DIY: *Conic Sections*-*Parabolas, Ellipses, Circles and Hyperbolas*

To review the Conic Sections, Identify them and sketch them from the given equations, watch the following set of YouTube videos. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. Introduction to Conic Sections
2. Parabolas-part1 (Note: this presenter uses "a" for the focal distance. In our solutions we will use "p", since this is the letter used in the COD textbook.
3. Parbolas-part2
4. Ellipse-part1
5. Ellipse-part2 :- The instructor has the questions which say "find the parabola" but he means ellipse
6. Hyperbola- part1
7. Hyperbola-part2 :- The instructor has the questions which say "find the parabola" but he means Hyperbola
8. Circle
9. Determining What Type of Conic Section from General Form
10. Eccentricity of a conic section

Summary of Formulas: see http://www.ttdk.com/dk/conic%20sections%20formulas.pdf

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Determine what type of conic the following represent. Graph it. Find center, focus/foci, vertex/vertices, equations of asymptotes, and directrix for each, where applicable:
   - a) $4x^2 + 3y^2 = 12$
   - b) $2x^2 + 9y^2 - 8x + 54y + 71 = 0$
   - c) $y^2 = 4x$
   - d) $x^2 = 16y$
   - e) $3x^2 + 4y^2 + 6x - 16y + 7 = 0$
   - f) $4x^2 - y^2 + 2y = 5$
   - g) $6x^2 - 7y^2 - 36x + 14y = -89$
   - h) $x^2 - 2x - 8y + 49 = 0$
   - i) $x^2 + y^2 - 20x + 2y + 100 = 0$
   - j) $y^2 - 12x - 4y + 28 = 0$

2. Graph each of the following relations. Each represents half of a conic section. Name the type of conic. Also mention their focus/foci, directrix, center, vertex/vertices, axis of symmetry (where applicable), domain and range:
   - a) $4x + 24 = \sqrt{(y-9)}$
   - b) $\frac{x}{2} = \sqrt{1 - \frac{(y-5)^2}{16}}$
   - c) $\frac{y}{3} = \sqrt{\frac{(x+1)^2}{4} - 1}$
   - d) $y = \sqrt{(x+4)} + 2$
   - e) $2x = -\sqrt{8y + 28 - 4y^2}$
   - f) $2y = -\sqrt{36 - 4x^2}$

3. Write the equation for each of the following:
   a) Parabola with vertex at (0,2) and directrix as y = -2
   b) Ellipse with center at (2,3) and vertices on the major axis at (-3,3) and (7,3) and minor axis of length 2
   c) Hyperbola with vertices at (0,2) and (2,2) and foci at (-1,2) and (3,2)
   d) Circle with center at(0,0) and radius 3
   e) Parabola with focus at (0,-1) and directrix as x = - 4
   f) Hyperbola with vertical transverse axis with center (0,0) and the asymptotes are $y = \pm\frac{2}{3}x$
   g) Ellipse with foci (3,2) and (3,-4) major axis of length 8.
   h) Circle with center at (1,2) and point (1,6) lies on the circle

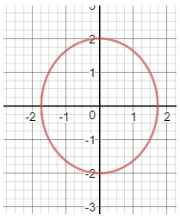
4. An arch of a bridge has the shape of the top half of an ellipse. The arch is 60ft wide and 11ft high at the center. Find the equation of the ellipse. Find the height of the arch 10ft from the center of the bottom.

5. A cannon shell follows a parabolic path. It reaches a max height of 100ft and land at a distance of 50ft from the cannon. Write the equation of the parabolic path the shell follows. (Note: your answer will depend on where you locate your coordinate axes. Many correct answers are possible.)

Answers:

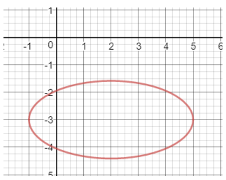
1.

a)



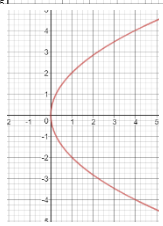
Ellipse with vertical major axis
Center: (0,0)
Foci: (0,1), (0, -1)
Vertices: (0,2) , (0 , -2)
Endpoints of Minor Axis: ($\sqrt{3}$,0), (-$\sqrt{3}$,0)

b)



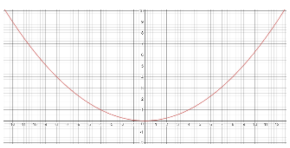
Ellipse with horizontal major axis
Center: (2, -3)
Foci: (2-$\sqrt{7}$, -3), (2+$\sqrt{7}$, -3)
Vertices: (-1, -3) , (5, -3)
Endpoints of Minor Axis: (2, -3+$\sqrt{2}$), (2, -3-$\sqrt{2}$)

c)



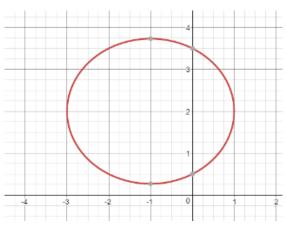
Parabola with horizontal axis
Vertex: (0, 0)
Focus: (1 ,0)
Directrix: x= -1

d)



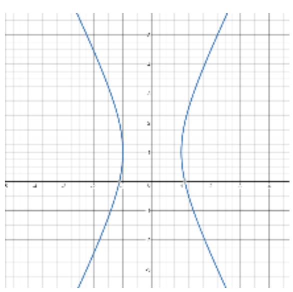
Parabola with vertical axis
Vertex: (0, 0)
Focus: (0,4)
Directrix: y= -4

e)



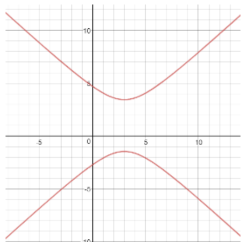
Ellipse with horizontal major axis
Center: (-1,2)
Foci: (0,2), (-2,2)
Vertices: (0,2) , (0 , -2)
Endpoints of Minor Axis: ($\sqrt{3}$,0), (-$\sqrt{3}$,0)

f)



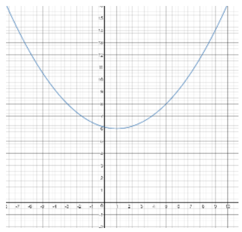
Hyperbola with horizontal transverse axis
Center: (0,1)
Foci: (-$\sqrt{5}$,1), ($\sqrt{5}$,1)
Vertices: (-1,1), (1,1)
Asymptotes: y=$\pm$2x +1

g)



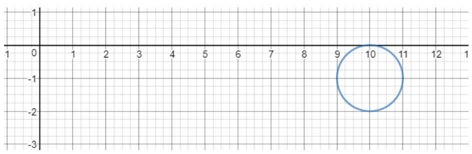
Hyperbola with vertical transverse axis
Center: (3,1)
Foci: $(3, 1+\sqrt{13})$, $(3, 1-\sqrt{13})$
Vertices: $(3, 1-\sqrt{7})$, $(3, 1+\sqrt{7})$
Asymptotes: $y=\pm\sqrt{\frac{6}{7}}(x-3)+1$

h)



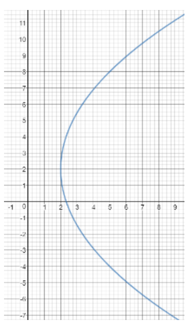
Parabola with vertical axis
Vertex: (1,6)
Focus: (1,8)
Directrix: y = 4

i)



Circle
Center: (10, -1)
Radius: 1

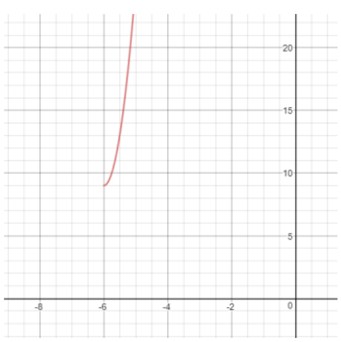
j)



Parabola with horizontal axis
Vertex: (2,2)
Focus: (5,2)
Directrix: x = -1

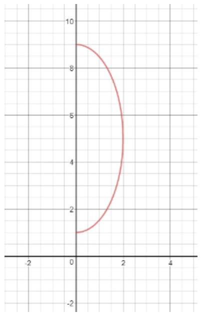
2.

a)



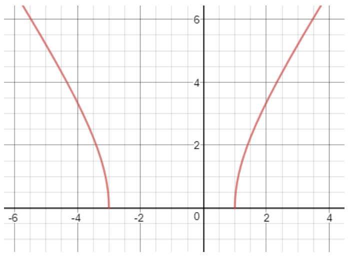
Right half of a Parabola with vertical axis
Vertex: (-6,9)
Focus: (-6,13)
Directrix: y=5
Axis of symmetry: x=-6
Domain: $[-6, \infty)$
Range: $[9,\infty)$

b)



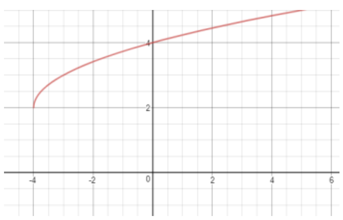
Right half of an Ellipse with vertical major axis
Center: (0, 5)
Foci: $(0,5+2\sqrt{3})$; $(0,5-2\sqrt{3})$
Vertices: (0, 1), (0, 9)
Endpoint of Minor Axis: (2, 5)
Domain: [0,2]
Range: [1, 9]

c)



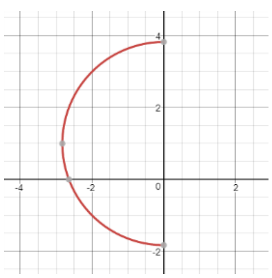
Upper half of a Hyperbola with horizontal transverse axis
Center: $(-1,0)$
Foci : $(-1+\sqrt{13},0)$, $(-1-\sqrt{13},0)$
Vertices : (1,0) , $(-3,0)$
Asymptotes: $y=\pm\frac{3}{2}\ (x+1)$
Domain: $(-\infty,-3]\ \cup[1,\infty)$
Range: $[0,\infty)$

d)



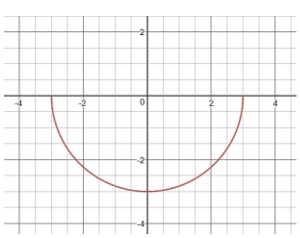
Upper half of a Parabola with horizontal axis
Vertex: $(-4,2)$ Axis: y = 2
Focus: $(-\frac{15}{4},2)$
Directrix: $x=-\frac{17}{4}$
Domain: $[-4,\infty)$
Range : $[2,\infty)$

e)



Left half of a Circle
Center: (0,1)
Radius: $2\sqrt{2}$
Domain: $[-2\sqrt{2},0]$
Range : $[1-2\sqrt{2},1+2\sqrt{2}]$

f)



Lower half of a Circle
Center: (0,0)
Radius: 3
Domain: [-3, 3]
Range: [-3, 0]

3.

a) $x^2 = 16(y-2)$

b) $\frac{(x-2)^2}{25} + (y-3)^2 = 1$

c) $(x-1)^2 - \frac{(y-2)^2}{3} = 1$

d) $x^2 + y^2 = 9$

e) $(y+1)^2 = 8\,(x+2)$

f) $\frac{y^2}{4} - \frac{x^2}{9} = 1$

g) $\frac{(x-3)^2}{7} + \frac{(y+1)^2}{16} = 1$

h) $(x-1)^2 + (y-2)^2 = 16$

4. Equation: $\frac{x^2}{900} + \frac{y^2}{121} = 1$ Upper half only would be $y = \frac{11}{30}\sqrt{900 - x^2}$

height at $x = 10$ is $\frac{22\sqrt{2}}{3} ft \approx 10.37 ft$

5. $x^2 = -\frac{25}{4}(y - 100)$

(Detailed solutions begin on next page.)

Detailed Solutions

1. a. $4x^2+3y^2=12$ The x^2 and y^2 terms have the same sign but different values, so this is an ellipse

Putting the equation in standard form: $\frac{4x^2}{12}+\frac{3y^2}{12}=\frac{12}{12}$

$\boxed{\frac{x^2}{3}+\frac{y^2}{4}=1}$ because the number under the y^2 is larger, the ellipse has a vertical major axis

$b^2=3$, $b=\sqrt{3}$ $\qquad a^2=4$, $a=2$

To find the distance from the center to a focus, c, use $a^2-b^2=c^2$

$4-3=1$

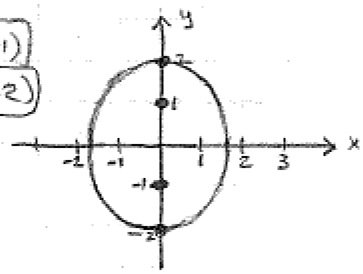
$c=1$

Center: $(0,0)$

Foci: $(0,0\pm c)=(0,0\pm1)=\boxed{(0,1),(0,-1)}$

Vertices: $(0,0\pm a)=(0,\pm2)=\boxed{(0,2),(0,-2)}$

Endpoints of minor axis $=(0\pm b,0)=\boxed{(\sqrt{3},0),(-\sqrt{3},0)}$



b. $2x^2+9y^2-8x+54y+71=0$ x^2,y^2 terms are same sign but different values, so ... ellipse

Using completing the square, put equation into standard form: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

(a^2 is the larger of a^2 and b^2)

$2x^2-8x \quad +9y^2+54y \quad =-71$

$2(x^2-4x+\quad)+9(y^2+6y+\quad)=-71+\_\_+\_\_$

$\frac{1}{2}(-4)=-2 \qquad \frac{1}{2}(6)=3$

$(-2)^2=4 \qquad 3^2=9$

$2(x^2-4x+4)+9(y^2+6y+9)=-71+2(4)+9(9)$

$\frac{2(x-2)^2}{18}+\frac{9(y+3)^2}{18}=\frac{-71+8+81=18}{18} \Rightarrow \boxed{\frac{(x-2)^2}{9}+\frac{(y+3)^2}{2}=1}$

$a^2=9$, $a=3$ $\qquad b^2=2$, $b=\sqrt{2}$

$a^2-b^2=c^2$

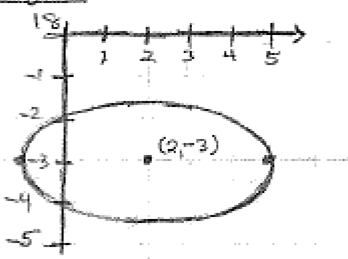
$9-2=7 \rightarrow c=\sqrt{7}$

Center: $(2,-3)$

Vertices: $(2\pm3,-3)$

Foci: $(2\pm\sqrt{7},-3)$

endpts. of minor axis: $(2,-3\pm\sqrt{2})$



1.c. $y^2 = 4x$ parabola with horizontal axis

$4 = 4p \quad p = 1$

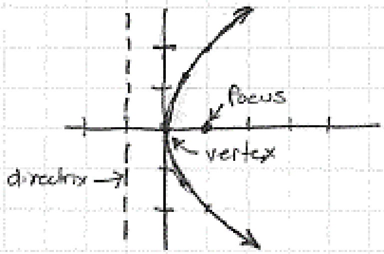
vertex: $(0,0)$

focus: p units to right of vertex $= (1,0)$

axis: $y = 0$

directrix: p units to left of vertex and perpendicular to axis.

$x = -1$



d. $x^2 = 16y$ parabola with vertical axis.

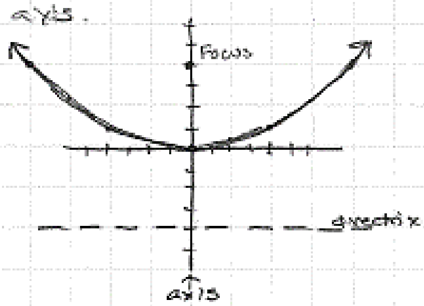
$4p = 16$

$p = 4$

vertex: $(0,0)$

focus: $(0,4)$

directrix: $(0,-4)$



e. $3x^2 + 4y^2 + 6x - 16y + 7 = 0$ ← ellipse

(x^2, y^2 same sign, different coefficients)

$3x^2 + 6x + 4y^2 - 16y = -7$

$3(x^2 + 2x \quad) + 4(y^2 - 4y \quad) = -7$

$\frac{1}{2}(2) = 1 \quad 1^2 = 1 \qquad \frac{1}{2}(-4) = -2 \quad (-2)^2 = 4$

$3(x^2 + 2x + 1) + 4(y^2 - 4y + 4) = -7 + 3(1) + (4)(4)$

$3(x+1)^2 + 4(y-2)^2 = 12$

$\frac{3(x+1)^2}{12} + \frac{4(y-2)^2}{12} = \frac{12}{12}$

$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{3} = 1$

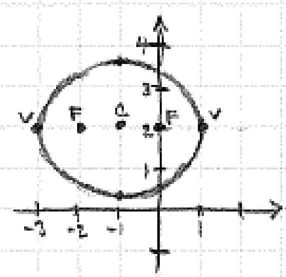
$a^2 = 4$, $a = 2$; $b^2 = 3$, $b = \sqrt{3}$

(horizontal major axis)

$c^2 = a^2 - b^2$

$= 4 - 3 = 1$

$c = 1$



center: $(-1, 2)$

vertices $(-1 \pm 2, 2) = (-3, 2), (1, 2)$

endpoints of minor axis: $(-1, 2 \pm \sqrt{3})$

Foci: $(-1 \pm 1, 2) = (-2, 2), (0, 2)$

1.f. $4x^2 - y^2 + 2y = 5$ Hyperbola (x^2, y^2 terms have opposite signs)

$4x^2 - (y^2 - 2y) = 5$ $\frac{1}{2}(-2) = -1$ $(-1)^2 = 1$

$4x^2 - (y^2 - 2y + 1) = 5 - 1$

$\frac{4x^2}{4} - \frac{(y-1)^2}{4} = \frac{4}{4} \Rightarrow \frac{x^2}{1} - \frac{(y-1)^2}{4} = 1$ x^2-term is (+), so horiz transverse axis

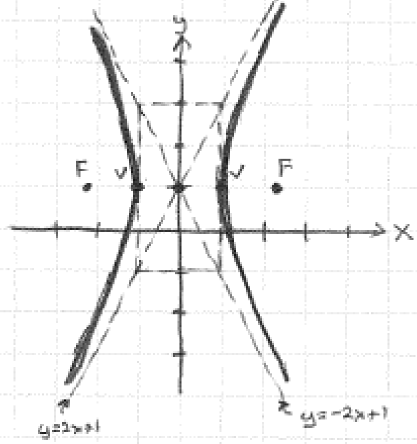
center: $(0,1)$

a^2 = number under positive term

$a^2 = 1$ $a = 1$

$b^2 = 4$ $b = 2$

$a^2 + b^2 = c^2$ $1 + 4 = c^2$ $c = \sqrt{5}$



asymptotes: slope $= \pm \frac{b}{a} = \pm 2$

pass through center $(0,1)$

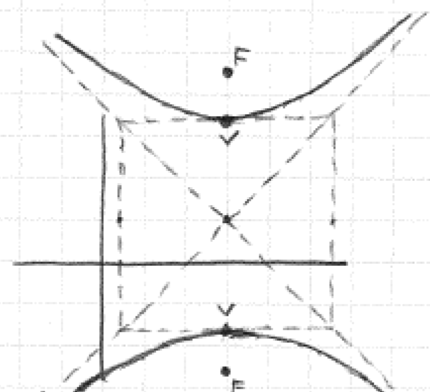
$y - 1 = \pm 2(x - 0)$

$y = \pm 2x + 1$ $\begin{cases} y = 2x + 1 \\ y = -2x + 1 \end{cases}$

vertices: $(0 \pm 1, 1) = (-1,1), (1,1)$

Foci: $(0 \pm \sqrt{5}, 1) \approx (\pm 2.2, 1)$

g. $6x^2 - 7y^2 - 36x + 14y = -89$ (hyperbola)



$6x^2 - 36x - 7y^2 + 14y = -89$

$6(x^2 - 6x) - 7(y^2 - 2y) = -89$

$6(x^2 - 6x + 9) - 7(y^2 - 2y + 1) = -89 + 54 - 7$

$\frac{6(x-3)^2}{-42} - \frac{7(y-1)^2}{-42} = \frac{-42}{-42}$

$-\frac{(x-3)^2}{7} + \frac{(y-1)^2}{6} = 1$ $\left(\frac{(y-1)^2}{6} - \frac{(x-3)^2}{7} = 1\right)$

(vertical transverse axis)

center: $(3,1)$

$a^2 = 6$ $a = \sqrt{6} \approx 2.45$

$b^2 = 7$ $b = \sqrt{7} \approx 2.6$

$c^2 = a^2 + b^2 = 13$ $c = \sqrt{13} \approx 3.6$

vertices: $(3, 1 \pm \sqrt{6})$

Foci: $(3, 1 \pm \sqrt{13})$

asymptotes: $m = \pm \frac{a}{b} = \pm \frac{\sqrt{6}}{\sqrt{7}} \approx .9$

passes through center $(3,1)$

$y - 1 = \pm \frac{\sqrt{42}}{7}(x - 3)$

$y = \pm \frac{\sqrt{42}}{7}(x-3) + 1$

1.h. $x^2-2x-8y+49=0$ parabola (only one variable has a squared term.)

$x^2-2x = 8y-49$ no y^2 term → vertical axis.

$x^2-2x+1 = 8y-48$

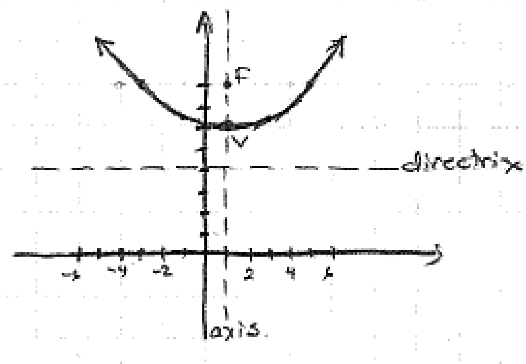
$(x-1)^2 = \underset{\nwarrow 4p}{8}(y-6).$

Vertex: $(1,6)$

focal distance $p = 8/4 = 2$

Focus: $(1, 6+2) = (1,8)$

directrix: $y = 6-2 \Rightarrow y=4$



i. $x^2+y^2-20x+2y+100=0$ circle: coefficients of x^2, y^2 are equal.

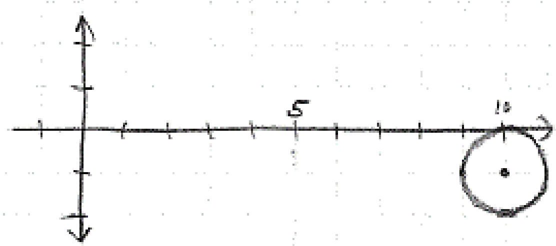
$x^2-20x+y^2+2y = -100$

$x^2-20x+100+y^2+2y+1 = -100+100+1$

$(x-10)^2+(y+1)^2 = 1$

center: $(10,-1)$

radius: 1



j. $y^2-12x-4y+28=0$ parabola with horizontal axis

$y^2-4y = 12x-28$

$y^2-4y+4 = 12x-28+4$

$(y-2)^2 = 12x-24$

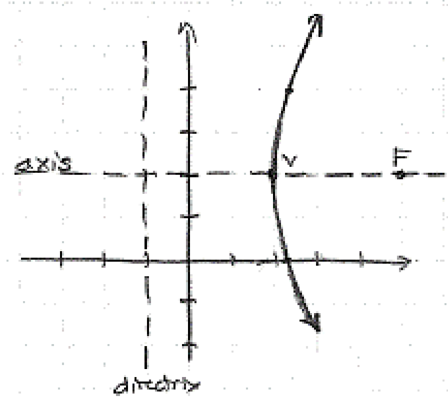
$(y-2)^2 = \underset{\nwarrow 4p}{12}(x-2)$

vertex: $(2,2)$

focal distance $p = \frac{12}{4} = 3$

focus: $(2+3, 2) = (5,2)$

directrix: $x = 2-3$ $\boxed{x=-1}$



2\. a. $4x+24=\sqrt{y-9}$ note: $4x+24 \geq 0$ (since $\sqrt{\text{anything}} \geq 0$)

$4x \geq -24$

$x \geq -6 \rightarrow$ right half of conic.

squaring both sides:

$$[4(x+6)]^2 = \left(\sqrt{y-9}\right)^2$$

$$16(x+6)^2 = y-9$$

$$(x+6)^2 = \frac{1}{16}(y-9)$$

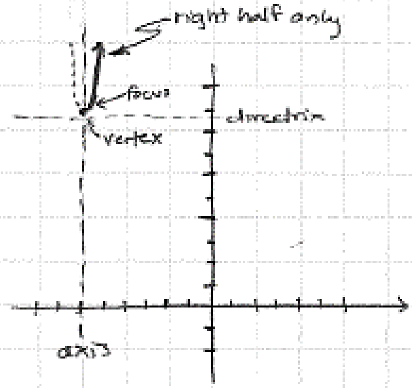
parabola w/ vertical axis

$4p = \frac{1}{16}$ $p = \frac{1}{64}$ = focal dist. (opens upward)

vertex: $(-6, 9)$

focus: $(-6, 9+\frac{1}{64})$

directrix: $y = 9 - \frac{1}{64}$



Domain: $[-6, \infty)$

Range: $[9, \infty)$

b. $\frac{x}{2} = \sqrt{1-\frac{(y-5)^2}{16}}$ note: $\frac{x}{2} \geq 0$ $x \geq 0 \rightarrow$ right side of conic

$$\frac{x^2}{4} = 1 - \frac{(y-5)^2}{16}$$

$$\frac{x^2}{4} + \frac{(y-5)^2}{16} = 1$$

$b^2=4$ $a^2=16$

$b=2$ $a=4$

$c^2=a^2-b^2=16-4=12$

$c=\sqrt{12}=2\sqrt{3}\approx 3.4$

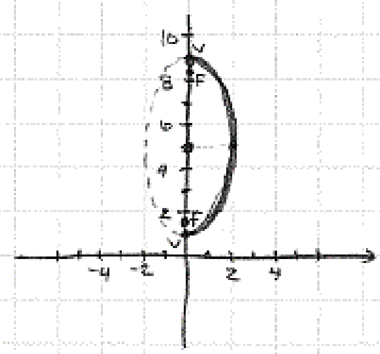
ellipse. center $(0,5)$

vertices: $(0, 5\pm 4)$

$(0,1), (0,9)$

foci: $(0, 5\pm 2\sqrt{3})$

$\approx (0, 1.6), (0, 8.4)$



endpoint of minor axis: $(2,5)$

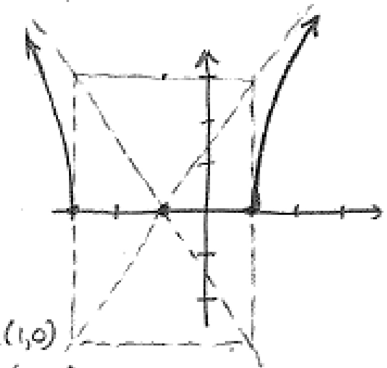
Domain: $[0, \infty)$

Range: $[-1, 9]$

2.c. $\frac{y}{3} = \sqrt{\frac{(x+1)^2}{4} - 1}$ note: $\frac{y}{3} \geq 0$ $y \geq 0$ (upper half of conic)

$\frac{y^2}{9} = \frac{(x+1)^2}{4} - 1$

$\frac{(x+1)^2}{4} - \frac{y^2}{9} = 1$ hyperbola w/ horiz. transerve axis



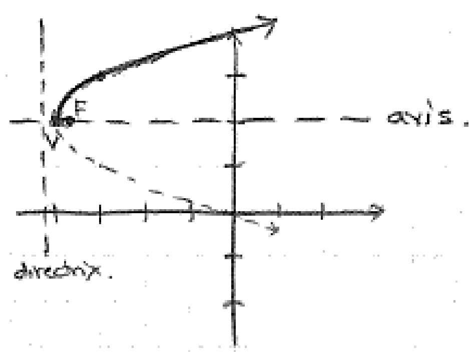
center: $(-1, 0)$
$a^2 = 4$ $a = 2$
$b^2 = 9$ $b = 3$
$c^2 = a^2 + b^2 = 4 + 9 = 13$
$c = \sqrt{13} \approx 3.6$

vertices: $(-1 \pm 2, 0) = (-3,0), (1,0)$
foci: $(-1 \pm \sqrt{13}, 0) \approx (-4.6,0), (2.6,0)$

Domain: $(-\infty, -3] \cup [1, \infty)$
Range: $[0, \infty)$

d. $y = \sqrt{x+4} + 2$ note: $y \geq 0 + 2$ $y \geq 2$ (upper half of conic)
$y - 2 = \sqrt{x+4}$
$(y-2)^2 = x + 4$ (parabola with horiz. axis)

vertex: $(-4, 2)$
focus: $4p = 1$ $p = \frac{1}{4}$
$(-4 + \frac{1}{4}, 2) = (-3.75, 2)$
directrix: $x = -4.25$
axis: $y = 2$



note: this could have been graphed by using the basic function $y = \sqrt{x}$, translated 4 units left, 2 units up)

Domain: $[-4, \infty)$
Range: $[2, \infty)$

2e. $2x = -\sqrt{8y+28-4y^2}$ note: $x \le 0$ (left side of conic)

$4x^2 = 8y - 4y^2 + 28$

$4x^2 + 4y^2 - 8y = 28$ (circle)

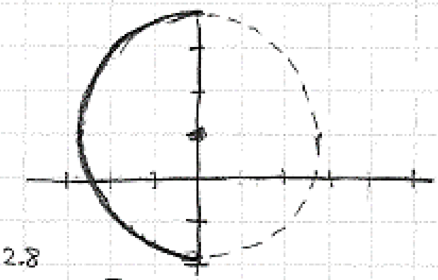
$4x^2 + 4(y^2 - 2y + 1) = 28 + 4$

$\frac{4x^2}{4} + \frac{4(y-1)^2}{4} = \frac{32}{4}$

$x^2 + (y-1)^2 = 8$

Center: $(0,1)$

radius $= \sqrt{8} = 2\sqrt{2} \approx 2.8$



Domain: $(-2\sqrt{2}, 0]$

Range: $[1-2\sqrt{2}, 1+2\sqrt{2}] \approx [-1.8, 3.8]$

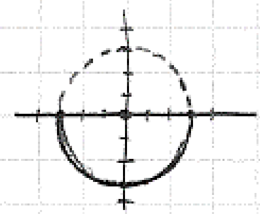
f. $2y = -\sqrt{36-4x^2}$ note: $y \le 0$ (lower half of conic)

$4y^2 = 36 - 4x^2$

$4x^2 + 4y^2 = 36$

$x^2 + y^2 = 9$

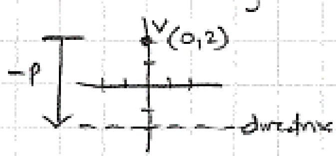
circle with center $(0,0)$ radius $= 3$



Domain: $[-3, 3]$

Range: $[-3, 0]$

3. a. Parabola with vertex $(0,2)$ and directrix $y = -2$



$p = 4$

vertical axis

focus: $(0, 2+4) = (0, 6)$

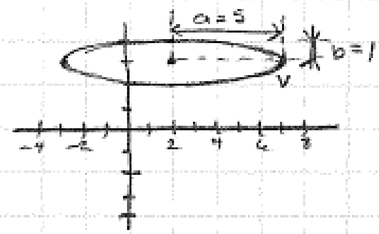
model equation: $(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4(4)(y-2)$

$\boxed{x^2 = 16(y-2)}$

3.b. Ellipse with center (2,3) and vertices (-3,3), (7,3) and length of minor axis = 2. (b=1)

Major axis horizontal, a = 5



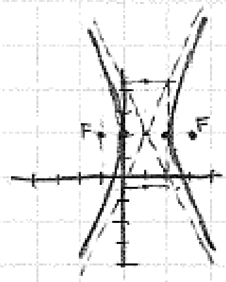
model equation: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

$$\boxed{\dfrac{(x-2)^2}{25} + \dfrac{(y-3)^2}{1} = 1}$$

c. hyperbola with vertices (0,2), (2,2)

foci (-1,2), (3,2) ↑

(horizontal transverse axis) center (1,2)



$a = 1 \qquad c = 2$

$a^2 + b^2 = c^2$

$1 + b^2 = 4$

$b^2 = 3 \qquad b = \sqrt{3} \approx 1.7$

model equation: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

$(h,k) = (1,2)$

$a^2 = 1 \quad b^2 = 3$

$$\boxed{\dfrac{(x-1)^2}{1} - \dfrac{(y-2)^2}{3} = 1}$$

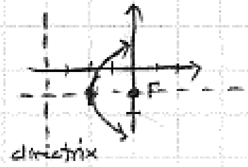
d. circle with center (0,0) and radius = 3

model equation: $(x-h)^2 + (y-k)^2 = r^2$

$(h,k) = (0,0) \quad r = 3 \implies \boxed{x^2 + y^2 = 9}$

e. parabola with focus (0,-1) and directrix x = -4

vertex: (-2,-1) horizontal axis y = -1



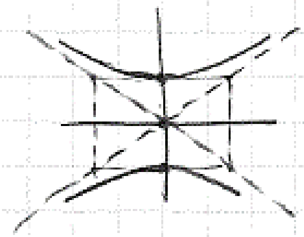
model equation:

$(y-k)^2 = 4p(x-h)$

$(h,k) = (-2,-1) \qquad p = 2$

$(y+1)^2 = 4(2)(x+2) \qquad \boxed{(y+1)^2 = 8(x+2)}$

3.f. Hyperbola with vertical transverse axis with center (0,0) and asymptotes $y = \pm \frac{2}{3}x$

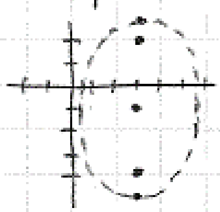


asymptotes: $m = \pm \frac{a}{b}$ $a=2$, $b=3$

model equation: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$$\boxed{\frac{y^2}{4} - \frac{x^2}{9} = 1}$$

g. Ellipse with foci (3,2) and (3,-4) and major axis of length 8



center is midpoint between foci (3,-1)

$a = \frac{1}{2}$ length of major axis $= 4$ (vertical major axis)

vertices $(3, -1 \pm 4) = (3,3), (3,-5)$

$c = \frac{1}{2}$ distance between foci $= 3$

$a^2 - b^2 = c^2$

$16 - b^2 = 9$

model eqn: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ $b^2 = 7$ $b = \sqrt{7} \approx 2.6$

$$\boxed{\frac{(x-3)^2}{7} + \frac{(y+1)^2}{16} = 1}$$

h. circle with center (1,2) and point (1,6) lies on circle

model eqn: $(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-2)^2 = r^2$

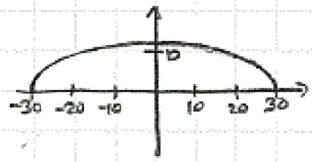
substitute (1,6) for (x,y) to find r

$(1-1)^2 + (6-2)^2 = r^2$

$0^2 + 4^2 = r^2$

$r = 4 \longrightarrow \boxed{(x-1)^2 + (y-2)^2 = 16}$

4. Placing the center of the elliptical arch at (0,0)



vertices: $(\pm 30, 0) \rightarrow a = 30$

endpt. of minor axis $(0, 11) \rightarrow b = 11$

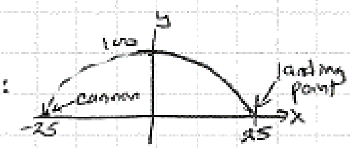
$$\frac{x^2}{30^2} + \frac{y^2}{11^2} = 1 \Rightarrow \boxed{\frac{x^2}{900} + \frac{y^2}{121} = 1}$$

Question: Find y when $x = 10$: $\frac{10^2}{30^2} + \frac{y^2}{11^2} = 1$

$$(900)(121)\left(\frac{100}{900} + \frac{y^2}{121}\right) = 1\,(900)(121)$$

$$121(100) + 900y^2 = 900(121)$$
$$900y^2 = 900(121) - 100(121)$$
$$= 800(121)$$
$$y^2 = \frac{800(121)}{900}$$
$$y = \sqrt{\frac{800(121)}{900}} = \frac{\cancel{10}^{1}(11)}{\cancel{30}_3}\sqrt{8} = \frac{11}{3} \cdot 2\sqrt{2}$$
$$y = \frac{22\sqrt{2}}{3} \approx \boxed{10.37 \text{ ft.}}$$

5. The coordinate axes are placed as shown:
so (0,100) is the vertex.
and (50,0) is the landing point.

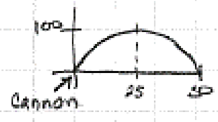


model eqn:
$$(x-h)^2 = 4p(y-k)$$
$$x^2 = 4p(y-100)$$

passes through $(50,0) \rightarrow$ $25^2 = 4p(0-100)$
$$625 = -400p$$
$$p = -\frac{625}{400} \qquad 4p = -\frac{625}{100} = -\frac{25}{4}$$

$$\boxed{x^2 = -\frac{25}{4}(y-100)} \quad \text{or} \quad y = -\frac{4}{25}x^2 + 100$$

Note: if axes are set up so vertex (25,100)
$$(x-25)^2 = -\frac{25}{4}(y-100)$$



Additional Resources

1. Go to http://www.kutasoftware.com/freeipc.html
2. Under **"Conic Sections"** find**:**
   - Parabolas
   - Circles
   - Ellipses
   - Hyperbolas

   You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets
3. For help please contact the ***Math Assistance Area*.**