DIY: *Solving Linear Equations*

To review Solving Linear Equations, watch the following set of YouTube videos introducing positive and negative integers, operations on integers, order of operations in an arithmetic expression, absolute values, place values and rounding. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

Solving basic linear equations:

1. https://www.youtube.com/watch?v=l3XzepN03KQ Introduction to solving linear equations: equations involving addition and subtraction.
2. https://www.youtube.com/watch?v=Qyd_v3DGzTM Solving linear equations involving multiplication or division
3. https://www.youtube.com/watch?v=LDIiYKYvvdA Solving 2-step linear equations

Multi-step linear equation solving:

4. https://www.youtube.com/watch?v=76E9K3JzjDM Variable on both sides of equation, example 1
5. https://www.youtube.com/watch?v=NwN7LM5AMzw example 2 (with decimals)
6. https://www.youtube.com/watch?v=nq95m-X_lp0 example 3 (with fractions)
7. https://www.youtube.com/watch?v=lC_rBjLc_us equations with fractions and parentheses

Applications of Linear Equations:

8. https://www.youtube.com/watch?v=4Ru1suvxv6o Distance, rate, and time problem
9. https://www.youtube.com/watch?v=QKiN9K2DXBI Mixture problem
10. https://www.youtube.com/watch?v=NCqjjoOw3Ck Work (Rate) problem
11. https://www.youtube.com/watch?v=M20if3pBHwU Solving literal equations (formulas)

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

In questions 1-8, solve for the variable.

1. a. $5x = 12$ b. $-3t = 15$ c. $\frac{3}{2}c = \frac{10}{7}$

2. a. $x + 4 = 13$ b. $5 - p = -7$ c. $\frac{2}{3} + t = \frac{1}{4}$

3. a. $2x - 3 = 4$ b. $5 - 6d = 23$ c. $\frac{x}{2} + 3 = \frac{5}{6}$ d. $\frac{3t-4}{2} = 8$

4. $2x + 3 = 7x - 5$

5. $3(s + 4) - 8 = 5s - 2$

6. $0.75(x + 10) + x = 0.5(2x + 10)$

7. $\frac{5}{2}(n - 4) = \frac{1}{3}n + 6$

8. a. $7x + 6 = 3(x + 4) + 2(2x - 3)$ b. $12a - 7 = 8(2a - 3) + (7 - 4a)$

Solve the following application problems:

9. How many ounces of a 10% acid solution would have to be mixed with a 40% acid solution to make 80 ounces of a 35% acid solution?

10. A car leaves an intersection at 12 noon heading east at 35 miles per hour. At 12:30 PM, a second car leaves the same intersection heading east also. How fast would the second car have to travel in order to catch up to the first car by 4 PM?

11. Alex can paint a room in 90 minutes. Brad can paint the same room in 60 minutes. How long will it take to paint the room if they work together?

12. The formula for the area of a trapezoid is $A = \frac{h}{2}(B + b)$. Solve the equation for b in terms of A, B, and h.

13. From electricity, the equation for the combined resistance of two resistors connected in parallel is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Solve this equation for R_1 in terms of R and R_2.

14. A boat can travel 20 miles upstream against a current with a speed of 3 mph in the same amount of time that the same boat can travel 30 miles downstream with the same current. What is the speed of the boat in still water (without the current)?

Answers:

1. a. $x = \frac{12}{5}$ b. t = -5 c. $c = \frac{20}{21}$

2. a. x = 9 b. p = 12 c. $t = -\frac{5}{12}$

3. a. $x = \frac{7}{2}$ b. d = -3 c. $x = -\frac{13}{3}$ d. $t = \frac{20}{3}$

4. $x = \frac{8}{5}$ *or* $1\frac{3}{5}$ 5. s = 3 6. $x = -\frac{10}{3}$ *or* $-3\frac{1}{3}$ 7. $n = \frac{96}{13}$ *or* $7\frac{5}{13}$

8.a. x = any real number b. no solution

9. need $13\frac{1}{3}$ ounces of the 10% solution (mixed with $66\frac{2}{3}$ ounces of the 40% solution

10. The second car would have to travel at 40 mph to catch the first car at 4 PM.

11. It would take 36 minutes for Alex and Brad to paint the room together.

12. $b = \frac{2A}{h} - B$ *or* $\frac{2A-hB}{h}$ 13. $R = \frac{R_1 R_2}{R_1 + R_2}$

14. The boat travels at 15 mph in still water.

Detailed Solutions

1. a. $5x = 12$. Divide both sides of equation by 5.

$$\frac{5x}{5} = \frac{12}{5} \Rightarrow \boxed{x = \frac{12}{5}}$$

b. $-3t = 15$ $\quad \frac{-3t}{-3} = \frac{15}{-3} \Rightarrow t = -\frac{15}{3} = \boxed{-5}$

c. $\frac{3}{2}c = \frac{10}{7}$ The least common denominator (LCD) for all terms is $2(7) = 14$. Multiply both sides of the equation by 14.

$$14\left(\frac{3}{2}c\right) = 14\left(\frac{10}{7}\right)$$

$$\frac{21c}{21} = \frac{20}{21} \Rightarrow c = \frac{20}{21}$$

alternate method: multiply both sides of the equation by the reciprocal of the coefficient of x. (reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$).

$$\frac{2}{3}\left(\frac{3}{2}c\right) = \frac{2}{3}\left(\frac{10}{7}\right)$$

$$\boxed{c = \frac{20}{21}}$$

2. a. $x + 4 = 13$

$-4 \quad -4$

$\boxed{x = 9}$

b. $5 - p = -7$

$-5 \quad -5$

$-p = -12$

$(-1)(-p) = (-1)(-12)$

$\boxed{p = 12}$

c. $\frac{2}{3} + t = \frac{1}{4}$

LCD = 12. Multiply each term by 12. (or $\frac{12}{1}$)

$$\frac{12}{1}\left(\frac{2}{3}\right) + 12t = \frac{12}{1}\left(\frac{1}{4}\right)$$

$$8 + 12t = 3$$

$-8 \quad -8$

$12t = -5$

$\frac{12t}{12} = \frac{-5}{12} \Rightarrow \boxed{t = -\frac{5}{12}}$

3. a. $2x - 3 = 4$

$+3 \quad +3$

$2x = 7$

$\frac{2x}{2} = \frac{7}{2} \Rightarrow \boxed{x = \frac{7}{2}}$

b. $5 - 6d = 23$

$-5 \quad -5$

$-6d = 18$

$\frac{-6d}{-6} = \frac{18}{-6}$

$d = -\frac{18}{6}$

$\boxed{d = -3}$

c. $\frac{x}{2} + 3 = \frac{5}{6}$

LCD = 6

$\overset{3}{\cancel{6}}\left(\frac{x}{\cancel{2}_1}\right) + 6(3) = \overset{1}{\cancel{6}}\left(\frac{5}{\cancel{6}_1}\right)$

$3x + 18 = 5$

$-18 \quad -18$

$3x = -13$

$\frac{3x}{3} = \frac{-13}{3}$

$\boxed{x = -\frac{13}{3}}$ ← note: we could also convert these answers to mixed numbers.

$-\frac{13}{3} = -4\frac{1}{3}$

d. $\frac{3t-4}{2} = 8$

$2\left(\frac{3t-4}{2}\right) = 2(8)$

$3t - 4 = 16$

$+4 \quad +4$

$3t = 20$

$\frac{3t}{3} = \frac{20}{3}$

$\boxed{t = \frac{20}{3}}$ → $\frac{20}{3} = 6\frac{2}{3}$

4. $2x + 3 = 7x - 5$

We must move the two x-terms to the same side and the two constant terms to the other side.

$2x + 3 = 7x - 5$

$-2x \quad -2x$

$3 = 5x - 5$

$+5 \quad +5$

$8 = 5x \longrightarrow \frac{5x}{5} = \frac{8}{5} \rightarrow \boxed{x = \frac{8}{5} \text{ or } 1\frac{3}{5}}$

5. $3(s+4) - 8 = 5s - 2$

First, remove ( ) by distributing the 3 to both inside terms

$3s + 3(4) - 8 = 5s - 2$

$3s + 12 - 8 = 5s - 2$

$3s + 4 = 5s - 2$

$-3s \quad -3s$

$4 = 2s - 2$

$+2 \quad +2$

$6 = 2s$

$\frac{6}{2} = \frac{2s}{2} \Rightarrow 3 = s$ or $\boxed{s = 3}$

6. $.75(x+10) + x = .5(2x+10)$

To remove decimals, we multiply each term on both sides of the equation by 100. This "moves" the decimal point 2 places to the right.

$.75(100) = 75 \qquad .5(100) = .50(100) = 50$

$100[.75(x+10)] + 100 \cdot x = 100[.5(2x+10)]$

$75(x+10) + 100x = 50(2x+10)$

$75x + 750 + 100x = 100x + 500$

$175x + 750 = 100x + 500$

$-100x \quad -100x$

$75x + 750 = 500$

$-750 \quad -750$

$75x = -250$

$\frac{75x}{75} = \frac{-250}{75}$

$x = -\frac{250 \div 25}{75 \div 25} = \boxed{-\frac{10}{3} \text{ or } -3\frac{1}{3}}$

7. $\frac{5}{2}(n-4) = \frac{1}{3}n + 6$

$\frac{5}{2}n - \frac{20}{2} = \frac{1}{3}n + 6 \quad$ LCD = 6

$3\left(\frac{5}{2}n\right) - 3\left(\frac{20}{2}\right) = 2\left(\frac{1}{3}n\right) + 6(6)$

$15n - 60 = 2n + 36$

$-2n \qquad -2n$

$13n - 60 = 36$

$+60 \qquad +60$

$13n = 96$

$\frac{13n}{13} = \frac{96}{13} \rightarrow$ **$n = \frac{96}{13}$ or $7\frac{5}{13}$**

8. a. $7x + 6 = 3(x+4) + 2(2x-3)$

$7x + 6 = 3x + 12 + 4x - 6$

$7x + 6 = 7x + 6$

This is an identity. It is true for any value of x.

So solution is **x = any real number.**

b. $12a - 7 = 8(2a - 3) + (7 - 4a)$

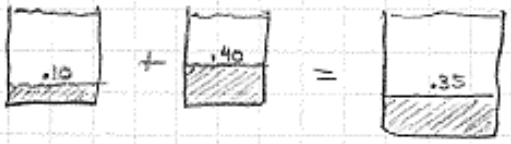
$12a - 7 = 16a - 24 + 7 - 4a$

$12a - 7 = 12a - 17$

$-12a \qquad -12a$

$-7 = -17$ ← This is never true. It is a contradiction.

So there is **no solution.**

9. 

80 oz.

let x = number of ounces of the 10% solution.

Since there is a total of 80 ounces in the mix, there must be $80-x$ ounces of the 40% solution.

	strength	× total volume	= vol. of acid	
10% soln.	.10	x	$.10x$	volume of acid in the 2 ingredients must equal
40% soln.	.40	$80-x$	$.40(80-x)$	
mix	.35	80	$.35(80)$	the volume of acid in the mixture.

equation: $.10x + .40(80-x) = .35(80)$

(multiply by 100) $10x + 40(80-x) = 35(80)$

$10x + 3200 - 40x = 2800$

$3200 - 30x = 2800$

$-3200 \quad -3200$

$-30x = -400$

$x = \frac{-400}{-30} = \frac{40}{3}$ oz. or $13\frac{1}{3}$ oz.

You must mix $13\frac{1}{3}$ oz. of the 10% solution with $(80 - 13\frac{1}{3}) = 66\frac{2}{3}$ oz. of the 40% solution to get 80 oz. of a 35% solution.

10. First car: speed (rate) = 35 mi/hr.

time = 4 hrs. (noon to 4 PM)

Second car: speed = x

time = 3.5 hrs. (12:30 - 4:00)

	rate	× time	= distance	
1st car	35	4	$35(4)$	distances are same
	x	3.5	$3.5x$	

equation: $35(4) = 3.5x$

$140 = 3.5x$

$x = \frac{140}{3.5} =$ 40 mi/hr.

The second car would have to travel at 40 mi/hr in order to catch up to the first car at 4 PM.

11. Alex paints 1 room in 90 min., so his rate is $\frac{1}{90}$ room/min.

Brad paints 1 room in 60 min., so his rate is $\frac{1}{60}$ room/min.

They both work for t minutes.

	rate × time = rooms painted (rooms/min.) × (min.)		
Alex	$\frac{1}{90}$	t	$\frac{t}{90}$
Brad	$\frac{1}{60}$	t	$\frac{t}{60}$

1 ← together they paint 1 room.

equation: $\frac{t}{90} + \frac{t}{60} = 1$

LCD = 180 $\qquad 180\left(\frac{t}{90}\right) + 180\left(\frac{t}{60}\right) = 180(1)$

$2t + 3t = 180$

$5t = 180$

$t = 180/5 =$ **36 minutes to paint 1 room working together.**

12. $A = \frac{h}{2}(B+b)$ Solving for b can be done in two ways.

① Start by multiplying by $\frac{2}{h}$ on both sides.

$\frac{2}{h}(A) = \frac{2}{h}\left(\frac{h}{2}\right)(B+b)$

$\frac{2A}{h} = B+b$

$-B \quad -B$

$\frac{2A}{h} - B = b$ *

② Start by multiplying both sides by 2 to clear fraction, but then multiply out the right side of the equation.

$2 \cdot A = \frac{h}{2}(B+b) \cdot 2$

$2A = h(B+b)$

$2A = hB + hb$

$-hB \quad -hB$

$2A - hB = hb$

$\frac{2A-hB}{h} = \frac{hb}{h} \Rightarrow b = \frac{2A-hB}{h}$ *

* these answers are equivalent.

13. Solve for R: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

1. multiply both sides of equation by LCD = RR_1R_2

$$RR_1R_2\left(\frac{1}{R}\right) = RR_1R_2\left(\frac{1}{R_1}\right) + RR_1R_2\left(\frac{1}{R_2}\right)$$

$$R_1R_2 = RR_2 + RR_1$$

2. Factor R out of the terms containing R. (They are already both on the right side of the equation.

$$R_1R_2 = R(R_2 + R_1)$$

3. Divide both sides of equation by (R_1+R_2)

$$\frac{R_1R_2}{R_1+R_2} = \frac{R(R_2+R_1)}{(R_1+R_2)} \Rightarrow \boxed{R = \frac{R_1R_2}{R_1+R_2}}$$

14. speed of boat in still water = x mph.
upstream, the current slows the boat to $x-3$ mph.
downstream, the current speeds the boat up to $x+3$ mph.

	rate	time	distance
upstream	$x-3$	$\frac{20}{x-3}$	20
downstream	$x+3$	$\frac{30}{x+3}$	30

$t = \frac{d}{r}$
times are same.

equation: $\text{time}_{\text{upstream}} = \text{time}_{\text{downstream}}$

$$\frac{20}{x-3} = \frac{30}{x+3}$$

multiply each side by $(x-3)(x+3)$: $(x-3)(x+3)\left(\frac{20}{x-3}\right) = (x-3)(x+3)\left(\frac{30}{x+3}\right)$

$$20(x+3) = 30(x-3)$$

(continuation)

$20x + 60 = 30x - 90$
$-20x \quad -20x$

$60 = 10x - 90$
$+90 \quad +90$

$150 = 10x$

$x = 15$ mph in still water

Additional Resources

1. Go To http://www.kutasoftware.com/free.html
2. Under "**Equations**" click on:
    - One-step equations
    - Two-step equations
    - Multi-step equations
3. For practice with application problems, click on:
    - Distance-rate-time word problems
    - Mixture word problems
    - Work word problems
    - Literal Equations (solving formulas)
4. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets

For help you can contact the ***Math Assistance Area***.