DIY: *Functions: Basics and Graphs*

To review function basics and graphing functions, watch the following set of YouTube videos explaining what functions are, their domain and range, and some basic or "parent" functions. For more information about specific types of functions, about function inverses and composites of functions, see other DIYs in this series.

They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=52tpYl2tTqk Intoduction to functions
2. https://www.youtube.com/watch?v=ObEucyZX464 Domain and Range of a function from of graph
3. https://www.youtube.com/watch?v=w81y25anEOM Finding Domain and Range algebraically
4. https://www.youtube.com/watch?v=8VgmBe3ulb8 Even and Odd Functions, graphically
5. https://www.youtube.com/watch?v=bOsNS3FsCPs Even and Odd Functions algebraically
6. https://www.youtube.com/watch?v=GALfCd-2XRQ&list=PL4FB17E5C77DCCE69&index=2
   Characteristics of functions: intercepts, increasing/decreasing
   (Note: When the presenter describes the "roots, zeros, or solutions of a function", he is talking about the x-intercepts which are x-values for which f(x) = 0 and the graph touches/crosses the x-axis.)

(The following two videos summarize some of the material from the DIY Review "Graphs of Basic Functions; Transformations)

7. https://www.youtube.com/watch?v=IP70kcTtWkI graphs of nine basic or "parent" functions
8. https://www.youtube.com/watch?v=Ccq1aQDCfVE transformations of functions

(see next page for sample problems.)

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

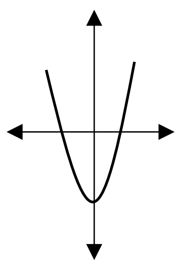
1. Determine if the following relations are functions.

   a) F={(1,0),(2,2),(8,10),(16,0)}

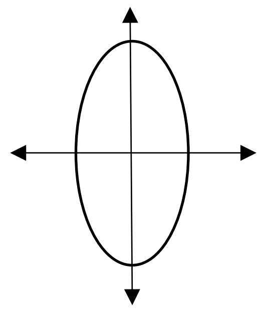
   b) G={(0,0),(1,2),(5,11),(1,0),(1,-1)}

   c) $f(x) = -\sqrt{x^2 - 25}$

   d)

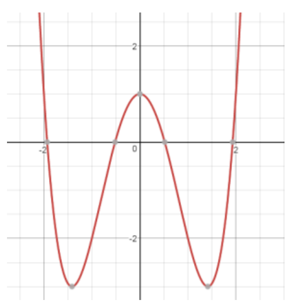


   e)

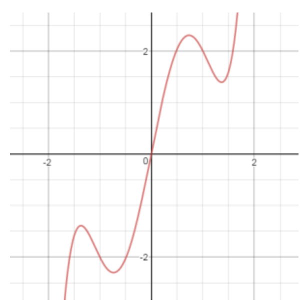


2. Classify the following functions as even, odd or neither:

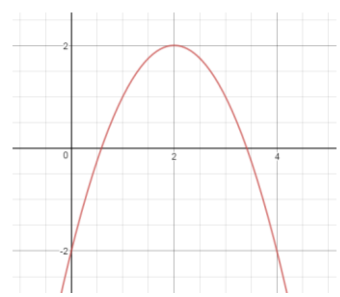
   a)



   b)



   c)



   d) $h(x) = \frac{1}{x}$

   e) $y = |x|$

   f) $y = x^3 - 3x + 1$

3. Find the domain and range of the following functions:

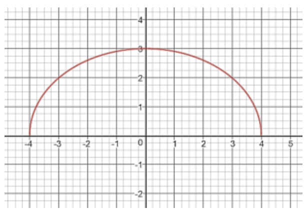
a) $y = \frac{-1}{3}x - \frac{7}{22}$ b) $f(x) = \sqrt{x-3} + 2$ c) $g(x) = \frac{x-1}{x+3}$

d) $h(x) = 4 - (x+1)^2$ e) $s(t) = 2^{t+1}$ f) $y = \ln(2x+5)$

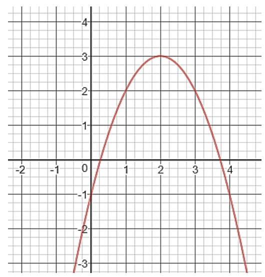
g) $R(x) = \{(-2,4), (0,4), (2,3)\}$

4. Find the domain and range of each of the functions graphed below.

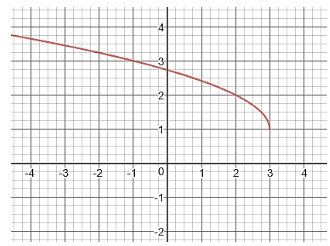
a.



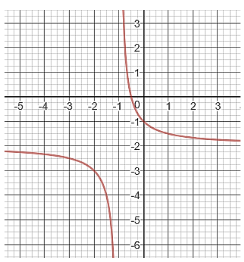
b.



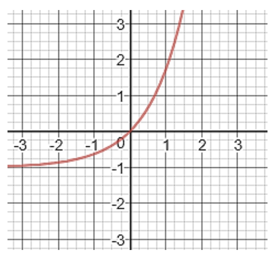
c.



d.



e.



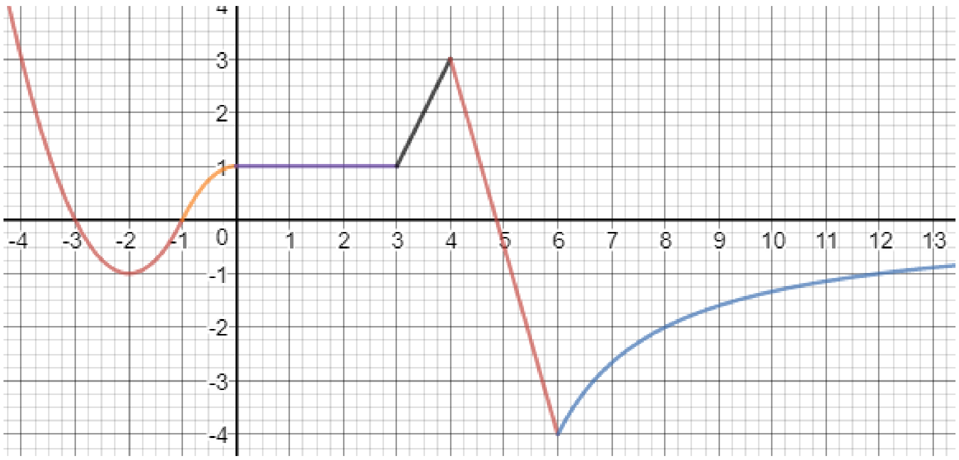
5. Given: $f(x) = \sqrt{x-3} + 2$ $g(x) = \frac{x-1}{x+3}$ $h(x) = \begin{cases} 2x+1 \; if \; x \le 2 \\ x^2 - 4 \; if \; x > 2 \end{cases}$

a. $f(4) =$ ____ b. $f(3) =$ ____ c. $f(2) =$ ____ d. $f(a) =$ ____

e. $g(0) =$ ____ f. $g(-3) =$ ____ g. $g(1) =$ ____ h. $g(x+1) =$ ____

i. $h(-1) =$ ____ j. $h(4) =$ ____ k. $h(2) =$ _____

6.



Answer the questions below about the graph of f(x) above.

a. Give all intervals on which f(x) is increasing.
b. Give all intervals on which f)x) is decreasing.
c. Give all intervals on which f(x) is constant.
d. Find the zeros of f(x) (another name for zeros is x-intercepts)

e. f(0) = ____ f. f(6) = ____ g. If f(a) = -2, then find all possible values of a.

Answers:

1.

a) Function b) Not a function c) Function
d) Function e) Not a function

2.

a) Even b) Odd c) Neither d) Odd e) Even f) Neither

3.

a) D : $(-\infty, \infty)$
R : $(-\infty, \infty)$

b) D : $[3, \infty)$
R : $[2, \infty)$

c) D : $(-\infty, -3) \cup (-3, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

d) D : $(-\infty, \infty)$
R : $(-\infty, 4]$

e) D : $(-\infty, \infty)$
R : $(0, \infty)$

f) D : $(-5/2, \infty]$
R : $(-\infty, \infty)$

g) D: $\{-2, 0, 2\}$
R: $\{3, 4\}$

4. a. D: $[-4, 4]$ R: $[0, 3]$

b. D: $(-\infty, \infty)$ R: $(-\infty, 3]$

c. D: $(-\infty, 3]$ R: $[1, \infty)$

d. D: $(-\infty, -1) \cup (-1, \infty)$ R: $(-\infty, -2) \cup (-2, \infty)$

e. D: $(-\infty, \infty)$ R: $(-1, \infty)$

5. a. 3 b. 2 c. not real d. $\sqrt{a-3}+2$ e. -1/3

f. undefined g. 0 h. $\frac{x}{x+4}$ i. -1 j. 12 k. 5

6. a. f(x) is increasing on $(-2,0)$, $(3, 4)$, and $(6, \infty)$

b. f(x) is decreasing on $(-\infty, -2)$ and $(4, 6)$

c. f(x) is constant on $(0, 3)$

d. -3, -1, ≈ 4.9

e. f(0) = 1

f. f(6) = -4

g. ≈ 5.4 or 8

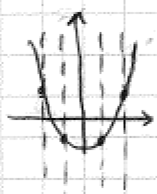
(Detailed solutions begin on next page)

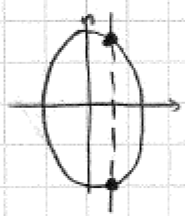
Detailed Solutions

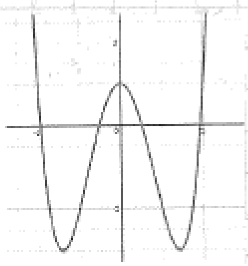
1. a. $F = \{(1,0), (2,2), (8,10), (16,0)\}$ <u>is</u> a function since all the x-values are different.

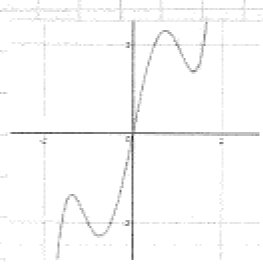
   b. $G = \{(0,0), (1,2), (5,11), (1,0), (1,-1)\}$ is <u>not</u> a function since the x-value $= 1$ is paired with <u>different</u> y-values.

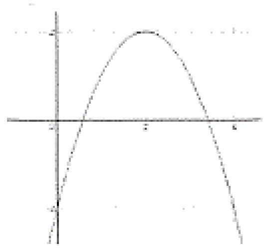
   c. $f(x) = -\sqrt{x^2 - 25}$ <u>is</u> a function since no x-value, substituted into this function, could give 2 different y-values.

   d. 
   <u>is</u> a function. It passes the vertical line test — there is no vertical line that can be drawn which would pass through this graph more than once.

   e. 
   is <u>not</u> a function. The vertical line drawn touches the graph in two places, which means for that x-value, there are two y-values.

2. a. 
   is an <u>even</u> function. The graph is symmetrical with respect to the y-axis, meaning that if the graph were folded along the y-axis, the two sides of the graph would coincide. If the point (a,b) is on the graph, so will $(-a,b)$.

   b. 
   is an <u>odd</u> function. It has symmetry with respect to the origin, meaning that if the graph were folded first along one axis, then again along the other axis, the two sides of the graph would coincide. If (a,b) is a point on the graph, then $(-a,-b)$ is also on the graph.

2.c. 

This function is <u>neither</u> odd nor even since it is not symmetric with respect to either the y-axis nor the origin.
(Note that the point (2,2) is on the graph, but neither (-2,2) nor (-2,-2) is on the graph).

d. $h(x) = \frac{1}{x}$ $\quad h(-x) = \frac{1}{-x} = -\frac{1}{x} = -h(x)$.

$h(x)$ is an $\boxed{\text{odd}}$ function

e. $y = |x|$
or $y(x) = |x|$ $\quad y(-x) = |-x| = |x| = y(x)$

since $y(-x) = y(x)$, this is an $\boxed{\text{even}}$ function.

f. $y(x) = x^3 - 3x + 1$ $\quad y(-x) = (-x)^3 - 3(-x) + 1$
$= -x^3 + 3x + 1 \neq y(x)$ (not even)

$-y(x) = -x^3 + 3x - 1$ so $y(-x) \neq -y(x)$
(not odd)

$y(x)$ is <u>neither</u> even nor odd.

3. a. $y = -\frac{1}{3}x - \frac{7}{22}$ This is a linear function (it's graph is a straight line).
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$.

b. $f(x) = \sqrt{x-3} + 2$ D: since we cannot take $\sqrt{\ }$ of a negative number, $x - 3 \geq 0$ $\quad x \geq 3$
$\boxed{D: \ [3, \infty)}$

R: $\sqrt{\text{anything}} \geq 0$ so $\sqrt{\ } + 2 \geq 2$
Range: $[2, \infty)$

3.c. $g(x) = \dfrac{x-1}{x+3}$ Here, the limitation on the domain stems from the concept that division by 0 is undefined.

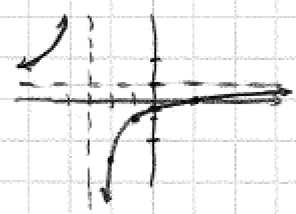
$x+3 \neq 0$

$x \neq -3$

D: $\boxed{(-\infty,-3) \cup (-3,\infty)}$

Range: (this will be explained in depth under "Graphing Rational Functions")

Graph:



$y \neq 1$ $\boxed{R: (-\infty,1) \cup (1,\infty)}$

if we let $g(x)=y$ and then solve the relation for x, we can see better why $y \neq 1$

$$y = \frac{x-1}{x+3} \rightarrow \begin{aligned} y(x+3) &= x-1 \\ xy+3y &= x-1 \\ xy-x &= -3y-1 \\ x(y-1) &= -3y-1 \\ x &= \frac{-3y-1}{y-1} \end{aligned}$$

If $y=1$, we would be dividing by zero.

d. $h(x) = 4-(x+1)^2$ D: all real numbers (no limitation on x-values)

$\boxed{(-\infty,\infty)}$

h(x): 

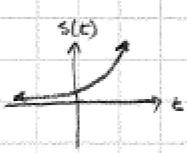
Range: since $(\ )^2 \geq 0$ then $-(\ )^2 \leq 0$ and $-(\ )^2+4 \leq 4$

so $-(x+1)^2+4 \leq 4$

$\boxed{R: (-\infty,4]}$

e. $s(t) = 2^{t+1}$ Domain: t can be any real number

$\boxed{D: (-\infty,\infty)}$



Range: any positive number raised to any power will still be positive. R: $\boxed{(0,\infty)}$

Ex. $t=5$ $s(5) = 2^{5+1} = 2^6 = 64$

$t=-5$ $s(-5) = 2^{-5+1} = 2^{-4} = \frac{1}{16}$

3.f. $y = \ln(2x+5)$

Domain: We can only take logs of positive numbers, so $2x+5>0$

$2x>-5$

$x>-5/2$

$\boxed{D: (-5/2, \infty)}$

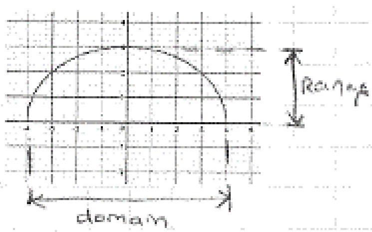
Range: all real numbers. $\boxed{(-\infty, \infty)}$

g. $R(x) = \{(-2,4), (0,4), (2,3)\}$

D: set of all x-coordinates $\boxed{\{-2,0,2\}}$

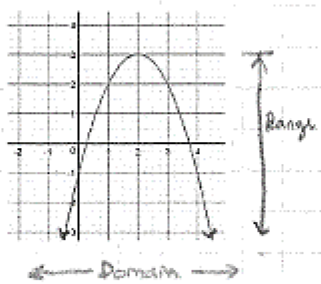
R: set of all y-coordinates $\boxed{\{3,4\}}$

4. a.



D: $[-4,4]$ R: $[0,3]$

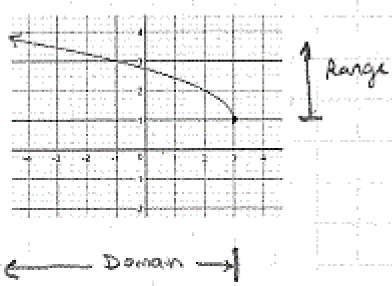
b.



D: $(-\infty, \infty)$

R: $(-\infty, 3]$

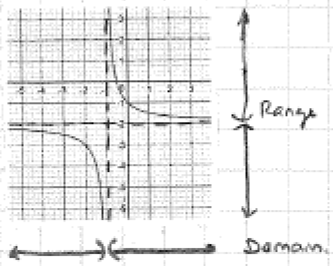
c.



D: $(-\infty, 3]$

R: $[1, \infty)$

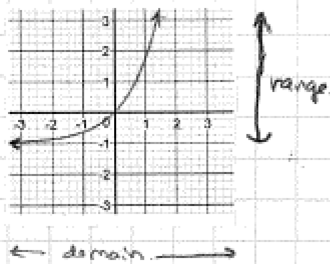
4\. d.



Domain: $(-\infty,-1)\cup(-1,\infty)$ $\quad(x\neq-1)$

Range: $(-\infty,-2)\cup(-2,\infty)$ $\quad(y\neq-2)$

e.



Domain: $(-\infty,\infty)$

Range: $(-1,\infty)$

5\. $f(x)=\sqrt{x-3}+2$ $\qquad g(x)=\dfrac{x-1}{x+3}$ $\qquad h(x)=\begin{cases}2x+1 & \text{if } x\leq 2\\ x^2-4 & \text{if } x>2\end{cases}$

a. $f(4)=\sqrt{4-3}+2$
$=\sqrt{1}+2$
$=1+2=3$

b. $f(3)=\sqrt{3-3}+2$
$=\sqrt{0}+2$
$=0+2=2$

c. $f(2)=\sqrt{2-3}+2$
$=\sqrt{-1}+2$
$\boxed{\text{(not real)}}$

d. $f(a)=\boxed{\sqrt{a-3}+2}$

e. $g(0)=\dfrac{0-1}{0+3}$
$=\boxed{\dfrac{-1}{3}}$

f. $g(-3)=\dfrac{-3-1}{-3+3}$
$=\dfrac{-4}{0}=\boxed{\text{undefined}}$

g. $g(1)=\dfrac{1-1}{1+3}=\dfrac{0}{4}=\boxed{0}$

h. $g(x+1)=\dfrac{(x+1)-1}{(x+1)+3}$
$=\dfrac{x+1-1}{x+1+3}$
$=\boxed{\dfrac{x}{x+4}}$

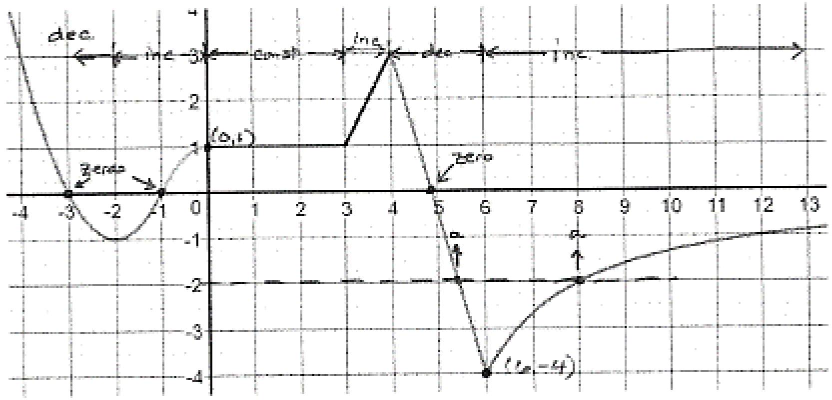
i. $h(-1)=2(-1)+1$
$=-2+1$
$=\boxed{-1}$

since $x=-1$ falls in the $x\leq 2$ interval, use $h(x)=2x+1$.

j. $\underset{x>2}{h(4)}=4^2-4=16-4$
$=\boxed{12}$

k. $\underset{x\leq 2}{h(2)}=2(2)+1$
$=5$

6.



a. f(x) is increasing (↗) on $\boxed{(-2,0),\ (3,4),\ \text{and } (6,\infty)}$

b. f(x) is decreasing (↘) on $\boxed{(-\infty,-2) \text{ and } (4,6)}$

c. f(x) is constant (⟷) on $\boxed{(0,3)}$

Note: these intervals are the x-values for which the y-values are inc./dec./const as one goes from left to right.

d. Zeros are x-values for which $y=0$ (where the graph crosses the x-axis)

$$\text{zeros} = \{-3, -1, \approx 4.9\}$$

estimate.

e. $f(0)=1$ (the point $(0,1)$)

f. $f(6)=-4$ point $(6,-4)$

g. If $f(a)=-2$ then a could be either $\boxed{\approx 5.4 \text{ or } 8}$

Additional Resources

Click on the links below to download worksheets for additional practice

1. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets.
   a. http://cdn.kutasoftware.com/Worksheets/Alg2/Evaluating%20Functions.pdf evaluating functions
   b. https://www.dublin.k12.ca.us/cms/lib/CA01001424/Centricity/Domain/172/Function%20worksheet.pdf Domain and Range

2. For help please contact the ***Math Assistance Area***.