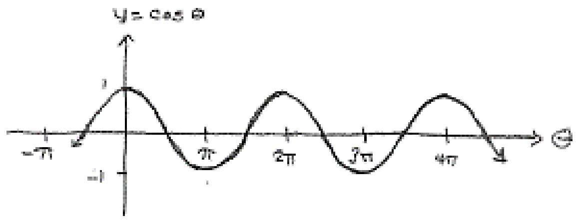


then the values repeat:



2b. $y = \cos\theta$ (this graph is the same as the sine graph but shifted $\pi/2$ to the right.)

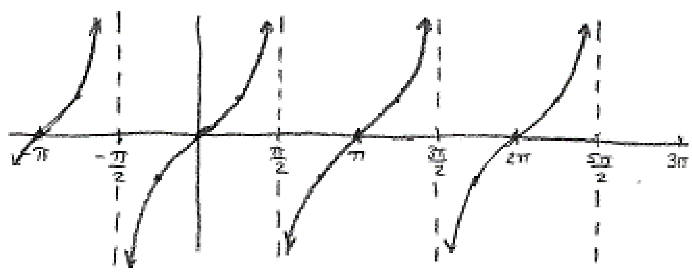
θ	0	$\pi/2$	π	$3\pi/2$	2π	(then it repeats)
y	1	0	-1	0	1	



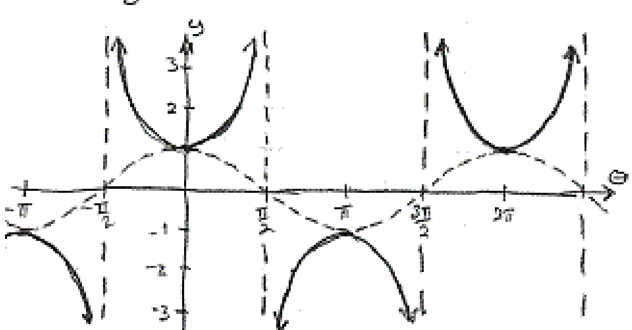
c. $y = \tan\theta$

θ	y
$-\pi/2$	(undefined)
$-\pi/4$	-1
0	0
$\pi/4$	1
$\pi/2$	(undefined)

(then the graph repeats)



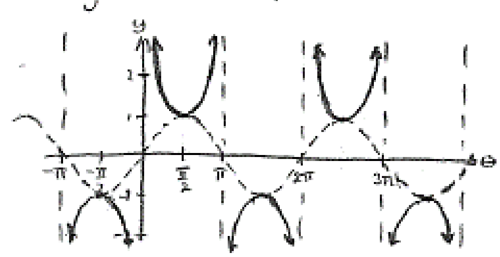
d. $y = \sec\theta$ (reciprocal of $\cos\theta$)



--- $y = \cos\theta$

— $y = \sec\theta$

e. $y = \csc\theta$ (reciprocal of $\sin\theta$)

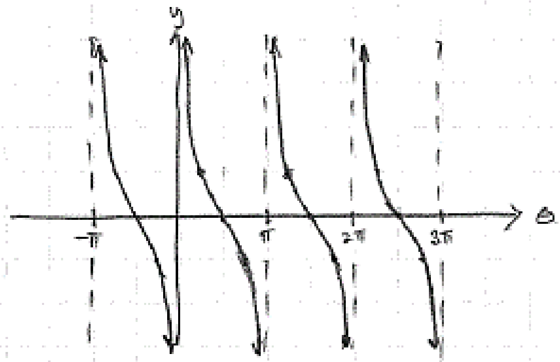


--- $y = \sin\theta$

— $y = \csc\theta$

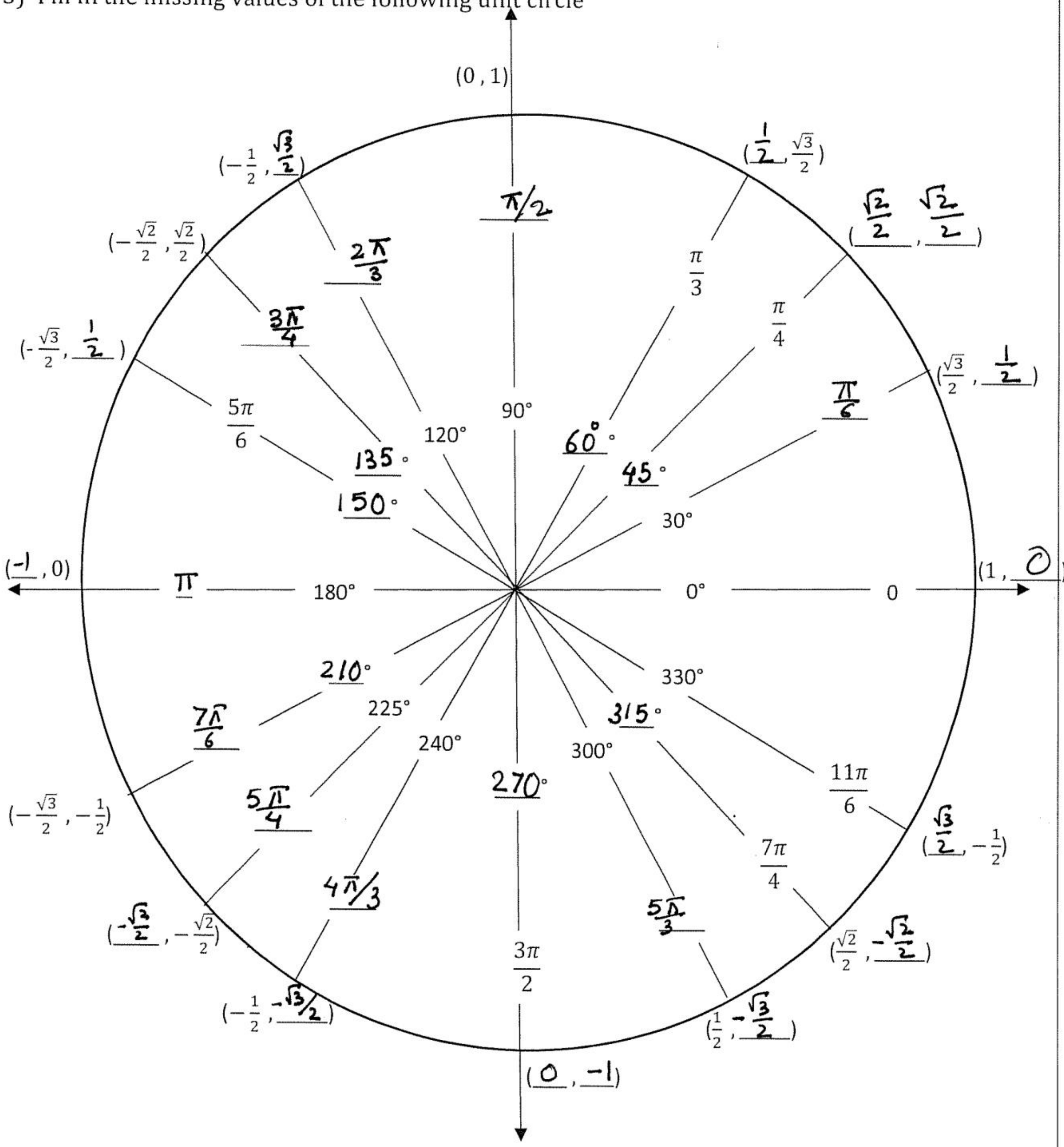
2.f. $y = \cot \theta$ (reciprocal of $\tan \theta$)

θ	$\tan \theta$	$\cot \theta$
0	0	undefined
$\frac{\pi}{4}$	1	1
$\frac{\pi}{2}$	undef.	0
$\frac{3\pi}{4}$	-1	-1
π	0	undef.
$\frac{5\pi}{4}$	1	1
$\frac{3\pi}{2}$	undef	0



(see next page for problem # 3.)

3) Fill in the missing values of the following unit circle

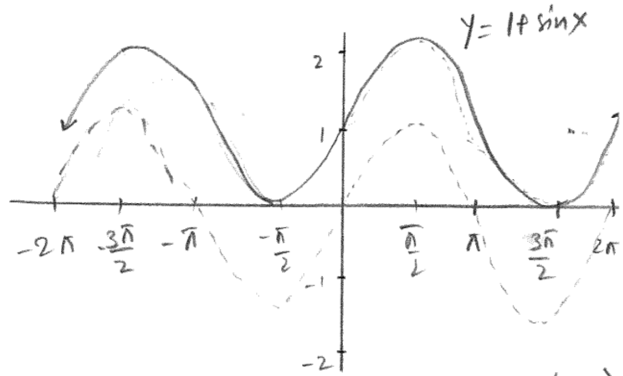


4. a. y = cosine x

b. y = cotangent x

c. y = sine x

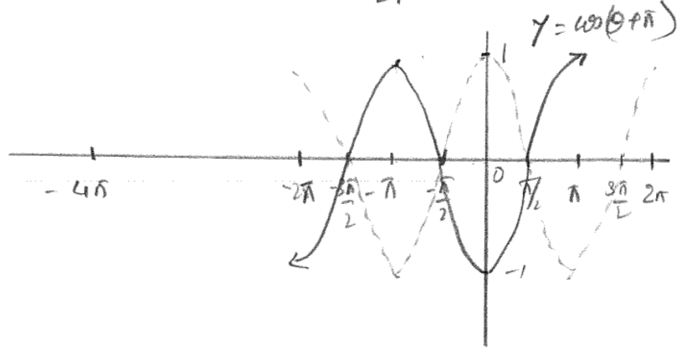
d. y = tangent x



5 a) $Y = 1 + \sin x$

steps :- 1) First we draw the graph of $Y = \sin x$

2) Move the graph one unit up.



5b) $Y = \cos(\theta + \pi)$

steps :- 1) first we draw the graph of $Y = \cos x$

2) Move π units to the left.

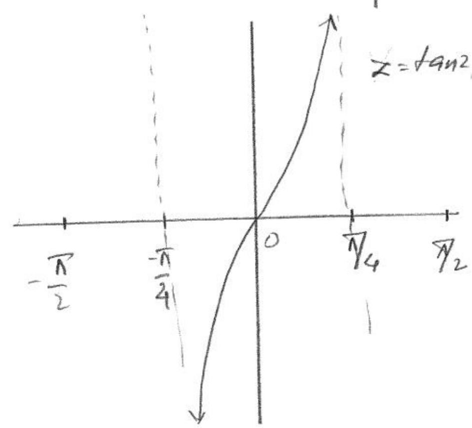
— 4 —

5c) $Z = \tan 2\beta$

steps =

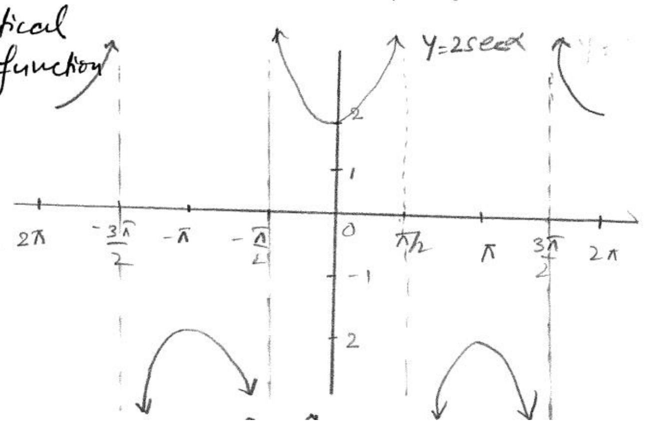
1) period of a tan graph is π therefore the period of $Z = \tan 2\beta$ is $\frac{\pi}{2}$

Thus there is a horizontal shrink to π/2



5d) $Y = 2\sec\alpha$

steps :- 1) We have a vertical strech of the sec α function by 2 units.



5e) $s = \csc(2t) + 3$

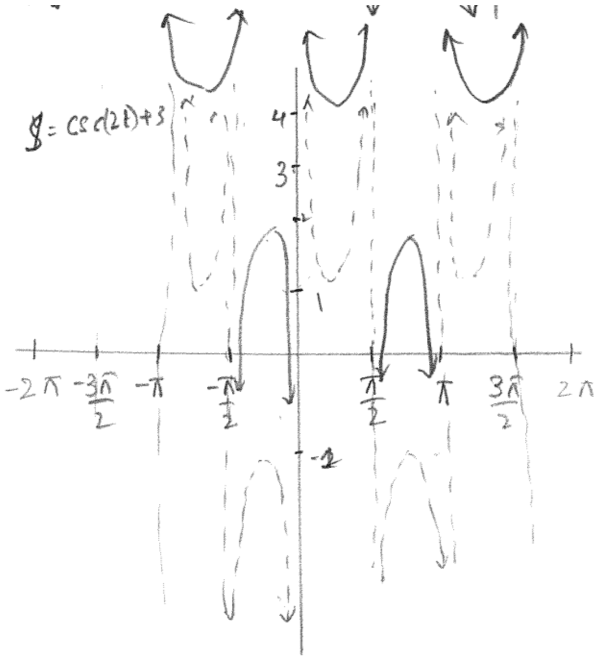
The period of csc is π

Therefore the period of $\csc 2t$ is $\frac{2\pi}{2} = \pi$ ~~$\frac{\pi}{2}$~~

Hence we have a horizontal shrink.

Steps:- 1) We draw the graph of $\csc 2t$

2) Move this graph up by 3 units.



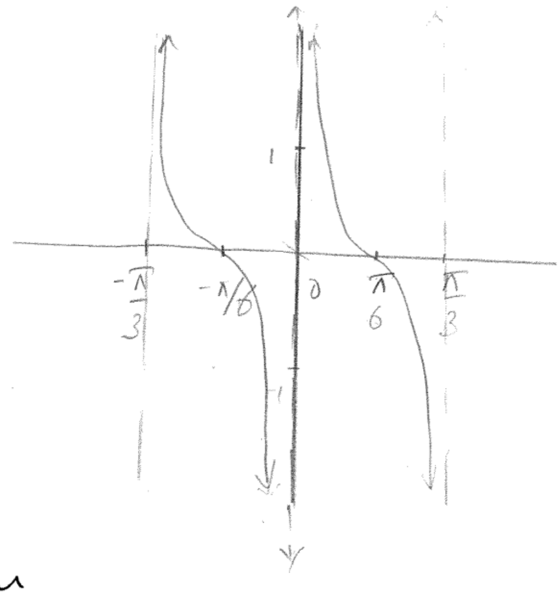
— 5 —

5f) $f(x) = \cot 3x$

The period of cot function is π therefore the period of $\cot 3x$ is

$$\frac{\pi}{3}$$

Thus we have a horizontal shrink in this case.

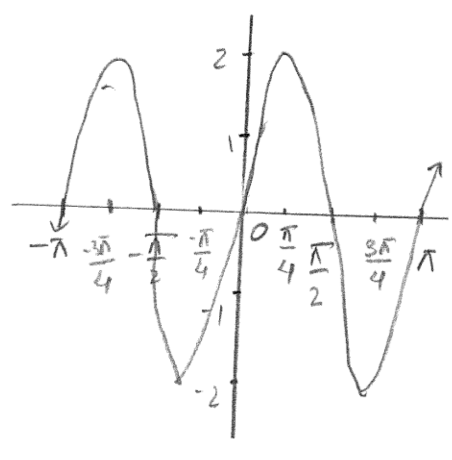


5g) $y = 2\sin(2x)$

The period of sine function is 2π, therefore the period of $\sin 2x$ will be

$$\frac{2\pi}{2} = \pi$$

Thus this will result in horizontal shrink. The function $\sin 2x$ is multiplied by 2 resulting in vertical strech.

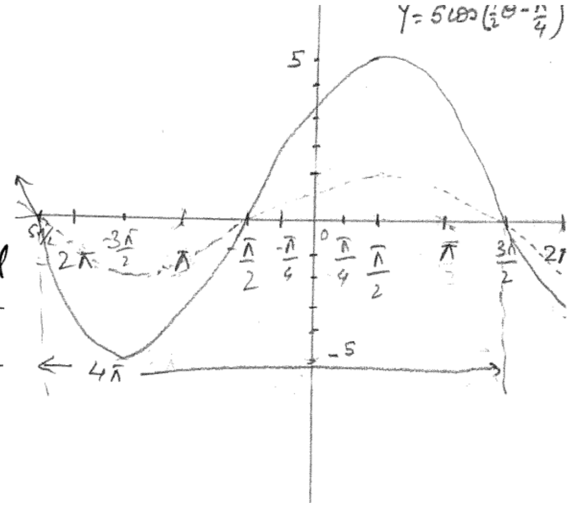


5h) $Y = 5\cos\left(\frac{1}{2}\theta - \frac{\pi}{4}\right)$

$Y = 5\cos\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)$

The period of this function is $\frac{2\pi}{1/2} = 4\pi$

Then we have a horizontal shift of $\frac{\pi}{2}$ to the right and a vertical strech due to multiplication by 5



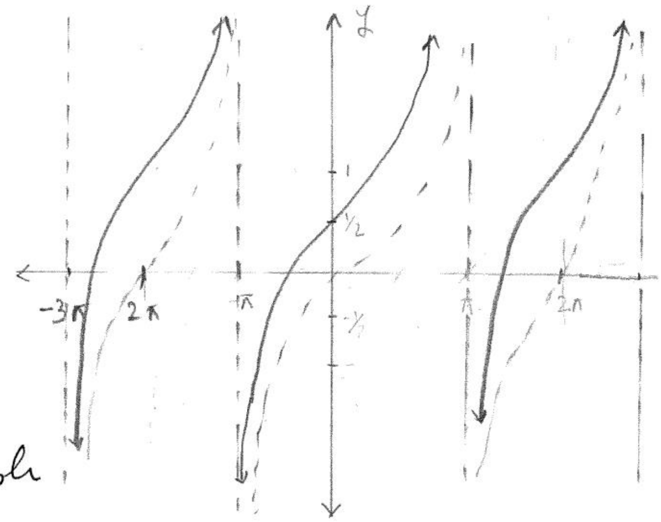
5i) $Y = \frac{1}{2} + \tan\frac{\theta}{2}$

period $= \frac{\pi}{b}$ where $b = \frac{1}{2}$

therefore period $= \frac{\pi}{1/2} = 2\pi$

Step 1:- We draw the graph of $\tan\frac{\theta}{2}$ with a period of 2π

2:- Shift the whole graph up $\frac{1}{2}$ unit



5j) $y = \cos(2x+\pi) - \frac{1}{4}$

$= \cos 2\left(x + \frac{\pi}{2}\right) - \frac{1}{4}$

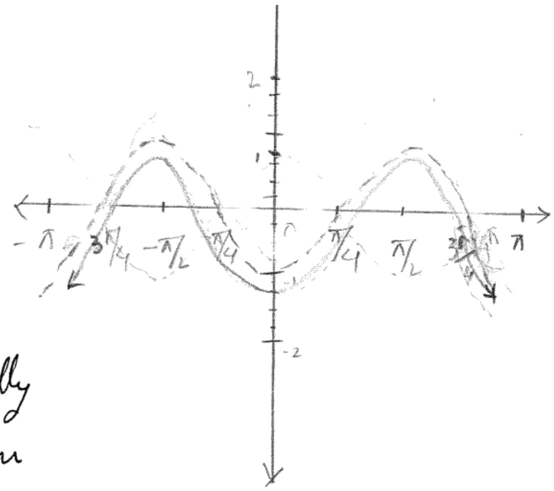
period $= \frac{2\pi}{b}$ where $b = 2$

therefore period $= \frac{2\pi}{2} = \pi$

steps 1:- We draw the graph of $\cos 2x$ with a period of π

2:- Shift the graph horizontally to the right by $\pi/2$

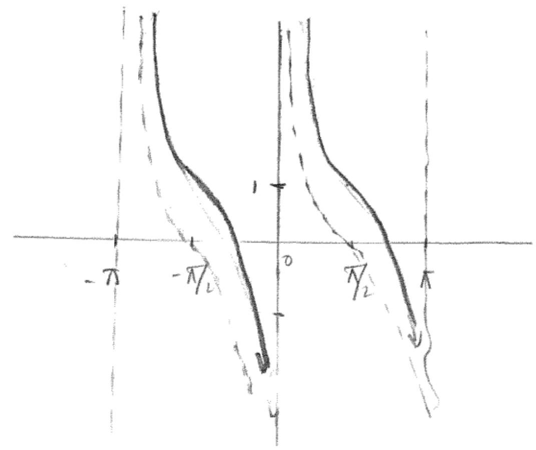
3:- Shift the graph down $1/4$ units



5k) $y = 1 + \frac{1}{\tan x} = 1 + \cot x$

steps:- 1) Draw the graph of $\cot x$

2:- Shift the whole graph up by 1 unit



5l) $y = 2 - \sin(4\theta + \pi)$

$= 2 - \sin 4\left(\theta + \frac{\pi}{4}\right)$

period $= \frac{2\pi}{b}$ where $b = 4$

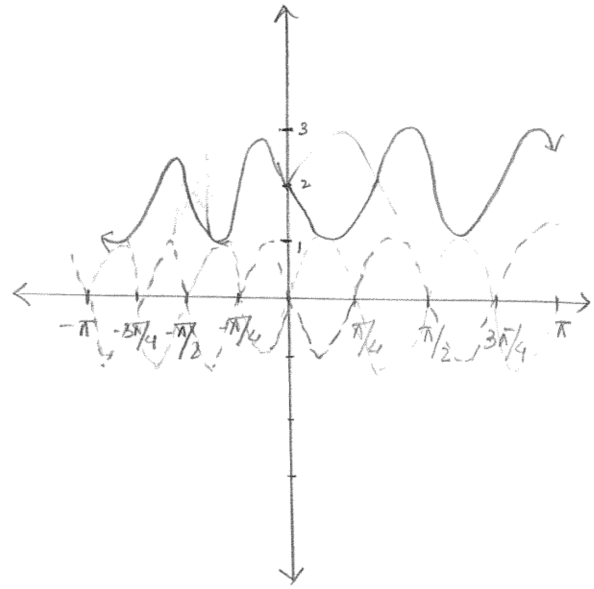
$= \frac{2\pi}{4} = \frac{\pi}{2}$

steps 1) Draw the graph of $\sin 4\theta$ with a period of $\frac{\pi}{2}$

2) shift the graph to the lef $\pi/4$

3) flip the graph about x-axis

4) Move the graph up 2 units.



6a) $6y = 3 + \cos(24x - 72\pi)$

$\frac{6y}{6} = \frac{3 + \cos 24(x - 3\pi)}{6}$

$y = \frac{1}{2} + \frac{1}{6}\cos 24(x - 3\pi)$

period $= \frac{2\pi}{b} = \frac{\cancel{2}\pi}{\cancel{24}_{12}} = \frac{\pi}{12}$

phase shift 3π to the right

amplitude is $1/6$

vertical translation $\frac{1}{2}$ upwards

6b) $23 - y = 24 + \sin 2\theta$

$\quad -23 \qquad -23$

$\frac{-y}{-1} = \frac{1 + \sin 2\theta}{-1}$

$y = -1 - \sin 2\theta$

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift 0

amplitude $= 1$

vertical translation is down 1 unit

6.c. $10 - 3\tan(x-2\pi) = 1-3y$

$3y = 1 - 10 + 3\tan(x-2\pi) = -9 + 3\tan(x-2\pi)$

$y = -3 + \tan(x-2\pi)$

period $= \pi$
amplitude: (not applicable to tangent functions)
phase shift: 2π to the right.
vertical translation: down 3.

d. $y = \sin(\pi\theta) + 23$

period $= \frac{2\pi}{\pi} = \boxed{2}$

amplitude $= 1$
no phase shift.
vertical shift $= 23$ (upward).

e. $5y = \tan\left(\frac{x}{2} + 4\pi\right) \rightarrow y = \frac{1}{5}\tan\frac{1}{2}(x+8\pi)$

period $= \frac{\pi}{1/2} = \boxed{2\pi}$

amplitude: not applicable to tangent functions
phase shift $= -8\pi$ (shifts to left)
vertical shift $= 0$

f. $y = -\frac{1}{4}\cos\left(\frac{3}{4}x + \frac{\pi}{8}\right) \rightarrow y = -\frac{1}{4}\cos\frac{3}{4}\left(x + \frac{\pi}{6}\right)$

period $= \frac{2\pi}{3/4} = \boxed{\frac{8\pi}{3}}$

amplitude $= \left|-\frac{1}{4}\right| = \frac{1}{4}$ (the negative will cause the graph to be reflected over the x-axis).

phase shift $= -\frac{\pi}{6}$ (shifted to left)

vertical translation $= 0$

7. Graph A: By its shape, this graph could be either a sine or cosine function, narrowing the possible correct functions to b, c, or d. If using sine, there would be no phase shift. If using cosine, there would be a phase shift of $\pi/2$ to the right or $-3\pi/2$ to the left. Choice b does not have the correct phase shift so can be eliminated. Choice c has a vertical shift, so cannot be the correct function for graph A.

This leaves choice d. $y = 2sin2\theta$ would have an amplitude of 2 and a period of π. This is the same as shown on graph A, so the correct function is **d.**

Graphs B and C: These graphs are also sine or cosine functions. **B** has a positive vertical shift (upward) while **C** has a negative vertical shift (downward ½ unit). Therefore, it would appear that function c corresponds to graph **B** and function b corresponds to graph **C**.

Checking further: although no vertical scale is shown on graph **B**, we can see that the function appears to have a period of 4π (looking at the points x = -2π and 2π). This would mean the coefficient of the argument of the sine function must be 1/2.

$$y = \frac{1}{3} + \sin\left(\frac{\theta}{2} + 2\pi\right) \ or \ y = \frac{1}{3} + sin\frac{1}{2}(\theta + 4\pi)$$

The phase shift for function c is -4π, which is exactly one period, so the graph would appear the same as if it had no phase shift. This also fits graph **B**. Graph **B** is function **c.**

For graph **C** with a cosine function, the period appears to be π, so the coefficient of the argument would be 2π/b = π so b = 2. Function b can be written as $y = -\frac{1}{2} + \cos 2(\theta + \pi)$. The amplitudes of the graphed function **C** and function b are both 1, and the phase shift of function b is π, which is again exactly one period, so the function would appear the same as if it had no phase shift. Graph **C** is function **b.**

Graphs D and F: These graphs are secant or cosecant function. Graph **D** appears to have no vertical shift and a period of 2π/3, so graph **D** must be function **e**.

Graph **F** has a vertical translation of -2 (downward) and a period of 2π, so graph **F** is function **a**.

Graph E: This graph is a tangent function hence graph **E** is function **f**

8. Characteristics:

a) amplitude $= 2$ $(a = 2)$

b) period $= 4\pi$ $\left(\frac{2\pi}{b} = 4\pi \rightarrow b = \frac{1}{2}\right)$

c) phase shift $= \frac{\pi}{2}$ to right $\left(d = \frac{\pi}{2}\right)$

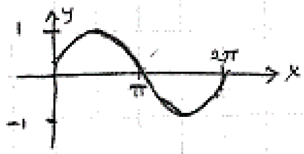
d) vertical translation $= \frac{1}{2}$ down $\left(c = -\frac{1}{2}\right)$

general sine model: $y = c + a \sin b(x - d)$

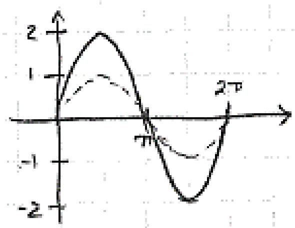
substituting: $\boxed{y = -\frac{1}{2} + 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)}$

To show each modification, see series of graphs below.

$y = \sin x$



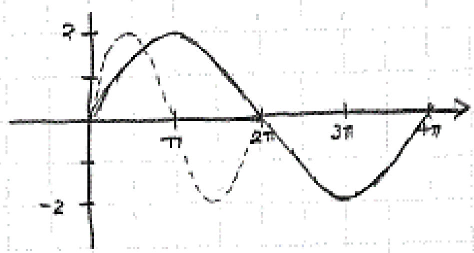
$y = 2 \sin x$



--- $y = \sin x$

— $y = 2 \sin x$

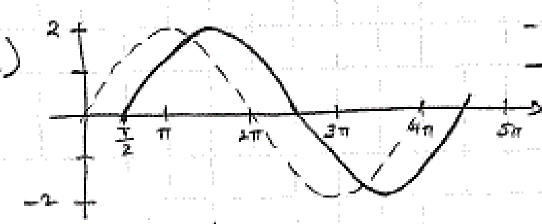
$y = 2 \sin \frac{1}{2} x$



--- $y = 2 \sin x$

— $y = 2 \sin \frac{1}{2} x$

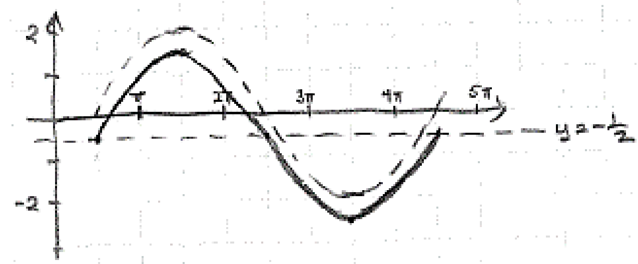
$y = 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$



--- $y = 2 \sin \frac{1}{2} x$

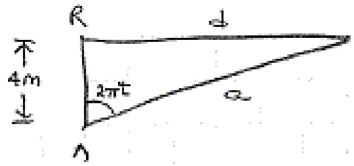
— $y = 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

$y = -\frac{1}{2} + 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$



--- $y = 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

—— $y = -\frac{1}{2} + 2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

9. 

given: $d = 4\tan 2\pi t$

a)

i. if $t=0$ $\quad d = 4\tan 0 = 0$ (beacon is aimed perpendicular to the wall). 

ii. if $t = 0.1$, $d = 4\tan 2\pi(.1) = 4(.7265) \approx \boxed{2.9\text{ m}}$ 

iii. $t = 0.2$ $\quad d = 4\tan 2\pi(.2) = 4\tan .4\pi = 4(3.07768)$

$d \approx \boxed{12.3\text{ m}}$ 

iv. $t = 0.8$ $\quad d = 4\tan 2\pi(.8) \approx 4(-3.07768) \approx \boxed{-12.3\text{ m}}$

v: the given equation is meaningless for $0.25 \le t \le 0.75$

why? at $t = .25$, the beacon is shining parallel to the wall. Between $t = .25$ and $t = .75$, the beacon is pointed <u>away</u> from the wall.

The equation would also be meaningless whenever $2\pi t$ gives an angle in quadrants II and III.

b) if $a = 4\left|\sec 2\pi t\right|$,

i. $t=0$ $\quad a = 4\left|\sec 0\right| = 4(1) = \boxed{4\text{ m}}$

ii. $t = 0.86$ $\quad a = 4\left|\sec 2\pi(.86)\right| = \dfrac{4}{\cos(2\pi(.86))} \approx \dfrac{4}{.6374} \approx \boxed{6.3\text{ m}}$

iii. $t = 1.24$ $\quad a = 4\left|\sec 2\pi(1.24)\right| = \dfrac{4}{\cos 2.48\pi} \approx \boxed{63.7\text{ m}}$

(beacon is almost parallel to the wall).

10. a) maximum height would occur when $\cos 20t = -1$.
Recall that $-1 \le \cos\theta \le 1$ for any θ, so S would be a maximum positive value when cosine $= -1$.

$$s(t) = -2\cos 20t$$

$$s_{max} = -2\,(-1) = 2 \text{ inches.}$$ (above equilibrium position.)

b) $\text{period} = \frac{2\pi}{20} = \boxed{\frac{\pi}{10} \text{ seconds}}$ (or seconds per cycle).

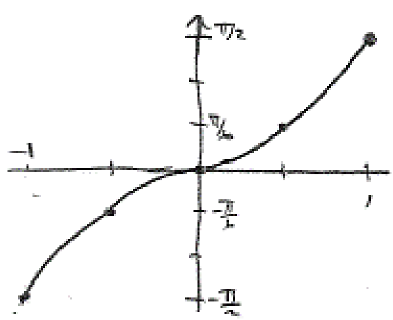
$\text{frequency} = \frac{1}{\text{period}} = \boxed{\frac{10}{\pi} \text{ cycles/second}}$

c) the weight would first be at its maximum height when $\cos 20t = -1$ the first time after $t=0$. That would occur when $20t = \pi$

$$\boxed{t = \frac{\pi}{20} \text{ sec.}}$$

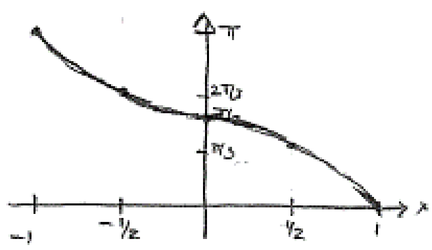
11. a. $y = \sin^{-1} x$ or $\begin{cases} x = \sin y \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2} \end{cases}$

x	-1	$-1/2$	0	$1/2$	1
y	$-\pi/2$	$-\pi/6$	0	$\pi/6$	$\frac{\pi}{2}$



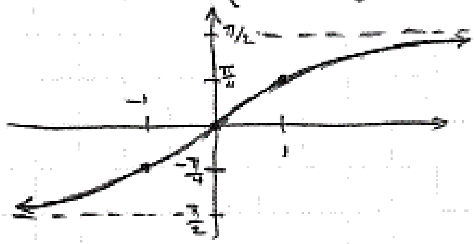
b. $y = \cos^{-1} x$ or $\begin{cases} x = \cos y \\ 0 \le y \le \pi \end{cases}$

x	-1	$-1/2$	0	$1/2$	1
y	π	$2\pi/3$	$\pi/2$	$\pi/3$	0



11. c. $y = \tan^{-1} x$ or $\begin{cases} x = \tan y \\ -\pi/2 < y < \pi/2 \end{cases}$

x	$(-\infty)$ undef	-1	0	1	$(+\infty)$ undef
y	$-\pi/2$	$-\frac{\pi}{4}$	0	$\pi/4$	$\pi/2$



12. a. $\sin^{-1}(\sin x) = x$

Note: if x is <u>not</u> in the interval $[-\pi/2, \pi/2]$, the answer would be different. For example, $\sin^{-1}\left(\sin \frac{3\pi}{2}\right)$

$= \sin^{-1}(-1) = -\pi/2$

b. $\arccos(\cos 2\pi) = \arccos(1) = \boxed{0}$

c. $\cos^{-1}(0.289) = \boxed{1.28 \text{ radians or } 73.2°}$

d. $\tan(\arctan 0) = \tan(0) = \boxed{0}$

e. $\text{arccot}(75) = \theta \Rightarrow \cot\theta = 75$

$\tan\theta = \frac{1}{75}$

$\theta = \arctan\left(\frac{1}{75}\right)$

$= \boxed{0.013 \text{ radians or } 0.764°}$

Additional Resources

Click on the links below to download worksheets for more practice:

1. Graphing trig functions
2. Inverse trig functions

   Alternatively;

1. Go To https://www.kutasoftware.com/freeipc.html
2. Under "**Trigonometry**" click on:
   - Graphing trig functions
   - Inverse trig functions
3. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets.
4. For help please contact the ***Math Assistance Area***.

References:

Lial, Margaret L., Hornsby, John, Schneider,David I. and Daniels, Callie J. Trigonometry. Pearson, 2009. Print